University of *Ljubljana*
Faculty *of Education*

# The role of Fermi problems in the concept of developing mathematical literacy among students

VIDA MANFREDA KOLAR, TATJANA HODNIK

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# Mathematical literacy

In order to respond to the rapidly changing world as successfully as possible, questions are repeatedly raised about the key competencies that a student should acquire during schooling.

The competencies that are aiming to reflect the changing world and are considered as students' abilities to act mathematically in this world are often reffered to as mathematical literacy.

Mathematical literacy is the activity of an individual capable of formulating, employing and interpreting mathematics in various contexts (OECD, 2019)

NA-MA POTI project (since 2018, the systematic development of mathematical literacy on the national level). In the project we identified two cornerstones of mathematical literacy:

- *mathematical thinking:* understanding and using mathematical concepts, procedures, strategies and communication as a basis for mathematical literacy

- *problem solving* in different contexts (personal, social, professional, scientific) that enable a mathematical approach.

The outcomes of *problem solving* are:

      - solving simple everyday problems (problems that do not demand mathematical modelling),

      - handling situations with <span style="color:red">mathematical modelling</span>,

      - and understanding mathematical praxes in different contexts.
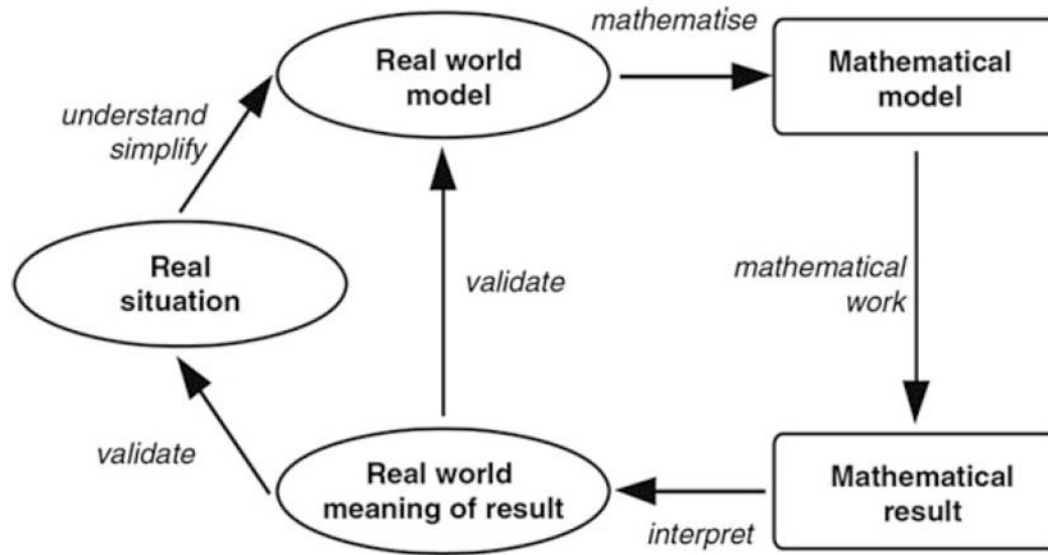
# Mathematical modelling

Mathematical modeling takes children beyond basic problem solving (an irrelevant context), to *authentic situations that need to be interpreted and described in mathematical ways (a meaningful context)* (Lesh, 2001).

Mathematical modelling always originates from a real-life problem, which is then described by a mathematical model and solved using this model. The entire process is then called modelling" (Greefrath & Vorhölter, 2016, p. 8).

# Traditional word problems vs. mathematical modelling

| | **Traditional word problems** | **Mathematical modelling** |
|---|---|---|
| Context | symbolically described situation. | meaningful situations |
| Initial data | Clearly defined | the initial situation is generally undefined, doubt about the choice of input data |
| Purpose | mathematical purpose– the aim is to solve a problem | purpose is not merely mathematical, but broader (confronting the unknown, anticipating, searching for data, testing different ideas) |
| Problem solving path | clearly defined: the correct choice of the procedure leads to the correct solution. | open: there is no single mapping between the choice of path and the correctness of the solution to the problem. The solutions are more or less acceptable. |

# Modelling process



Modelling cycle of Kaiser and Stender (2013, p. 279)

# Modelling process
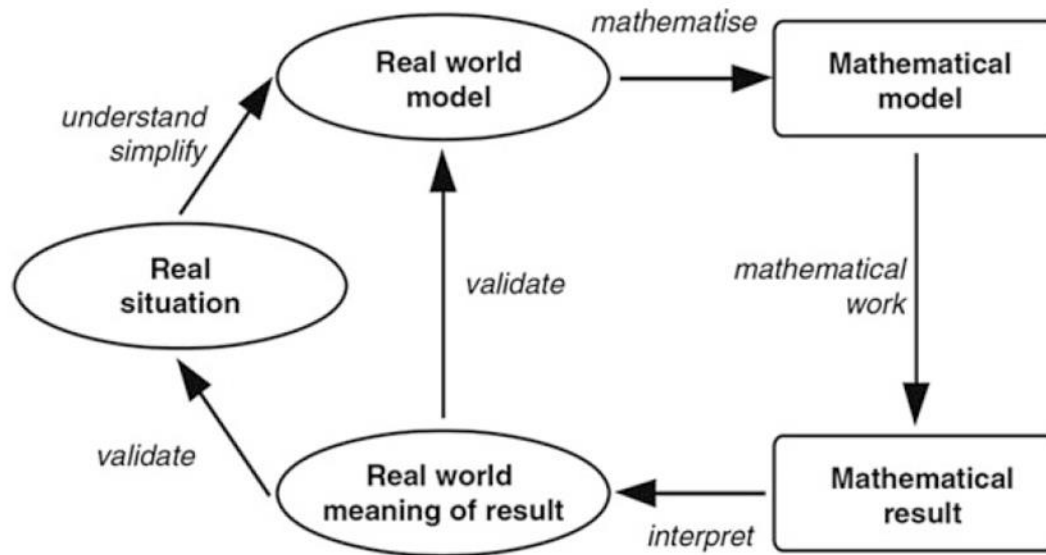
A representation of the real world, which—although simplified—matches the original and allows the application of mathematics.



Modelling cycle of Kaiser and Stender (2013, p. 279)

# Fermi problems

- A multi-step problem that can be solved in different ways.

- Initial problem is taken from real life (unlike other problems), it does not provide any initial data that the solver would have at his disposal. The solver must ask himself questions about what data makes sense to determine and how he will get it.

- Solution path: based on the use of estimating and rounding data that are commonly known.

# Solving of Fermi problems – a modelling proces?

| | |
|---|---|
| **Context** | realistic |
| **Initial data** | undefined |
| **Problem solving path** | undefined |
| **Solution** | more or less acceptable |
| **Cycle** | one or more cycles |
| **Process** | structuring, simplifying, mathematizing, interpreting, validating |

# Research

- Faculty of Education

- Primary education students: 70

- In the course of Didactic of mathematics, students were introduced to Fermi's known problems, solved them in a group within the course and justified them from the point of view of developing mathematical literacy.

- Creating own examples of Fermi problems for a fifth grader + solution

The aim:

- to design a model for evaluating /identifying a good Fermi problem for a fifh-grader.

- to determine the set of quality Fermi problems for fifth-graders on the basis of the developed model.

# Presentation of model criteria

| Criteria | The value of the variable | |
|---|---|---|
| **complexity of mathematization** | High (H) | Low (L) |
| **complexity of determining data** | High (H) | Low (L) |
| **linguistic aspect** | Appropriate (1) | Inappropriate (0) |
| **number of modelling cycles** | Unicyclic (1) | Multicyclic (> 1) |

# Complexity of mathematization: the path of solving the problem – transparent/non transparent?

**High**

How much time does a 5th grader spend on a mobile phone in one year?

How many popcorn is in a cinema that is completely sold out?

**Low**

What is the area of all sockets in one house?

# Complexity of data determination: initial stage - complexity of the data estimating process (data variability)

## High

In what time would you list all the numbers from 1 to 1 000 000?

## Low

How many times in one hour does a class with thirty students blink?

Linguistic aspect: use of terminology, clarity of the text, questions

**Inappropriate**

- How many rolls of toilet paper do we need to cover the entire surface of the Earth with it?

# Number of cycles: number of steps in the modeling process

## Unicyclic

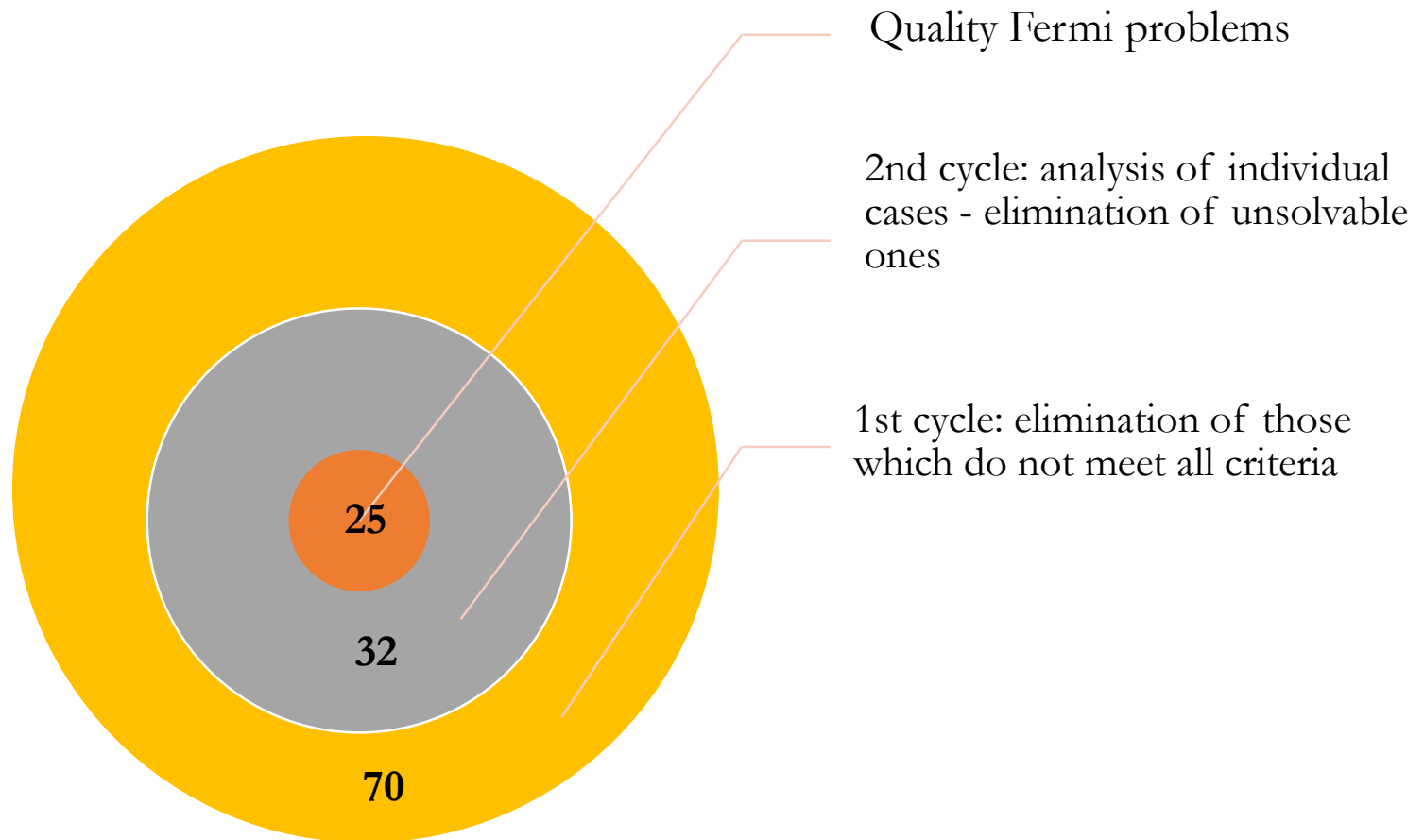- How many coloured pencils on average do the students of one class of the primary school have in total?

## Multicyclic

- How long would the row be if all the coloured pencils of the students of one class of the primary school were placed one after the other?

# Model for defining a quality Fermi problem

| | |
|---|---|
| **Complexity of mathematization** | high |
| **Complexity of data determination** | high |
| **language** | appropriate |
| **Number of cycles** | more than one |

# Evaluation of the model

Quality Fermi problems

2nd cycle: analysis of individual cases - elimination of unsolvable ones

1st cycle: elimination of those which do not meet all criteria

**25**

**32**
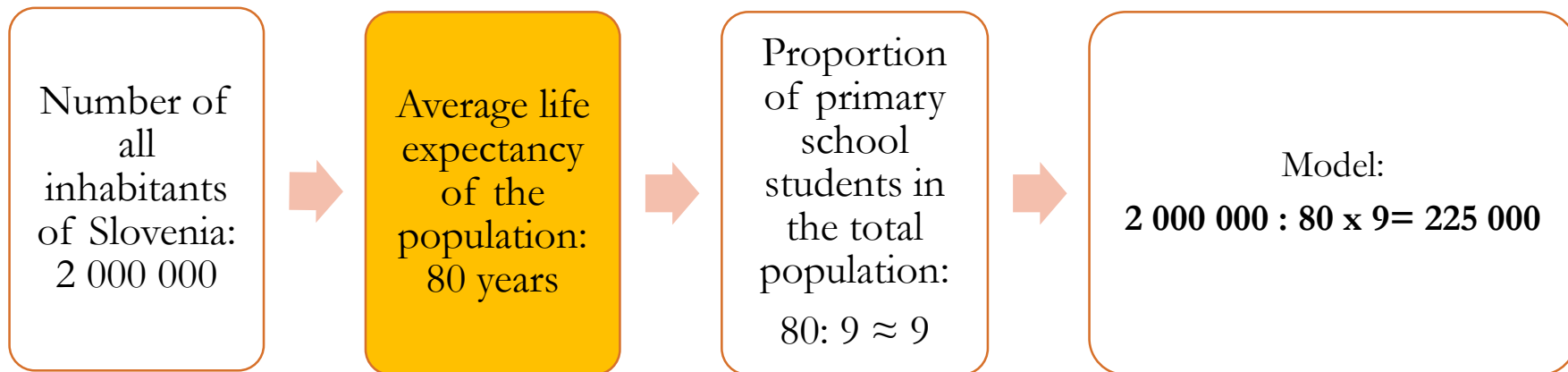
**70**

# Problems dropped after the 2nd cycle

- Unable to determine data - we do not have a reference point (number of hairs on the head)

- Impossible to determine the data: the fifth-grader does not have enough knowledge (Lake Bled area, length of the equator)

<span style="color:red">3rd cycle: Consideration of the strength of the formulated model criteria - one criterion is not met</span>
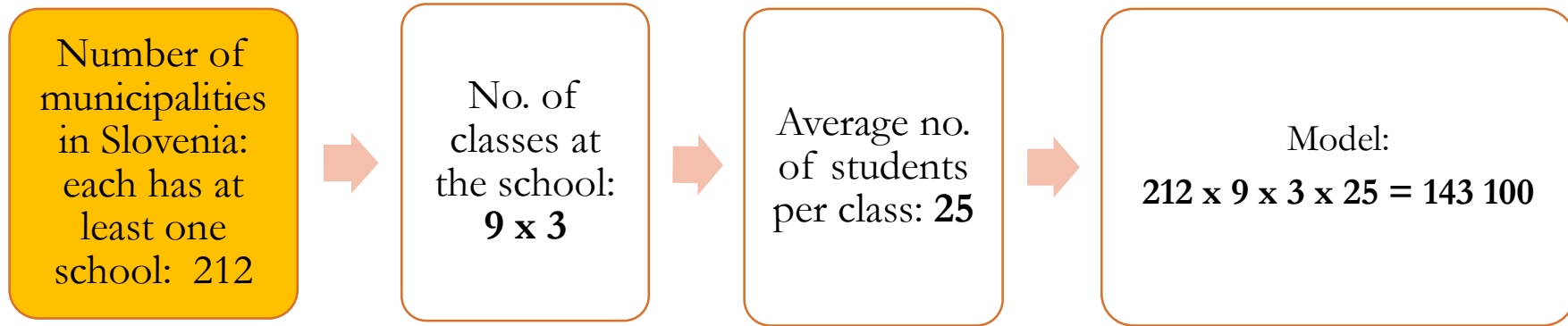
High complexity of mathematization + **low complexity of data determination**

> How many primary school students are there in
>
> Slovenia?

# Model 1

| Number of all inhabitants of Slovenia: 2 000 000 | → | Average life expectancy of the population: 80 years | → | Proportion of primary school students in the total population: 80: 9 ≈ 9 | → | Model: **2 000 000 : 80 x 9= 225 000** |

# Model 2

| Number of municipalities in Slovenia: each has at least one school: 212 | → | No. of classes at the school: **9 x 3** | → | Average no. of students per class: **25** | → | Model: **212 x 9 x 3 x 25 = 143 100** |

In school year 2019/20: 190 156 pupils in primary school

## 4th cycle: Consideration of the strength of the formulated modal criteria - one criterion is not met

**Low complexity of mathematization** + high complexity of data determination

> How many meters of coloured pencils are in the students' pencilcases in the elementary school of Hotedrščica?

# Final evaluation of the model



after 2nd cycle

after 3rd cycle

after 4th cycle

25

28

30
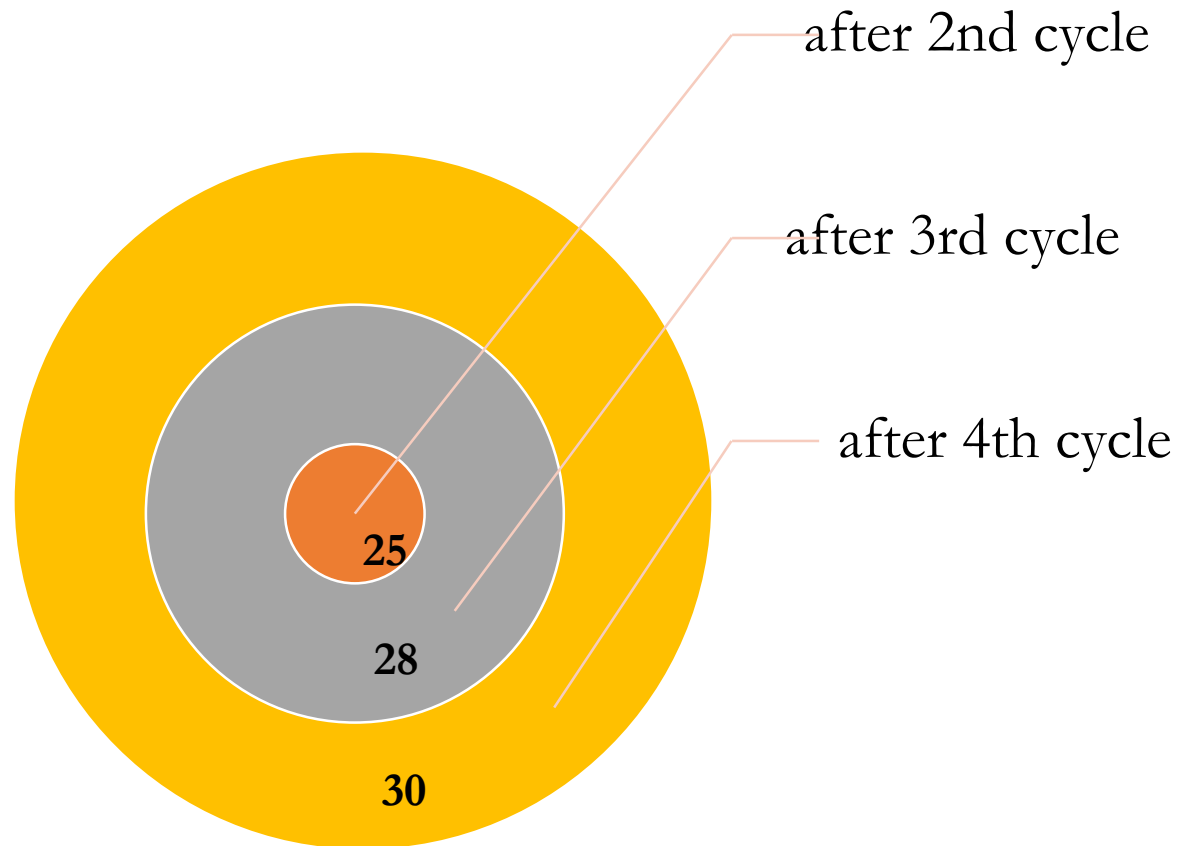
# Examples of solutions for quality Fermi problems

## How many popcorn is in a cinema that is completely sold out (we have in mind a medium-sized cinema)?

1. **How many people are in cinema** that is completely occupied?
   The cinema has 10 rows. Each row has 15 seats: 150 seats, or. 150 people.

2. At the cinema, we can buy popcorn bags in three different sizes: S, M, L. We need to find out **how many people will buy popcorn and what size bag they will choose**.
   Let's assume that only 100 of all people will buy popcorn. Of these, 30 will buy S size, 50 will buy M size, and 20 will buy L size.

3. **How many popcorns are in each bag?** (we help ourselves with glasses):
   Guess:
   How many popcorns can fit in a plastic jar (60 popcorns).
   How many jars can be packed into each bag (S: 6, M: 10, L: 14).
   Calculate:
   S: 60 x 6 = 360 popcorn.
   M: 60 x 10 = 600 popcorn.
   L: 60 x 14 = 840 popcorn.

4. **How much popcorn is in the cinema** under the above assumptions:.
   30 x 360 = 10 800 (S); 50 x 600 = 30 000 (M); 20 x 180 = 16 800 (L):
   Total: 57 600 popcorn.

# How long would it take a fifth grader to list all the multiples of the number 4 from 0 to 100,000 in order?

1. Students first **measure** how much time it takes them to say multiples of the number 4 from 0 to 40 (rounding time to tens).

2. They **calculate** how many times 40 goes into 100 000: 2500.

3. They **convert** time in seconds to minutes: divide by 60.

Student's description: „I myself spent 30 seconds listing multiples of the number 4 from 0 to 80, which I then multiplied by 1250 and got 37 500 seconds, which is 625 minutes or 10.4 hours."

# Discussion

1. Scientific contribution: a model for evaluating good Fermi problems – our set of criteria covers two perspectives:
   - content (input data, complexitxy of a question)
   - process (modelling cycle)

   Additional criteria (narrowing the set of problems):
   - the final result has a useful value in real life
   - motivational problem from the perspective of a student

   | How many primary school students are there in Slovenia? | How many popcorn is in a cinema that is completely sold out? |
   |---|---|

# Discussion

2.  Model transferability: the applicability of a model in evaluating the quality of other types of mathematical tasks.

3.  Multifaceted role of modeling in mathematics:
    - Content aspect (developing mathematical literacy)
    - Organizational aspect (evaluation aid for teachers)

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