Uniqueness for a cross-diffusion system resulting from seawater intrusion problems

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Introduction

- Modeling
- Global existence and uniqueness results in confined case

Well-posedness of the seawater intrusion problem

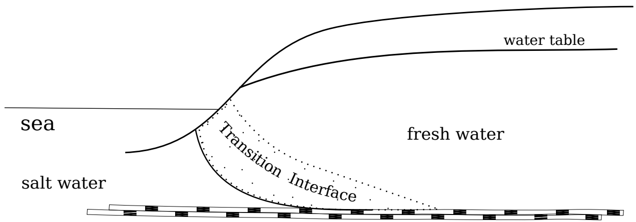
- Entropy method
- Global existence in the case of sharp-diffuse interface approach
- Regularity result and Uniqueness
- Boundedness of the solution

Applications

- P_1 Lagrange finite element method, Numerical simulations
- Hydraulic conductivity identification

Free and confined aquifer

- Confined aquifer : The lower and upper surfaces of the aquifer are impermeable.
- Free aquifer : The upper surface is constituted with a permeable layer. This aquifer are rechargeable in water with the raining falls but more sensitive to the pollution problem.



Fundamental laws

- The classical Darcy law for porous media gives

$$q = -K\,\nabla(\rho g H), \quad H = \frac{P}{\rho g} + z, \quad K = \frac{\kappa}{\mu},$$

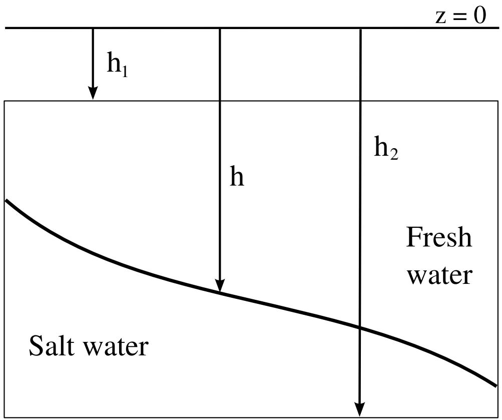
where q is the Darcy's flux, H the hydraulic head, K is the hydraulic conductivity and κ is the permeability tensor of the porous medium.

- The conservation of mass is given by the following equation :

$$\frac{\partial(\phi\rho)}{\partial t} + \nabla\cdot(\rho q) = \rho Q,$$

where ϕ is the porosity of the medium and Q denotes a generic source term (for production and replenishment).

Notations



$2D$ models for unconfined aquifer

- $2D$ Sharp interface model:

$$\phi\frac{\partial h}{\partial t} - \nabla\cdot(\alpha K(h-h_2)\nabla h) - \nabla\cdot(K(h-h_2)\nabla h_1) = Q_s,$$

$$\phi\frac{\partial h_1}{\partial t} + \nabla(K(h_2-h_1)\nabla h_1) + \nabla\cdot(\alpha K(h-h_2)\nabla h) = Q_f + Q_s.$$

- $2D$ Sharp-diffuse interface model:

$$\phi\frac{\partial h}{\partial t} - \nabla\cdot(\alpha K(h-h_2)\nabla h) - \nabla\cdot(\delta\nabla h) - \nabla\cdot(K(h-h_2)\nabla h_1) = Q_s,$$

$$\phi\frac{\partial h_1}{\partial t} + \nabla(K(h_2-h_1)\nabla h_1) - \nabla\cdot(\delta\nabla h_1) + \nabla\cdot(\alpha K(h-h_2)\nabla h) = Q_f + Q_s.$$

(C. Choquet, M. Dhiédhiou, C.R., SIAM J. Appl. Math. 2016.)

$2D$ models for confined aquifer

$$\phi\partial_t h - \nabla\cdot\Big(\alpha K\,(h_2 - h)\nabla h\Big) - \nabla\cdot(\delta\nabla h) + \nabla\cdot\Big(K\,(h_2 - h)\nabla f\Big) = -Q_s, \qquad (1)$$

$$-\nabla\cdot\Big(h_2 K\nabla f\Big) + \nabla\cdot\Big(K\,(h_2 - h)\nabla h\Big) = Q_f + Q_s. \qquad (2)$$

- We introduce function T_s defined by $T_s(h) = h_2 - h \quad \forall h \in (\delta_1, h_2)$ which is extended continuously and constantly outside (δ_1, h_2). The extension of T_s for $h \le \delta_1$ enables to ensure a thickness of freshwater zone always greater than δ_1 inside the aquifer.
- Assuming that there exist two positive real numbers, $0 < K^- \le K^+$, such that

$$0 < K^-|\xi|^2 \le K\xi\cdot\xi = \sum_{k,l=1}^{N} K_{kl}\xi_k\xi_l \le K^+|\xi|^2, \quad \forall\xi \in \mathbb{R}^N \setminus \{0\}. \qquad (3)$$

- Functions $(h_D, f_D) \in \big(L^2(0,T;H^1(\Omega)) \cap H^1(0,T;(H^1(\Omega))')\big) \times L^2(0,T;H^1(\Omega))$ while the function $h_0 \in H^1(\Omega)$. They satisfy conditions on the hierarchy of interfaces depth:

$$0 < \delta_1 \le h_D \le h_2 \text{ a.e. in } \Gamma\times(0,T), \quad 0 < \delta_1 \le h_0 \le h_2 \text{ a.e. in } \Omega.$$

Global existence in confined case

C. Choquet, J. Li, C.R., (EJDE, 2015)

Theorem:

Assume a low spatial heterogeneity for the hydraulic conductivity:

$$K_+ < \frac{h_2}{h_2 - \delta_1} \inf\Big(\sqrt{\frac{\delta K_-}{3\,h_2}}, K_-\Big).$$

Then for any $T > 0$, problem (1)-(2) admits a weak solution (h, f) satisfying

$$(h - h_D, f - f_D) \in W(0,T) \times L^2(0,T; H_0^1(\Omega)).$$

Furthermore the following maximum principle holds true:

$$0 < \delta_1 \leq h(t,x) \leq h_2 \quad \text{for a.e. } x \in \Omega \text{ and for any } t \in (0,T).$$

Global existence in confined case

Remark:

- We emphasize that the depth h is uniformly bounded as shown by the maximum principle. This result is specific to confined aquifers, it is no longer valid in the case of free aquifers for which we can face the situation where the aquifer overflows.
- The uniqueness result is a consequence of a $L^p(0,T;W^{1,p}(\Omega))$, $p>2$, regularity result proved for the solution of (1)-(2). This regularity is a generalization of the Meyers regularity results given in elliptic case and extended in parabolic case by Bensoussan et al..
  (C. Choquet, J. Li, C.R., (EJDE, 2017))

General Cross-Diffusion system

$$\partial_t u_i = \sum_{k=1}^{N} \frac{\partial}{\partial x_k}\left(\sum_{j=1}^{m} K_{i,j}^{k}(u) \frac{\partial u_j}{\partial x_k}\right) =: \nabla \cdot J_i, \quad \text{in } \Omega_T, \quad \text{for } i = 1, ..., m, \tag{4}$$

- Seawater intrusion problem (N=m=2)

$$K := K^1 = K^2 = \begin{pmatrix} \nu u_1 + \delta & \nu u_1 \\ \nu u_2 & u_2 + \delta \end{pmatrix}$$

J. Alkhayal, S. Issa, M. Jazar, R. Monneau (ESAIM Control Optim. Calc. Var., 2018)

Definition of the nonnegative entropy function Ψ:

$$\Psi(a) - \frac{1}{e} = \begin{cases} a\, ln(a) & \text{if } a > 0, \\ 0 & \text{if } a = 0, \\ +\infty & \text{if } a < 0. \end{cases}$$

Remark: $u_i \geq 0,\ i = 1, 2$ implies that the hierarchy between interfaces is preserved $h_1 \leq h \leq h_2$.

The boundedness by entropy method

N. Zamponi, A. Jüngel (since 2013), L. Desvillettes, T. Lepoutre, A. Moussa, A. Trescases (2015), Daus E., Milisić J.P. , N. Zamponi (2019, 2020)...

The main assumption is that system (4) has a formal gradient flow structure

$$\partial_t u - div\big(B\nabla Dh(u)\big) = f(u),$$

- $h : \mathcal{D} \to [0, \infty)$ is a function assumed to be convex (entropy density)
- B is a positive semi-definite matrix (depending on K and h), more precisely $B = K(u)(D^2h(u))^{-1} \in \mathbb{R}^{n\times n}$ and $D^2h(u) \in \mathbb{R}^{n\times n}$ is the Hessian of h.

Remark: For the seawater intrusion problem

- $h(u) = u_1(log u_1 - 1) + u_2(log u_2 - 1)$, $\mathcal{D} =]0, \infty)^2$, $Dh^{-1}(u) = (e^{u_1}, e^{u_2})$.
- The matrix $B(u)$ is positive semi-definite for all $u \in \mathcal{D}$

Shigesada-Kawasaki-Teramoto system for population dynamics

J. Kim (1984), H. Amann (1993), A. Yagi (1993), L. Chen, A. Jüngel (2004, 2016, 2017, 2018), M. Bendahmane, B. Perthame (2009), L. Desvillettes, T. Lepoutre (2014, 2015), M. Pierre, G. Rolland (2012); S. Kouachi, K.E. Yong, R.D. Parshadz (2014), D. Pham, R. Temam (2017, 2018)...

$$K := \begin{pmatrix} \alpha_{10} + 2\,\alpha_{11}\,u_1 \;+\; \alpha_{12}\,u_2 & \alpha_{12}\,u_1 \\ \alpha_{21}\,u_2 & \alpha_{20} + 2\,\alpha_{22}\,u_2 \;+\; \alpha_{21}\,u_1 \end{pmatrix}$$

The unknown u_i, for $i = 1, 2$, stands for the population density of the i-th species.

- **Uniqueness for SKT system in the diffusive case**: Hypothesis : $\alpha_{i0} \neq 0$, $i = 1, 2$
  D. Pham, R. Temam, Adv. Nonlinear Anal. (2017)

$$K(u)\xi.\xi \geq \alpha\left((u_1 + u_2)|\xi|^2 + |\xi|^2\right), \quad \text{with} \quad 0 < \alpha < min(\alpha_{ij}). \tag{5}$$

$$\partial_t u_i - \nabla \cdot \left(\delta_i \nabla u_i + u_i \sum_{j=1}^{2} K_{i,j} \nabla u_j \right) = Q_i(u) \quad \text{in } \Omega_T, \text{ for } i = 1, 2. \tag{6}$$

The tensors $K_{i,j}$ describe the permeability of the underground.
For any $1 \leq i, j \leq m$, there exist two positive real numbers, $0 < K_{i,j}^- \leq K_{i,j}^+$, such that

$$0 < K_{i,j}^- |\xi|^2 \leq K_{i,j} \xi \cdot \xi = \sum_{k,l=1}^{N} (K_{i,j})_{kl} \xi_k \xi_l \leq K_{i,j}^+ |\xi|^2, \quad \forall \xi \in \mathbb{R}^N \setminus \{0\}. \tag{7}$$

Remark :

- The SKT system reads (for $N = m = 2$)

$$\partial_t u_i - \nabla \cdot \left(\delta_i \nabla u_i + u_i (2\alpha_{ii} \nabla u_i + \alpha_{ij} \nabla u_j) + \alpha_{ij} u_j \nabla u_i \right) = Q_i(u), \; i = 1, 2, \; j \neq i.$$

1) The loss of a nonlinearity involving $u_j \nabla u_i$, $j \neq i$ implies that the form chosen for (6) does not satisfy the assumption (5).
2) The terms $u_j \nabla u_i \cdot \nabla u_i$ and $u_j \nabla u_j \cdot \nabla u_j$ may be used with the Cauchy-Schwarz and Young inequalities for containing the term $u_j \nabla u_j \cdot \nabla u_i$ when deriving *a priori* estimates for the SKT system and there is no hope to get similar results with System (6).

Global in time existence

C. Choquet, C. R., L. Rosier

$$\partial_t u_i - \nabla \cdot \left(\delta_i \nabla u_i + T_\ell(u_i) \sum_{j=1}^{2} K_{i,j} \nabla u_j \right) = Q_i(u) \quad \text{in } \Omega_T, \text{ for } i = 1, 2. \tag{8}$$

with $T_\ell(u) = u$ continuously and constantly extended outside the interval $[0, \ell]$.

Theorem 1 :

Assume that the tensor satisfy:

$$\frac{(K_{1,2}^+)^2}{K_{1,1}^-} < \frac{4\delta_2}{\ell}, \quad \frac{(K_{2,1}^+)^2}{K_{2,2}^-} < \frac{4\delta_1}{\ell}.$$

Pick $u_i^0 \in L^2(\Omega)$ with $0 \le u_i^0$ a.e. in Ω. Then for any $T > 0$, the problem (8) admits a weak solution $(u_i)_{i=1,2} \in W(0,T)^2$. Furthermore, the following maximum principle holds true:

$$0 \le u_i(t,x) \quad \text{for a.e. } x \in \Omega, \text{ for all } t \in (0,T) \text{ and for all } i = 1, 2.$$

Idea of the Proof :

Definition of the map $\mathcal{F} = (\mathcal{F}_1, \mathcal{F}_2)$

For the fixed point strategy, we define an application $\mathcal{F} : W(0,T)^2 \to W(0,T)^2$ by

$$\mathcal{F}(\bar{u}_1, \bar{u}_2) = \big(\mathcal{F}_1(\bar{u}_1, \bar{u}_2), \mathcal{F}_2(\bar{u}_1, \bar{u}_2)\big) = (u_1, u_2),$$

where (u_1, u_2) is the solution of the following initial boundary value problem

$$\begin{aligned}
\partial_t u_1 - \nabla \cdot \Big(\delta_1 \nabla u_1 + T_\ell(\bar{u}_1) K_{1,1} \nabla u_1 + T_\ell(\bar{u}_1) K_{1,2} \nabla \bar{u}_2\Big) &= Q_1(u),\\
\partial_t u_2 - \nabla \cdot \Big(\delta_2 \nabla u_2 + T_\ell(\bar{u}_2) K_{2,2} \nabla u_2 + T_\ell(\bar{u}_2) K_{2,1} \nabla \bar{u}_1\Big) &= Q_2(u),\\
(u_1, u_2) = (0,0) \quad \text{in } (0,T) \times \Gamma,&\\
(u_1(0,x), u_2(0,x)) = (u_1^0(x), u_2^0(x)) \quad \text{in } \Omega.&
\end{aligned}$$

Regularity result

We denote by

$$X_p = L^p(0, T; W_0^{1,p}(\Omega)) \text{ and } Y_p = L^p(0, T; W^{-1,p}(\Omega)),$$

The space Y_p is endowed with the norm $||f||_{Y_p} = \inf_{\mathrm{div}_x g = f} ||g||_{(L^p(\Omega_T))^N}$. Given $F \in Y_p$, there is a unique solution $u \in X_p$ of the following initial boundary value problem

$$\partial_t u - \Delta u = F \text{ in } \Omega_T,$$
$$u = 0 \text{ on } (0, T) \times \Gamma, \; u(0, x) = 0 \text{ in } \Omega.$$

We set $\Lambda^{-1} = \partial_t - \Delta$, so that $u = \Lambda(F)$. Let g be defined by

$$g(p) := ||\Lambda||_{\mathcal{L}(Y_p; X_p)}.$$

N. G. Meyers (1963), A. Bensoussan, J. L. Lions, G. Papanicolaou (1978).

Let $Au = -\sum_{i,j=1}^{N} \frac{\partial}{\partial x_i}\left(A_{ij}(t,x)\frac{\partial u}{\partial x_j}\right)$. We assume that that there exists $\gamma > 0$ s.t. $\sum_{i,j=1}^{N} A_{i,j}(t,x)\xi_i\xi_j \geq \gamma|\xi|^2, \quad \forall x \in \Omega \text{ and } \xi \in \mathbb{R}^N$. We set $\beta := \max_{1\leq i,j\leq n} ||A_{i,j}||_{L^\infty(\Omega_T)}$.

Lemma :

Let $f \in L^2(0,T,H^{-1}(\Omega))$, $u^0 \in H$, and $u \in L^2(0,T;H_0^1(\Omega))$ be the solution of

$$\begin{cases} \frac{\partial u}{\partial t} + Au = f \quad in \quad \Omega_T, \\ u(0) = u^0. \end{cases}$$

Then there exists $p > 2$, depending on γ, β and Ω, such that if $u^0 \in W_0^{1,p}(\Omega)$ and $f \in L^p(0,T;W^{-1,p}(\Omega))$, then $u \in L^p(0,T;W_0^{1,p}(\Omega))$. Furthermore, there exists a constant $C(\gamma,\beta,p) > 0$ such that

$$||u||_{L^p(0,T;W_0^{1,p}(\Omega))} \leq C(\gamma,\beta,p)(||f||_{L^p(0,T;W^{-1,p}(\Omega))} + ||u^0||_{W_0^{1,p}(\Omega)}). \tag{9}$$

Remark : $C(\gamma,\beta,p) \leq \frac{g(p)}{(1-k(p))\beta}, \quad k(p) = g(p)(1-\mu)$, with $\mu = \gamma/\beta$.

- We apply the previous Lemma to tensors $(\delta_i \mathcal{I}d + T_\ell(\bar{u}_i)K_{i,i})$, $i = 1, 2$, then $\gamma_i = \delta_i$, $\beta_i = \delta_i + \ell\, K^+_{i,i}$ for $i = 1, 2$.
- Let $p > 2$ such that

$$k_i(p) := g(p)\Big(\frac{\ell\, K^+_{i,i}}{\delta_i + \ell\, K^+_{i,i}}\Big) < 1, \;\; i = 1, 2, \tag{10}$$

Proposition

Let (u_1, u_2) be a solution of Problem (8) and let $p > 2$ such that (10) holds. Assume that $(\ell, \delta_1, \delta_2)$ and the tensors K satisfy

$$K^+_{i,j} < \frac{1}{g(p)\ell}\Big(1 - g(p)\frac{\ell\, K^+_{i,i}}{\delta_i + \ell\, K^+_{i,i}}\Big)\, \beta_i, \quad i = 1, 2, \, i \neq j, \tag{11}$$

and that $(u_1^0, u_2^0) \in (W^{1,p}(\Omega))^2$. Then ∇u_1 and ∇u_2 belong to $(L^p(\Omega_T))^N$.

Uniqueness

- The L^4-regularity of the gradient of the solution combined with Gagliardo-Nirenberg inequality for $p = 4$ allows to prove the uniqueness.

Theorem 2 :

We assume that the parameters $(\ell,\, \delta_1,\, \delta_2)$ and the tensor K satisfy

$$K^+_{i,i} < \frac{1}{g(4)-1}\,\frac{\delta_i}{\ell},\ i=1,2,\ \frac{(K^+_{1,2})^2}{K^-_{1,1}} < \frac{3\,\delta_2}{\ell} \quad \text{and} \quad \frac{(K^+_{2,1})^2}{K^-_{2,2}} < \frac{3\,\delta_1}{\ell}.$$

If $(u_{1,0}, u_{2,0}) \in W^{1,4}(\Omega)^2$, then the solution (u_1, u_2) is unique in $W(0,\, T)^2$.

Theorem 3 :

Assume the assumptions in Theorem 2 fulfilled. Assume $0 \leq u_i^0 \leq \ell$ a.e. in Ω and $0 \leq u_{i,D} \leq \ell$ a.e. in Ω_T. There exists source terms $Q_i \in L^2(0, T; (H^1(\Omega)'))$, $i = 1, 2$, such that the system (6) completed by the initial and boundary conditions admits a unique bounded weak global solution.

Proof : Step 1. Existence of a weak solution for a penalized problem

Let the function U_ℓ defined in $\mathbb{R}$ by $U_\ell(x) = \max(\ell, x)$. and let $\epsilon > 0$.

$$\partial_t u_i^\epsilon - \nabla \cdot \left(\delta_i \nabla u_i^\epsilon + T_\ell(u_i^\epsilon) \sum_{j=1}^{2} K_{i,j} \nabla u_j^\epsilon\right) + \frac{1}{\epsilon} \Delta U_\ell(u_i^\epsilon) = Q_i(u^\epsilon) \text{ in } \Omega_T, \tag{12}$$

$$u_i^\epsilon = u_{i,D}, \quad \text{in } (0,T) \times \Gamma, \qquad u_i^\epsilon(0,x) = u_i^0(x) \quad \text{in } \Omega. \tag{13}$$

Step 2. Uniform estimates of any solution of the penalized problem

Step 3. Letting the penalization blow up

P_1 Lagrange Finite element scheme.

If h_b^n et $h_{1,b}^n$ are in $(\mathcal{I}_b(h_D) + V_b^k) \times (\mathcal{I}_b(h_{1,D}) + V_b^k)$,

$$0 \leqslant h_{1,b}^n \leqslant h_b^n \leqslant h_2,$$

Semi-implicit in time scheme :
Find $(h_{1,b}^{n+1}, h_b^{n+1}) \in (\mathcal{I}_b(h_{1,D}) + V_b^k) \times (\mathcal{I}_b(h_D) + V_b^k),\ \forall w \in V_b^k.$

$$\phi \frac{h_{1,b}^{n+1} - h_{1,b}^n}{\delta t} - \nabla.(\delta \nabla h_{1,b}^{n+1}) - \nabla.(T_f(h_b^n - h_{1,b}^n)\nabla h_{1,b}^{n+1})$$

$$-\nabla.(T_s(h_b^n))\nabla(h_{1,b}^{n+1} + h_b^n) = {Q_s}^{n+1} + {Q_f}^{n+1}$$

$$\phi \frac{h_b^{n+1} - h_b^n}{\delta t} - \nabla.(\delta \nabla h_b^{n+1}) - \nabla.(T_s(h_b^n)\nabla(h_b^{n+1} + h_{1,b}^{n+1})) = {Q_s}^{n+1}$$

Error estimates for FEM .

Theorem 4:

If $\left(\phi - \frac{2\,h_2^2 K_+}{\delta}(2\,K_+ + K_-)C(b)^2\delta t\right) > 0$, there exists a constant $C > 0$, s.t. for any solution (h, h_1) in $Y(\Omega_T) = C^2([0,T], L^2(\Omega)) \cap C^1([0,T], H^1(\Omega))$. Moreover, we have

$$\max_{0\le n\le N} \|h(t^n) - h_b^n\|_{L^2} \le C\,(b + \delta t) max(\|h\|_{Y(\Omega_T)}, \|h_1\|_{Y(\Omega_T)}),$$

$$\max_{0\le n\le N} \|h_1(t^n) - h_{1,b}^n\|_{L^2} \le C\,(b + \delta t) max(\|h\|_{Y(\Omega_T)}, \|h_1\|_{Y(\Omega_T)}),$$

$$[\frac{1}{\delta t}\sum_{n=1}^{N} \|h(t^n) - h_b^n\|_{H^1}^2]^{\frac{1}{2}} \le C\,(b + \delta t) max(\|h\|_{Y(\Omega_T)}, \|h_1\|_{Y(\Omega_T)}),$$

$$[\frac{1}{\delta t}\sum_{n=1}^{N} \|h_1(t^n) - h_{1,b}^n\|_{H^1}^2]^{\frac{1}{2}} \le C\,(b + \delta t) max(\|h\|_{Y(\Omega_T)}, \|h_1\|_{Y(\Omega_T)}).$$

Non confined case, with Dirichlet boundary conditions

We couple the problem with the tides effects. For this simulation we use the parameters in Cooper'64 after a rescaling to our small aquifer.

- H.H. Cooper, *A hypothesis concerning the dynamic balance of fresh water and salt water in a coastal aquifer*, U.S. Geological Survey Water-Supply Paper 1613-C, 1–12, 1964.

We impose a Dirichlet boundary condition on the left boundary $\{x = 0\}$ for the saltwater elevation h. Its value is computed with the classical tide-produced change model for the artesian head of Ferris'51. We compare the interface h obtained with FE method with a reference solution, here derived from the analytic formula of Ferris'51.

- J. G. Ferris, *Cyclic fluctuations of water level as a basis for determining aquifer transmissibility*, Int. Assoc. Sci. Hydrology Publ., Vol. 1, 97–101, 1951.

Non confined case

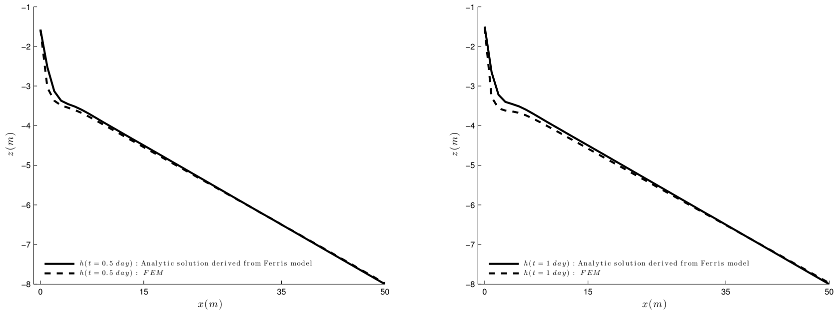


Figure : Comparison between FE solution and analytic solution derived from Ferris model. Times t=0,5 day (on the left) and t=1 day (on the right), N=200.

Existence and characterization of an optimal control

The inverse problem is formulated by an optimization problem whose cost function measures the squared difference between experimental interfaces depths and those given by the model. We introduce the following control problem:

$$(\mathcal{P}(\mathcal{K}))\begin{cases} \text{Find } K^* \in U_{adm} \text{ such that} \\ \mathcal{J}(K^*) = \inf_{K\in U_{adm}} \mathcal{J}(K), \end{cases}$$

with $\mathcal{J}(K) = \frac{1}{2}||h_1(K) - h_{1,obs}||^2_{L^2(\Omega_T)} + \frac{1}{2}||h(K) - h_{obs}||^2_{L^2(\Omega_T)}$, where $(h_1(K), h(K))$ is the weak solution of system $P(K)$ and $(h_{1,obs}, h_{obs})$ are the observed depths.

Theorem

There exists at least one optimal control for the problem $(\mathcal{O})$.

M. H. Tber, M. E. Talibi, D. Ouazar (2007, 2008), Aya Mourad, C. R., JOTA, 2019.

Description of Numerical simulations

- The aquifer is figured by a parallelepiped $(x, y) \in [0, 100] \times [0, 20], z \in [-20, 0]$.
- In first step, we take K_{exact} for the exact value of the hydraulic conductivity, then the saltwater/freshwater interface depth h and the depth of the interface between dry zone and saturated zone h_1 are computed by solving the exact problem associated with this value of K_{exact}; these numerical values of h and h_1 have been considered as observed data.
- Then starting from an arbitrary initial estimate of this parameter, we compute the optimal solution by the parameters identification procedure.

Experiment

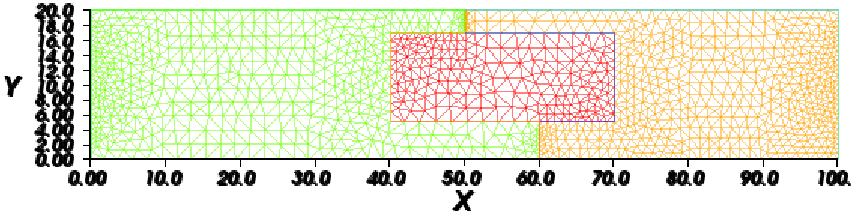


Figure : Schematization of the aquifer in experiment.

Experiment

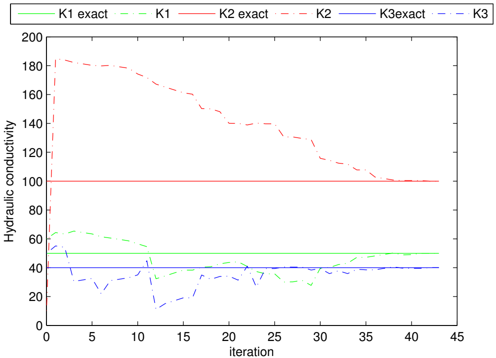
We choose a very wrong initial value of the second hydraulic conductivity K_2. The choice of K_2 (instead of the others) will have the greatest impact on the global identification procedure.

Table : Hydraulic condutivities values in experiment.

case	Number of wells	exact values	initial values	identified values
i	6	K_1= 50 m/d K_2= 100 m/d K_3= 40 m/d	K_1= 60 m/d K_2= 10 m/d K_3= 50 m/d	K_1= 50.008 m/d K_2= 99.91 m/d K_3= 40.05 m/d
ii	6	K_1= 50 m/d K_2= 5 m/d K_3= 40 m/d	K_1= 60 m/d K_2= 50 m/d K_3= 50 m/d	K_1= 50.07 m/d K_2= 5.0003 m/d K_3= 39.82 m/d

Experiment 3

The number of iterations needed to reach convergence is of course higher than in the previous experiment: 20 iterations for experiment 2 versus 40 for experiment 3.



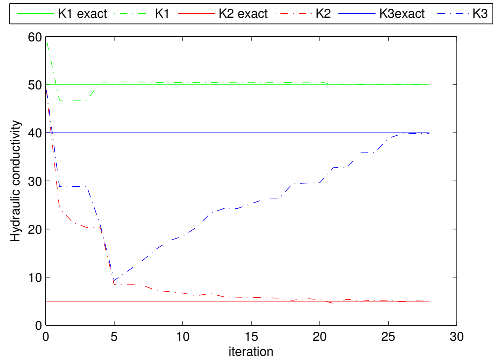


Figure : Graph representing the convergence of hydraulic conductivity in case i (on the left) case ii (on the right)

Thank you for your attention !