Optimal Drawdowns in Insurance

joint work with Leonie Brinker

Hanspeter Schmidli

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8th ECM, Portorož June 22nd, 2021



- Introduction
- The Classical Risk Model
- The Diffusion Approximation



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Definition of Drawdown

For a surplus process X_t denote by

$$ar{X}_t = \max\{ar{x}, \sup_{s \leq t} X_s\}$$

the running maximum. The drawdown

 $D_t = \bar{X}_t - X_t$

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is the deviation from the running maximum. We allow a past maximum \bar{x} .

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• Large drawdowns are a reputational risk

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- Large drawdowns are a reputational risk
- Investors compare with current maximum (risk for the manager)

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- Large drawdowns are a reputational risk
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- Goal is to keep surplus near maximum (stabilsation) which simplifies planning future strategies

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- Large drawdowns are a reputational risk
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- Goal is to keep surplus near maximum (stabilsation) which simplifies planning future strategies
- We try to keep the drawdown below some level d
- Drawdown below the critical level only for a short time

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Reinsurance

The insurer buys proportional reinsurance with retention level $b_t \in [0, 1]$ at time t. That is, the insurer pays $b_t Y$, the reinsurer $(1 - b_t)Y$ of a claim of size Y. The reinsurer uses an expected value principle with safty loading θ . We assume that reinsurance is more expensive than first insurance in order that the problem below is not trivial. The insurer choses continuously a reinsurance strategy $\{b_t\}$.

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The Optimisation Problem

The value of a reinsurance strategy b is

$$V^b(x) = \operatorname{I\!E} \left[\int_0^\infty \mathrm{e}^{-\delta t} \mathrm{I\!I}_{D^b_t > d} \, \mathrm{d}t
ight] \, .$$

We are interested in the optimal value

 $V(x) = \inf_{b} V^{b}(x)$

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and, if it exist, the optimal strategy b^* .

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The Classical Risk Model



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The Cramér–Lundberg Model

Let

$$X_t = \bar{x} - x + ct - \sum_{k=1}^{N_t} Y_k ,$$

where N is a Poisson process with rate λ and iid claim $\{Y_k\}$ with expected value μ . We write $c = (1 + \eta)\lambda\mu$ for some $\eta > 0$.

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where N is a Poisson process with rate λ and iid claim $\{Y_k\}$ with expected value μ . We write $c = (1 + \eta)\lambda\mu$ for some $\eta > 0$. After reinsurance,

$$X_t^b = \bar{x} - x + \int_0^t c(b_s) \, \mathrm{d}s - \sum_{k=1}^{N_t} b_{T_k} - Y_k \; ,$$

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where $c(b) = c - (1 - b)(1 + \theta)\lambda\mu = (b\theta - (\theta - \eta))\lambda\mu$.

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The Classical Risk Model		

We get the drawdown process

$$D_t^b = x + \sum_{k=1}^{N_t} b_{T_k-} Y_k - \int_0^t c(b_s) \, \mathrm{d}s + (\bar{X}_t^b - \bar{x}) \; .$$

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$$D_t^b = x + \sum_{k=1}^{N_t} b_{T_k-} Y_k - \int_0^t c(b_s) \, \mathrm{d}s + (\bar{X}_t^b - \bar{x}) \; .$$

That is

• jumps upwards, (downwards) deterministic paths

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The Classical Risk Model		

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That is

- jumps upwards, (downwards) deterministic paths
- reflection in zero

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The Classical Risk Model		

We get the drawdown process

$$D_t^b = x + \sum_{k=1}^{N_t} b_{T_k-} Y_k - \int_0^t c(b_s) \, \mathrm{d}s + (\bar{X}_t^b - \bar{x}) \; .$$

That is

- jumps upwards, (downwards) deterministic paths
- reflection in zero
- we now restrict to $b_t \in [b^0, 1]$ with $b^0 = (1 \eta/\theta)$, such that $c(b_t) \ge 0$.

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The Diffusion Approximation

With simplified notation, the diffusion approximation to the classical model is $X_t = \bar{x} - x + \eta t + \sigma W_t$ for some Brownian motion W. After reinsurance

$$X_t^b = \bar{x} - x + \int_0^t \{b_s \theta - (\theta - \eta)\} \, \mathrm{d}s + \sigma \int_0^t b_s \, \mathrm{d}W_s \, .$$

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The Diffusion Approximation

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$$X_t^b = \bar{x} - x + \int_0^t \{b_s \theta - (\theta - \eta)\} \, \mathrm{d}s + \sigma \int_0^t b_s \, \mathrm{d}W_s \, .$$

The drawdown process becomes

$$D_t^b = x - \int_0^t \{b_s \theta - (\theta - \eta)\} \, \mathrm{d}s - \sigma \int_0^t b_s \, \mathrm{d}W_s + (\bar{X}_t^b - \bar{x}) \; .$$

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General Results

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Lipschitz Continuity

Lemma

The function V is increasing with $0 \le V(x) \le \delta^{-1}$ for all $x \in [0, \infty)$, fulfils $\lim_{x\to\infty} V(x) = \delta^{-1}$ and is Lipschitz continuous with

$$|V(x) - V(y)| \leq rac{\lambda + \delta}{\delta c(1)} |x - y| \; .$$

In particular, V is absolutely continuous and differentiable almost everywhere.

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General Results

Proof.

For $0 \le y < x$ choose a strategy \tilde{b} with $V^{\tilde{b}}(y) < V(y) + \varepsilon$. For initial capital x define h = (x - y)/c(1), $b_t = \tilde{b}_{t-h}$ if $T_1 \land t \ge h$ and $b_t = 1$, otherwise. Then

$$egin{aligned} &V(x)-V(y)-arepsilon\leq V^b(x)-V^{ ilde{b}}(y)\ &\leq &\int_0^h \mathrm{e}^{-\delta t}\,\mathrm{d}t-(1-\mathrm{e}^{-(\lambda+\delta)h})V^{ ilde{b}}(y)+(1-\mathrm{e}^{-\lambda h})\delta^{-1}\ &\leq &(\lambda+\delta)h/\delta=rac{\lambda+\delta}{\delta c(1)}(x-y)\;, \end{aligned}$$

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General Results

Proof.

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The other statements are clear.

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General Results

Splitting of the Problem

Let

 $\vartheta_d = \inf\{t \ge 0 : D_t \le d\}, \qquad \vartheta^d = \inf\{t \ge 0 : D_t > d\}$

be the first entrance times. Then by considering the process until the stopping time

$$egin{array}{rcl} V(x) &=& \operatorname{I\!E}[\mathrm{e}^{-\deltaartheta^d}\,V(D_{artheta^d})]\;, & x\leq d\;, \ V(x) &=& \operatorname{I\!E}[\delta^{-1}(1-\mathrm{e}^{-\deltaartheta_d})+\mathrm{e}^{-\deltaartheta_d}\,V(d)]\;, & x>d\;. \end{array}$$

We can solve the two problems separately.

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The Solution

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Starting in the Critical Area

Problem: Maximise $\mathbb{E}^{x}[e^{-\delta \vartheta_{d}}]$.

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The Solution

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Starting in the Critical Area

Problem: Maximise $\mathbb{E}^{x}[e^{-\delta \vartheta_{d}}]$.

For x > d, reaching d one has to pass $y \in (d, x)$. Conclusion: $\mathbb{E}^{x}[e^{-\delta \vartheta_{d}}]$ is an exponential function.

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For any subinterval of a fixed length, the same quantity has to be maximised. Conclusion: the optimal strategy is constant.

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For any subinterval of a fixed length, the same quantity has to be maximised. Conclusion: the optimal strategy is constant.

 $V(x) = \delta^{-1} - (\delta^{-1} - V(d))e^{-\gamma(x-d)} \text{ where } \gamma \text{ is the positive solution to } c(1)\gamma - \lambda \mathbb{E}[1 - e^{-\gamma Y}] = \delta.$

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The Solution

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Starting in the Non-Critical Area

Problem: Minimise $\operatorname{I\!E}[\mathrm{e}^{-\delta\vartheta^d}V(D_{\vartheta^d})]$ with V(d) unknown.

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Starting in the Non-Critical Area

Problem: Minimise $\operatorname{I\!E}[\mathrm{e}^{-\delta \vartheta^d} V(D_{\vartheta^d})]$ with V(d) unknown.

Replace V(d) by $C \in (0, \delta^{-1})$, $V_C(x) = \inf_b \mathbb{E}[e^{-\delta \vartheta^d} V_C(D_{\vartheta^d})]$.

Lemma

There exists
$$C_0 \in (0, \delta^{-1})$$
 such that $V_C(d) \stackrel{\geq}{=} C$ iff $C \stackrel{\leq}{=} C_0$.

It turns out that $C_0 = V(d)$.

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The HJB Equation

Theorem

 $V_C(x)$ solves for $x \leq d$ the HJB equation

 $\inf_{b\in [b^0,1]}\lambda\int_0^\infty V_C(x+by)\,\mathrm{d}G(y)-c(b)V_C'(x)-(\lambda+\delta)V_C(x)=0\;.$

Let $b_C(x)$ be a measurable version of the maximiser. Then the strategy $b_C(D_t^C)$ is optimal.

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The HJB Equation II

Theorem

V(x) is the unique bounded continuous solution to the HJB equation

 $\inf_{b\in [b^0,1]}\lambda\int_0^\infty V(x+by)\,\mathrm{d}G(y)-c(b)V'(x)-(\lambda+\delta)V(x)=-1\!\!\mathrm{I}_{x>d}\;.$

Let b(x) be a measurable version of the maximiser. Then the strategy $b(D_t^*)$ is optimal.

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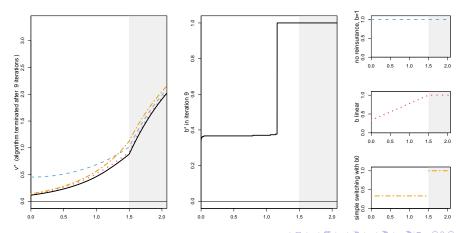
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Exponentially Distributed Claims



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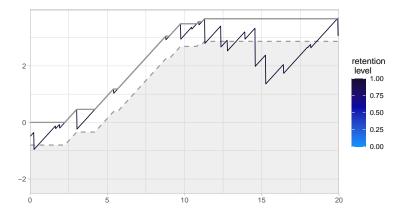
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Exponentially Distributed Claims: No Reinsurance



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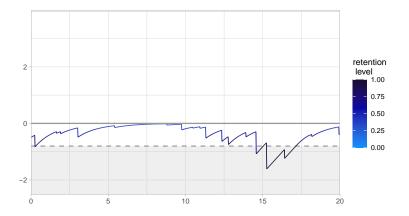
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Exponentially Distributed Claims: Linear Reinsurance



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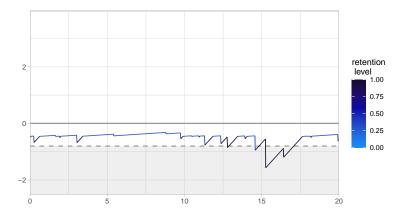
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Exponentially Distributed Claims: Optimal Reinsurance



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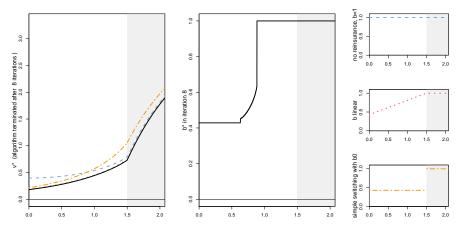
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Pareto Distributed Claims



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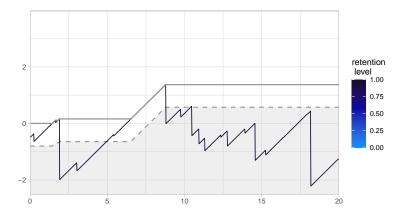
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Pareto Distributed Claims: No Reinsurance



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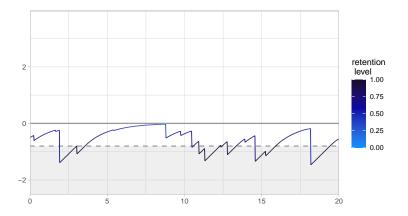
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Pareto Distributed Claims: Linear Reinsurance



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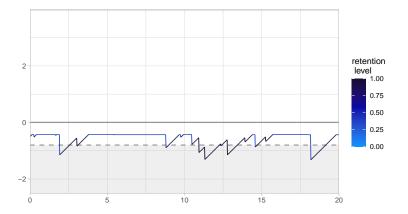
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Pareto Distributed Claims: Optimal Reinsurance



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Splitting of the Problem

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Splitting of the Problem

Splitting of the Problem

As for the classical model

$$\begin{split} V(x) &= & \operatorname{I\!E}[\operatorname{e}^{-\delta\vartheta^d}V(d)] \;, \qquad \qquad x \leq d \;, \\ V(x) &= & \operatorname{I\!E}[\delta^{-1}(1-\operatorname{e}^{-\delta\vartheta_d})+\operatorname{e}^{-\delta\vartheta_d}V(d)] \;, \quad x > d \;. \end{split}$$

In the critical area $x > d \ b = 1$ and thus

$$V(x) = \delta^{-1} \{1 - (1 - \delta V(d)) e^{-\kappa(x-d)}\}$$

for
$$\kappa > 0$$
 solving $\frac{1}{2}\sigma^2\kappa^2 + \eta\kappa = \delta$.

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The Diffusion Approximation

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The HJB Equation

Theorem

V(x) is the unique bounded continuously differentiable solution to

$$(\theta - \eta)V'(x) - \delta V(x) + \inf_{b \in [0,1]} \left\{ \frac{1}{2} b^2 \sigma^2 V''(x) - \theta b V'(x) \right\} = -\mathbb{I}_{x > d} .$$

Proof.

Explicit solution to the HJB and verification theorem.

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Construction of the Solution

A non-trivial solution must be strictly convex. If $b \neq 1$,

$$\frac{\theta^2 V'(x)^2}{2\sigma^2 V''(x)} + \delta V(x) = (\theta - \eta) V'(x) .$$

The function $x \mapsto -\ln V'(x)$ is strictly decreasing with inverse function Y. Thus $V'(Y(z)) = e^{-z}$. Plugging this into the equation and differentiation leads to differential equation and an explicit solution. There is $x_0 \in (0, \infty]$ such that

$$b(x) = rac{ heta V'(x)}{\sigma^2 V''(x)} \leq 1 \;, \quad x \in [0, x_0] \;.$$

Compound V(x) on $[0, x_0 \land d]$ with the solution with b(x) = 1 to a smooth solution.

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The Behaviour at Zero

Theorem

The strategy $b(D_t^*)$ is optimal. Under the optimal strategy \bar{X}_t^* is constant.

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The Diffusion Approximation

Value Function and Optimal Strategy

Drawdowns

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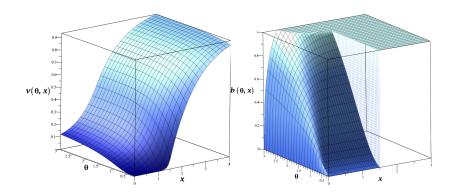
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Drawdowns

The Diffusion Approximation

Value Function and Optimal Strategy

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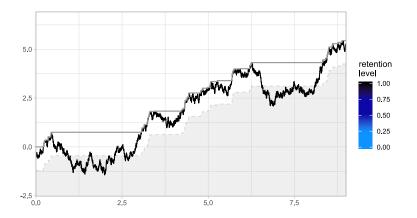
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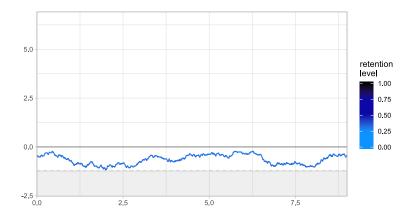
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Thank you for your attention

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