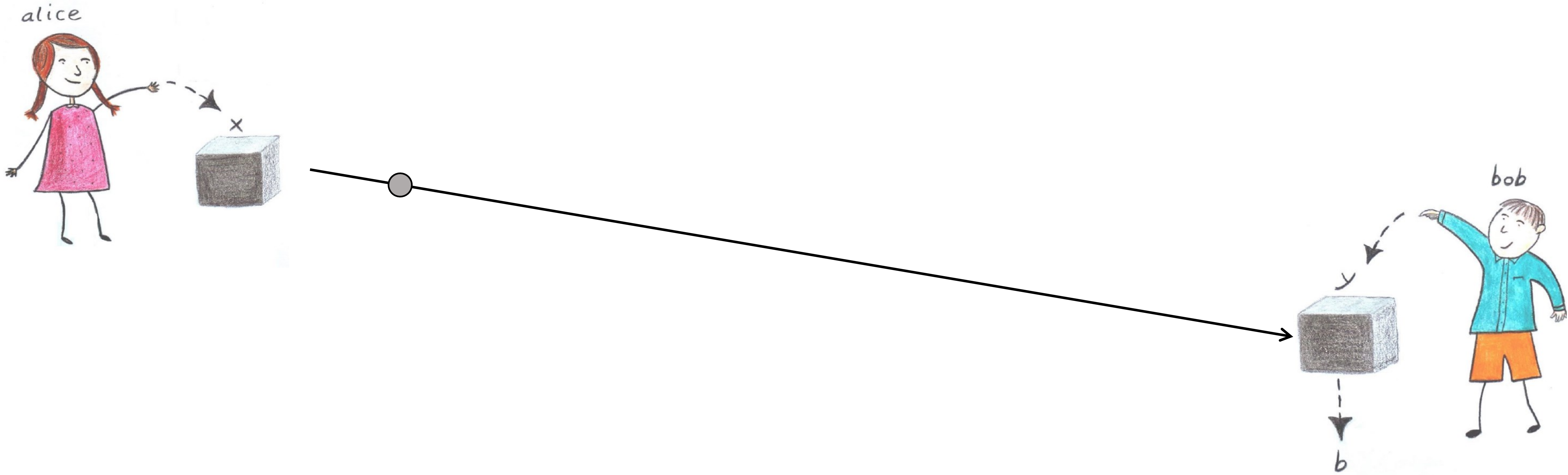
Quantum polynomial optimisation problems for dimension d variables, with symmetries



A. Tavakoli, D. Rosset, M-O. Renou
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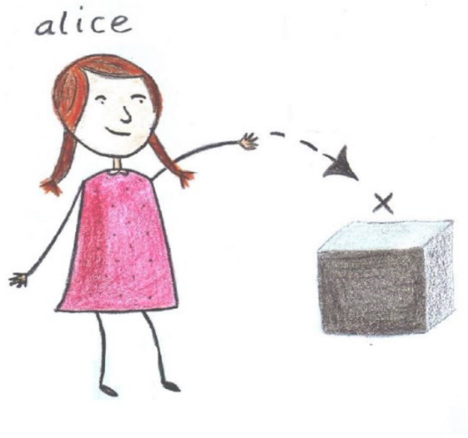
8ECM 2021
Computational aspects of commutative and noncommutative positive polynomials

Quantum polynomial optimisation problems for dimension $\boldsymbol{d}$ variables, with symmetries



I. The Quantum Random Access code

II. The NV relaxation

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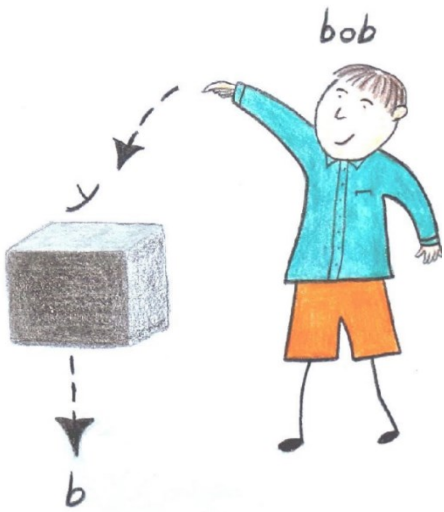
IV. Symmetrisation of the NV hierarchy



The Random Access Code (RAC) game

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- Two players: Alice, in Madrid. Bob, in Barcelona

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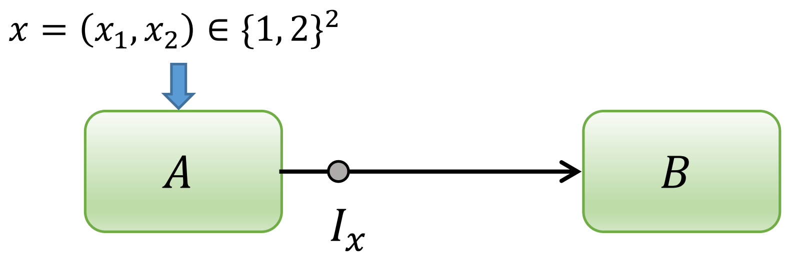

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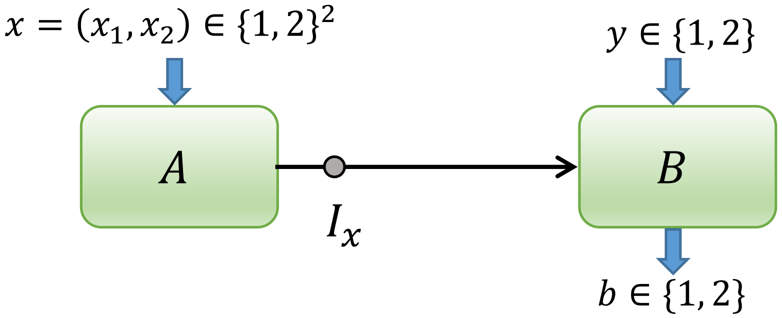

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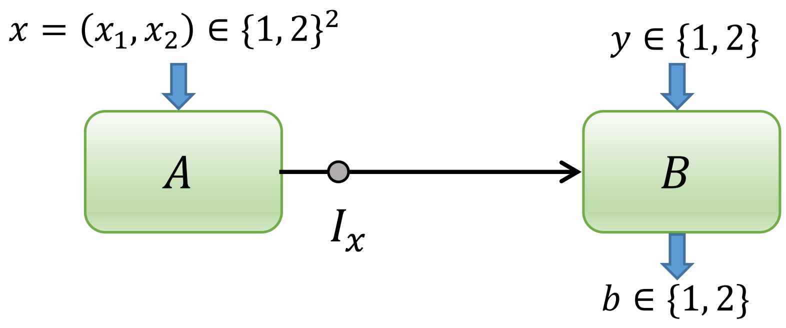

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Score

- This is done several time: $\boldsymbol{p(b|x,y)}$

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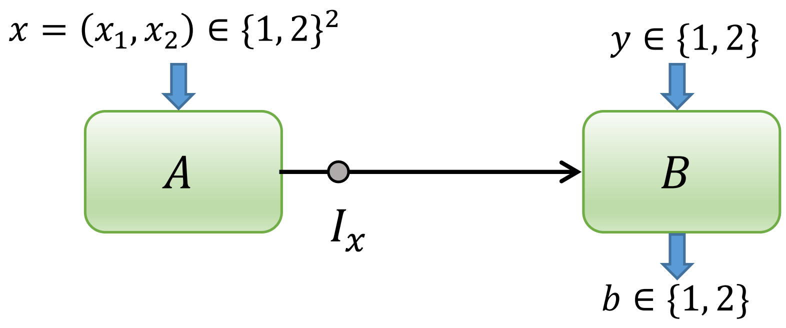

$$\boldsymbol{b = x_y?}$$

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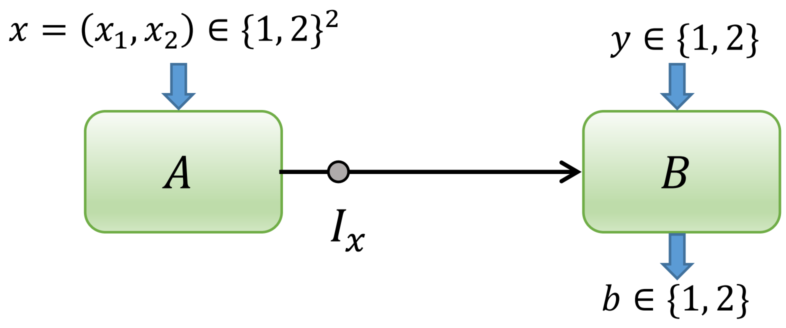
Score

- This is done several time: $\boldsymbol{p(b|x,y)}$
- Score of $\boldsymbol{A\&B}$: probability that $\boldsymbol{B}$ guesses $\boldsymbol{x_y}$:

$$\boldsymbol{S = p(b = x_y) = \sum_{xyb} \delta_{b=x_y} p(b|x,y)}$$

The Random Access Code (RAC) game



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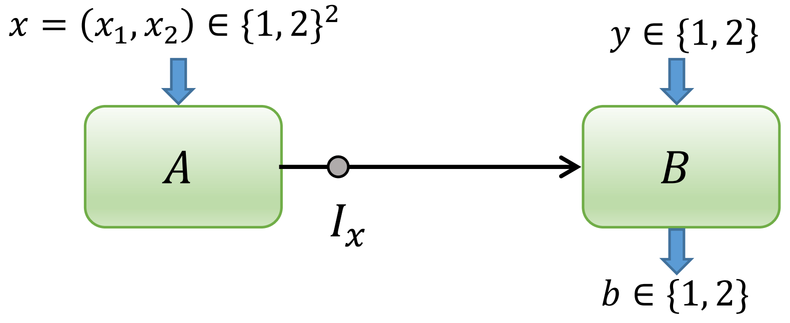
Trivial strategy

The Random Access Code (RAC) game



$\boldsymbol{b = x_y}$?

Trivial strategy

- $\boldsymbol{A}$ sends $\boldsymbol{I_x = x}$
  - ➢ $\boldsymbol{B}$ always guesses $\boldsymbol{x_y}$

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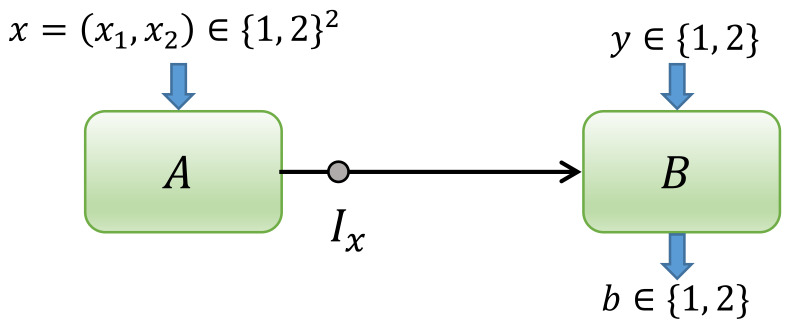

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Trivial strategy

- $\boldsymbol{A}$ sends $\boldsymbol{I_x = x}$
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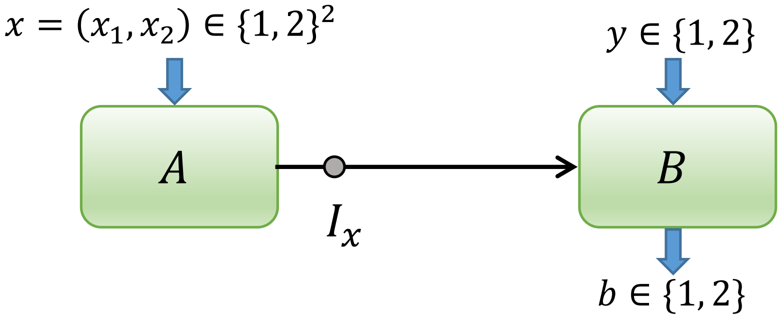
Restricted game

- $\boldsymbol{A}$ receives $\boldsymbol{x}$**,** sends **restricted** $\boldsymbol{I_x}$ of **dimension** 2

The Random Access Code (RAC) game

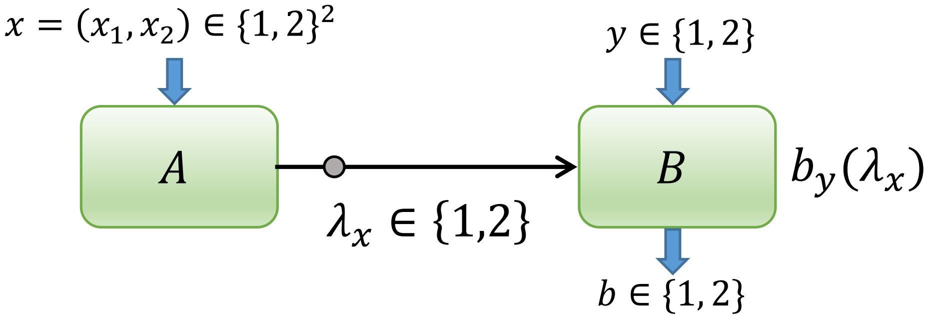


$$\boldsymbol{b = x_y}?$$

The Random Access Code (RAC) game

- $\boldsymbol{A}$ receives $\boldsymbol{x}$, sends **restricted** $\boldsymbol{I_x}$ of dimension 2
- $\boldsymbol{B}$ receives $\boldsymbol{y}$, $\boldsymbol{I_x}$, outputs $\boldsymbol{b}$
- Score : probability that $\boldsymbol{B}$ guesses $\boldsymbol{x_y}$ $\boldsymbol{S = p(b = x_y)}$

The Random Access Code (RAC) game



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Classical strategies

- $\boldsymbol{I_x \coloneqq \lambda_x}$ is a **bit** of information depending on $\boldsymbol{x}$
- $\boldsymbol{B}$ outputs a classical function of $\boldsymbol{\lambda_x}$, $\boldsymbol{b_y(\lambda_x)}$

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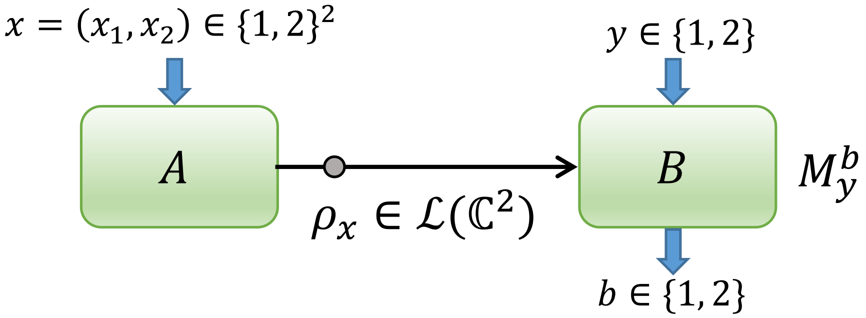

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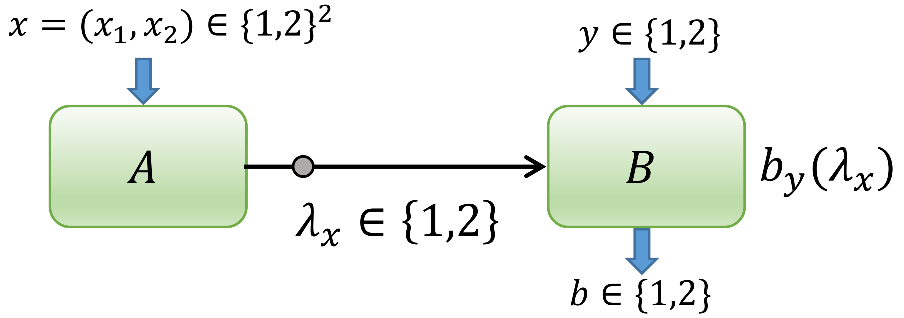
Quantum strategies

- $\boldsymbol{I_x \coloneqq \rho_x}$ is a **qubit** of information depending on $\boldsymbol{x}$
- $\boldsymbol{B}$ performs a quantum measurement $\boldsymbol{M_y^b}$ of $\boldsymbol{\rho_x}$

RAC game: classical strategies

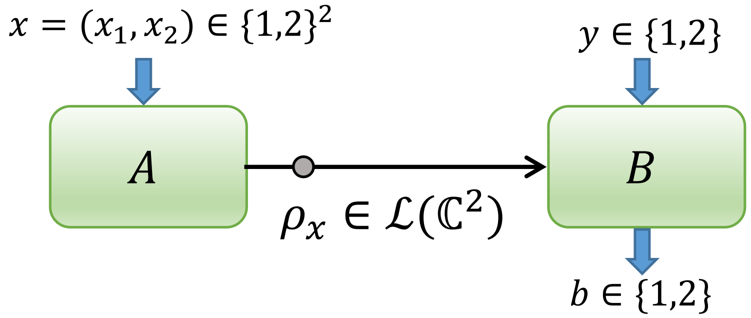


$\boldsymbol{b = x_y}$?

Classical RAC game: Maximal score

- Optimal strategy: $\boldsymbol{\lambda_x = x_1,\ b_y(\lambda_x) = \lambda_x}$
  - ➢ $\boldsymbol{S_{max}^{\text{classical}} = 3/4}$

RAC game: quantum strategies

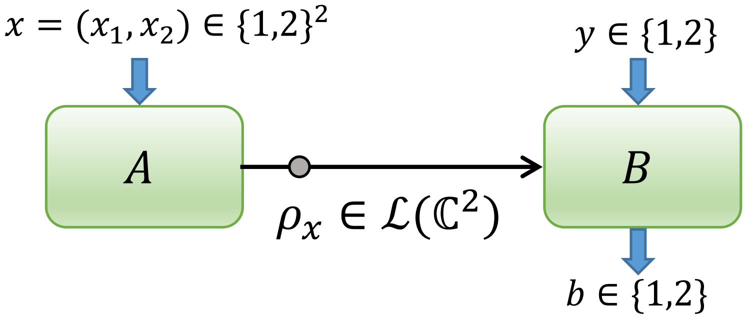


$\boldsymbol{b} = \boldsymbol{x_y}?$

Quantum RAC game

- $\boldsymbol{A}$ receives $\boldsymbol{x}$, sends qubit $\boldsymbol{\rho_x}$

RAC game: quantum strategies



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- $\boldsymbol{A}$ receives $\boldsymbol{x}$, sends qubit $\boldsymbol{\rho_x}$
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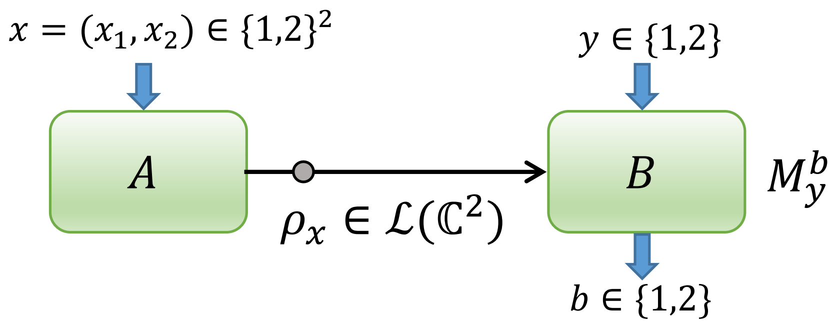
RAC game: quantum strategies



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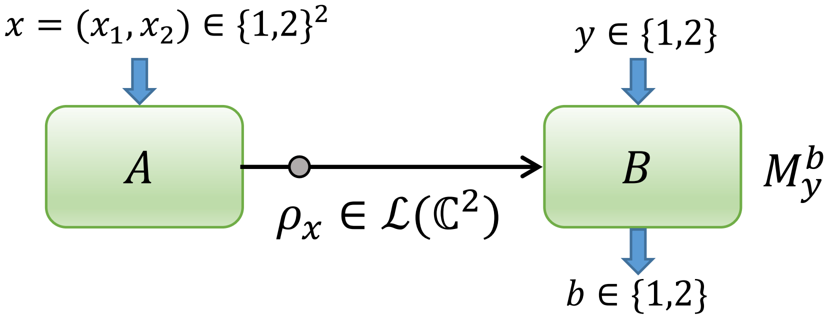

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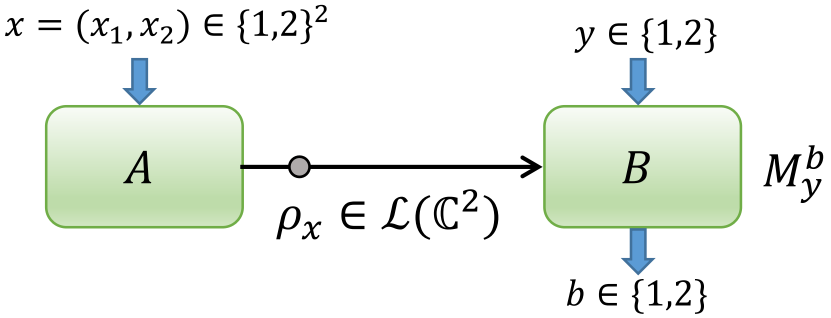

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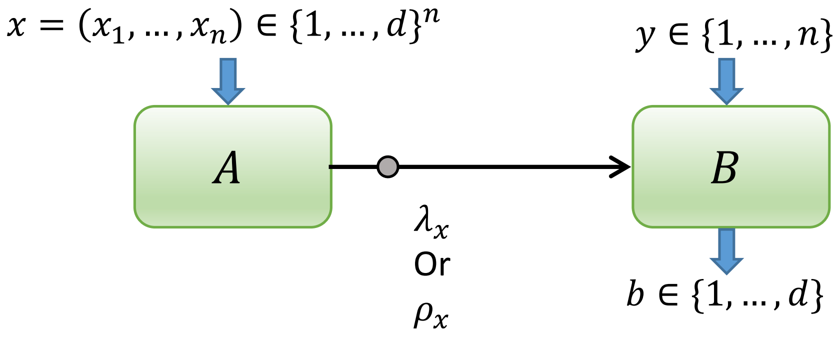
Maximal score

- $\boldsymbol{S_{max}^{Q}} \approx \mathbf{0,85}$

Generalised $(\boldsymbol{n}, \boldsymbol{d})$-RAC game



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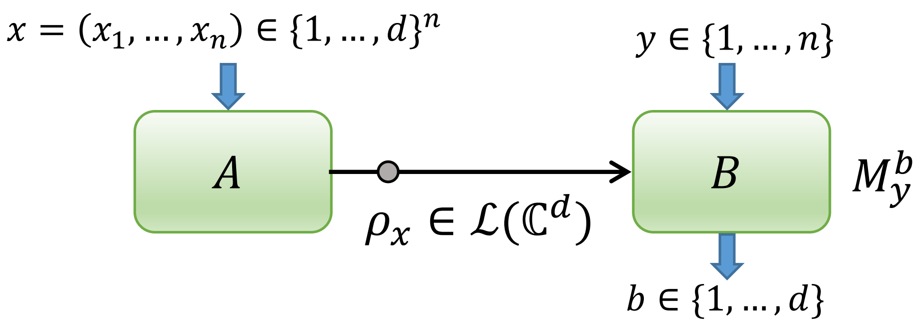
The $(\boldsymbol{n}, \boldsymbol{d})$-RAC game

- A direct generalisation:
  - $\boldsymbol{x} = (\boldsymbol{x_1}, \dots, \boldsymbol{x_n}) \in \{\boldsymbol{1}, \dots, \boldsymbol{d}\}^{\boldsymbol{n}}, \boldsymbol{y} \in \{\boldsymbol{1}, \dots, \boldsymbol{n}\}$
  - restricted $\boldsymbol{I_x}$ of dimension $\boldsymbol{d}$

Generalised (n, d)-RAC game



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$$S_{max}^{Q} = \max_{M_y^b, \rho_x \in \mathcal{F}} S(\{\rho_x, M_y^b\})$$

$\mathcal{F} : \rho_x, M_y^b \succcurlyeq 0$, **dimension d**, $\mathrm{Tr}(\rho_x) = 1, \forall y, \sum_b M_y^b = I_d$

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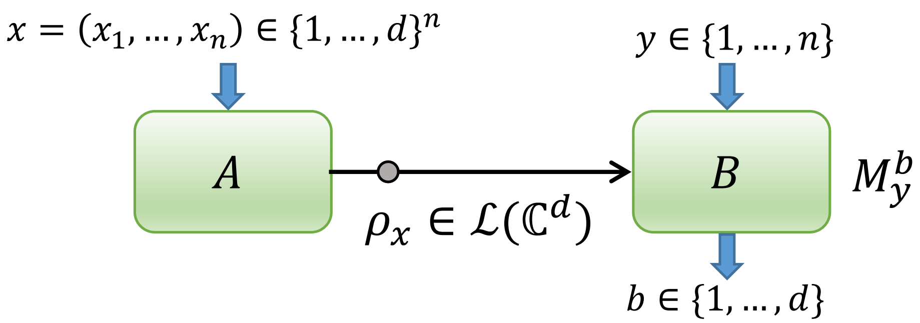

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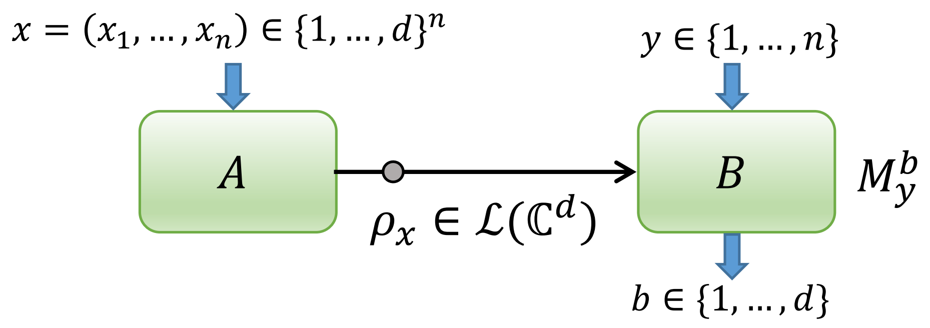
- In general, S_{max}^{Q} is **not** known

Generalised (n, d)-RAC game



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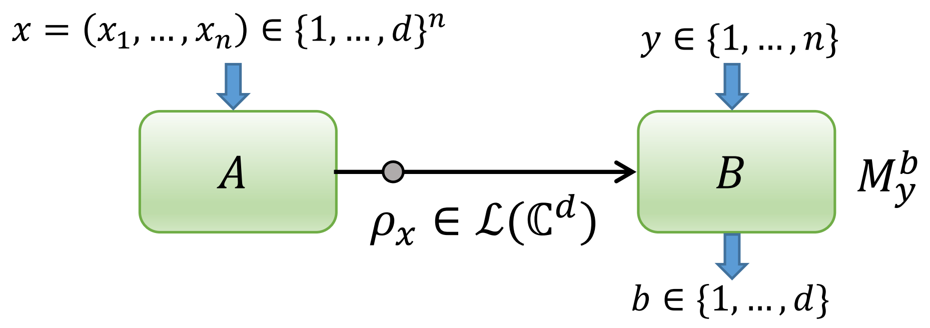

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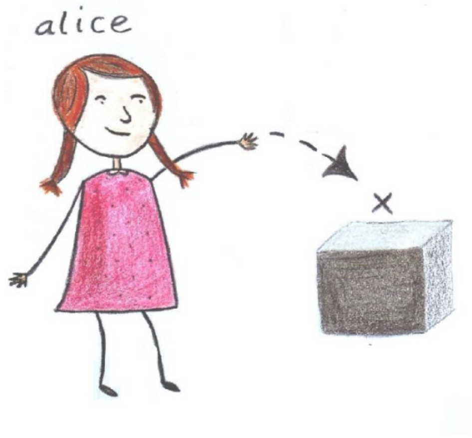
- In general, S_{max}^{Q} is **not** known
- Dimension d non commuting polynomial optimisation problem
  - ➢ Lower bounded by explicit solutions
  - ➢ Upper bounds?

Quantum polynomial optimisation problems for dimension $\boldsymbol{d}$ variables, with symmetries



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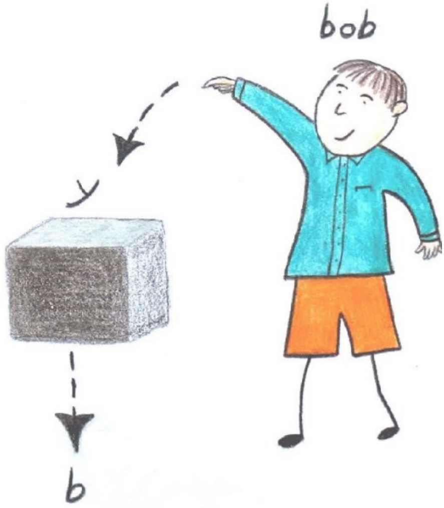

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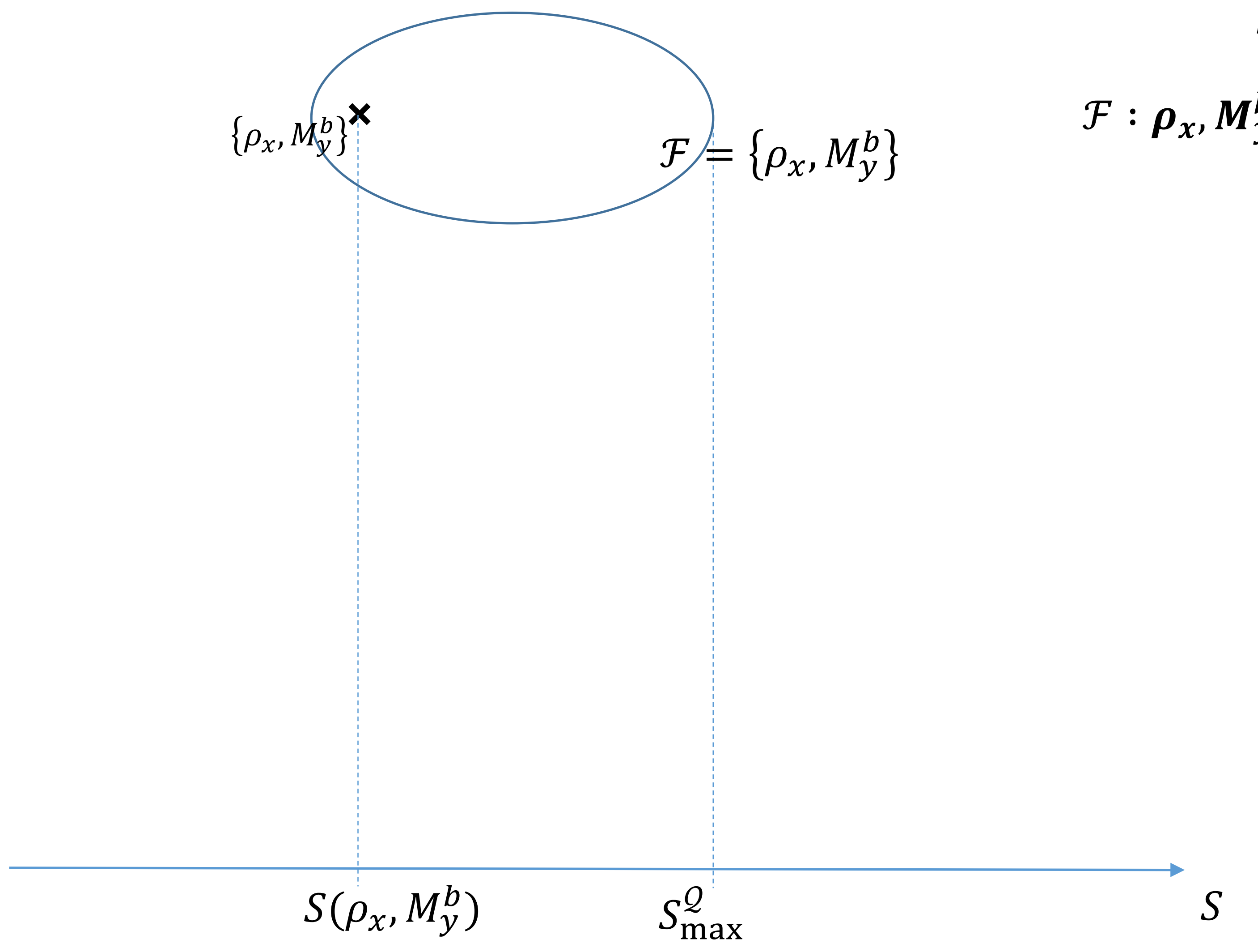

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M. Navascués, S. Pironio, and A. Acín, Phys. Rev. Lett. 98, 010401 (2007)

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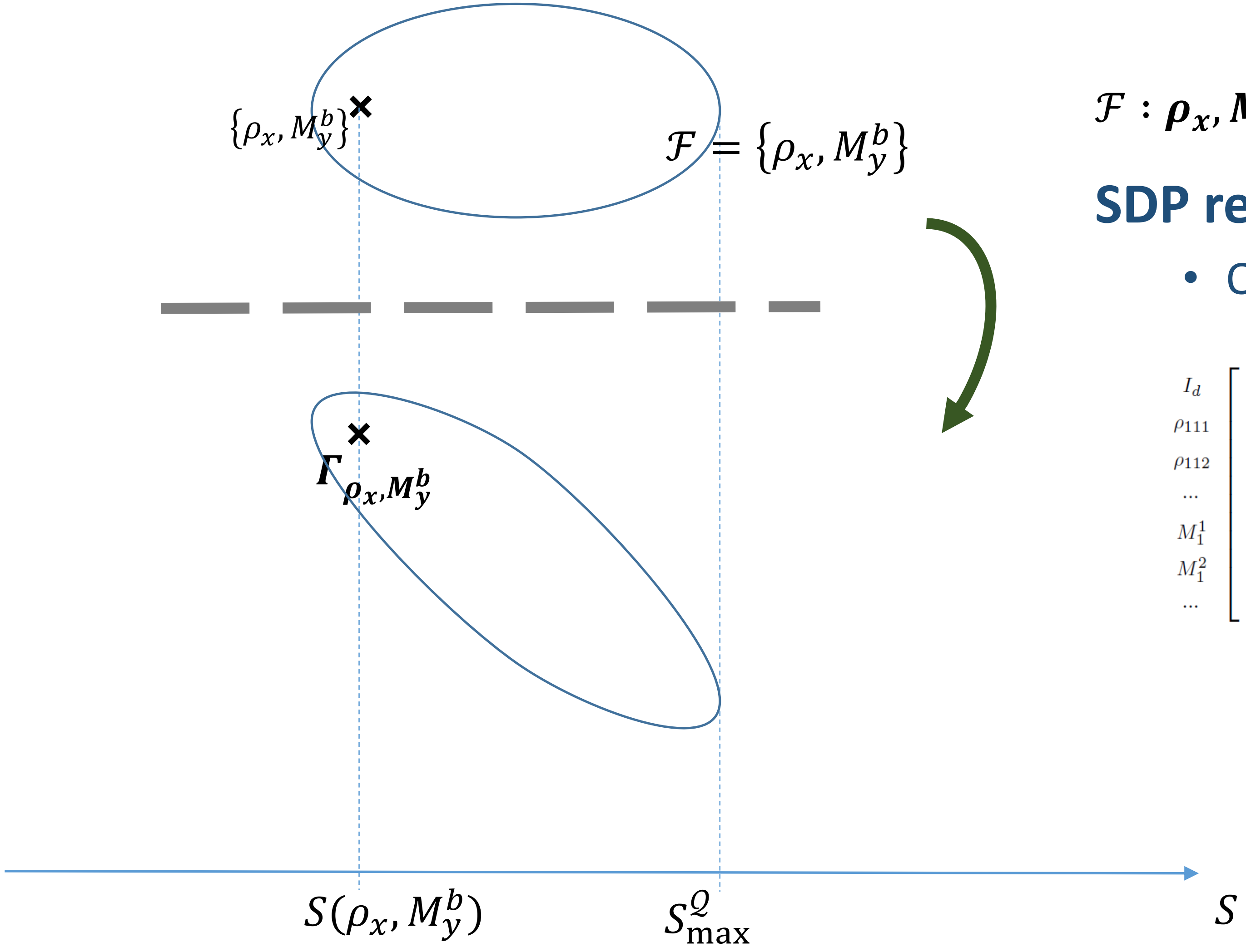

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SDP relaxation

- Order l moment matrix $\{\rho_x, M_y^b\} \mapsto \Gamma_{\rho_x, M_y^b}$

	I_d	ρ_{111}	ρ_{112}	...	M_1^1	M_1^2	...
I_d	d	1	1	$\cdots$	$\mathrm{Tr}(M_1^1)$	$\mathrm{Tr}(M_1^2)$	$\cdots$
ρ_{111}	1	$\mathrm{Tr}(\rho_{111}\cdot\rho_{111})$	$\mathrm{Tr}(\rho_{111}\cdot\rho_{112})$	$\cdots$	$\mathrm{Tr}(\rho_{111}\cdot M_1^1)$	$\mathrm{Tr}(\rho_{111}\cdot M_1^2)$	$\cdots$
ρ_{112}	1	$\mathrm{Tr}(\rho_{112}\cdot\rho_{111})$	$\mathrm{Tr}(\rho_{112}\cdot\rho_{112})$	$\cdots$	$\mathrm{Tr}(\rho_{112}\cdot M_1^1)$	$\mathrm{Tr}(\rho_{112}\cdot M_1^2)$	$\cdots$
...	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
M_1^1	$\mathrm{Tr}(M_1^1)$	$\mathrm{Tr}(M_1^1\cdot\rho_{111})$	$\mathrm{Tr}(M_1^1\cdot\rho_{112})$	$\cdots$	$\mathrm{Tr}(M_1^1\cdot M_1^1)$	$\mathrm{Tr}(M_1^1\cdot M_1^2)$	$\cdots$
M_1^2	$\mathrm{Tr}(M_1^2)$	$\mathrm{Tr}(M_1^2\cdot\rho_{111})$	$\mathrm{Tr}(M_1^2\cdot\rho_{112})$	$\cdots$	$\mathrm{Tr}(M_1^2\cdot M_1^1)$	$\mathrm{Tr}(M_1^2\cdot M_1^2)$	$\cdots$
...	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

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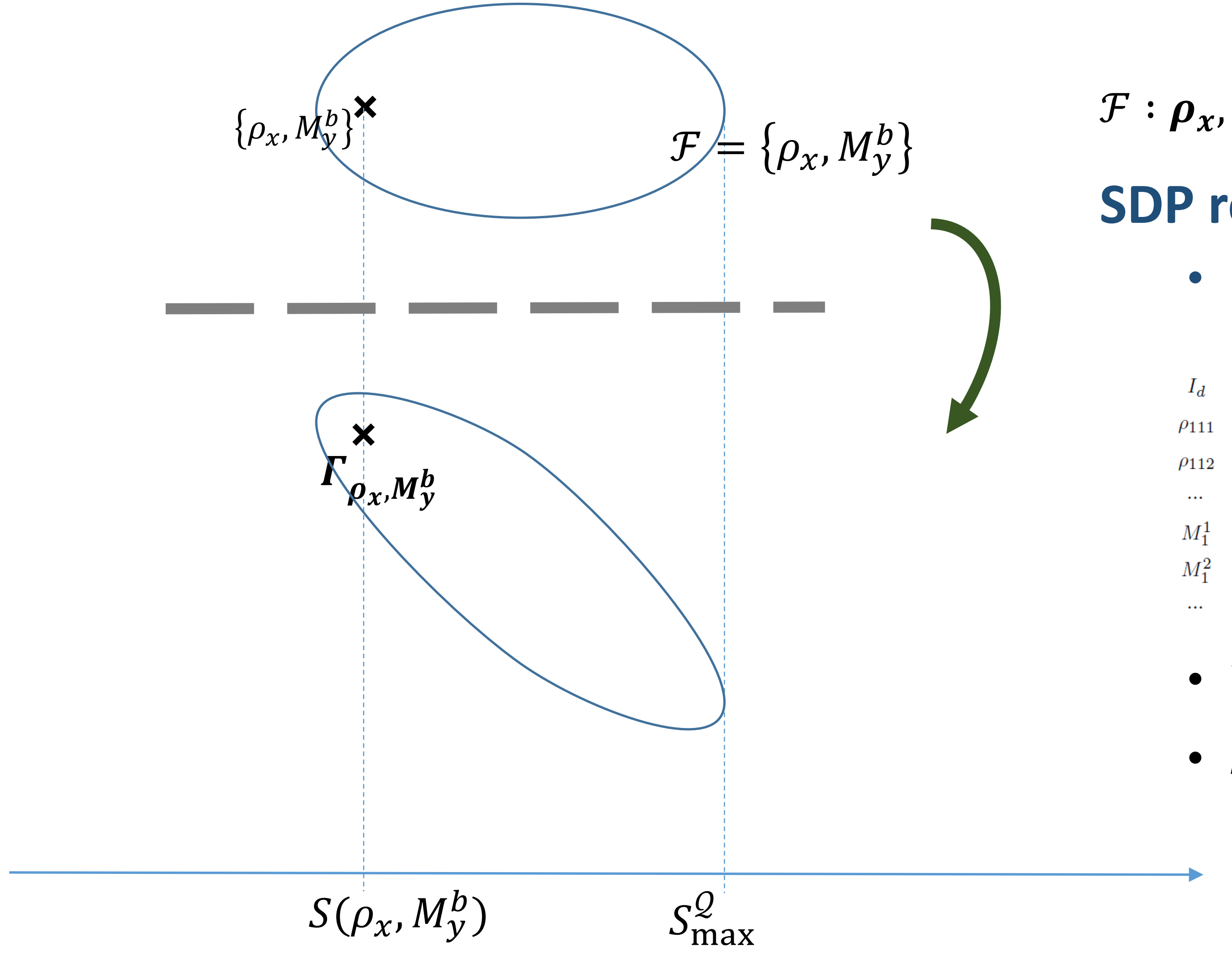

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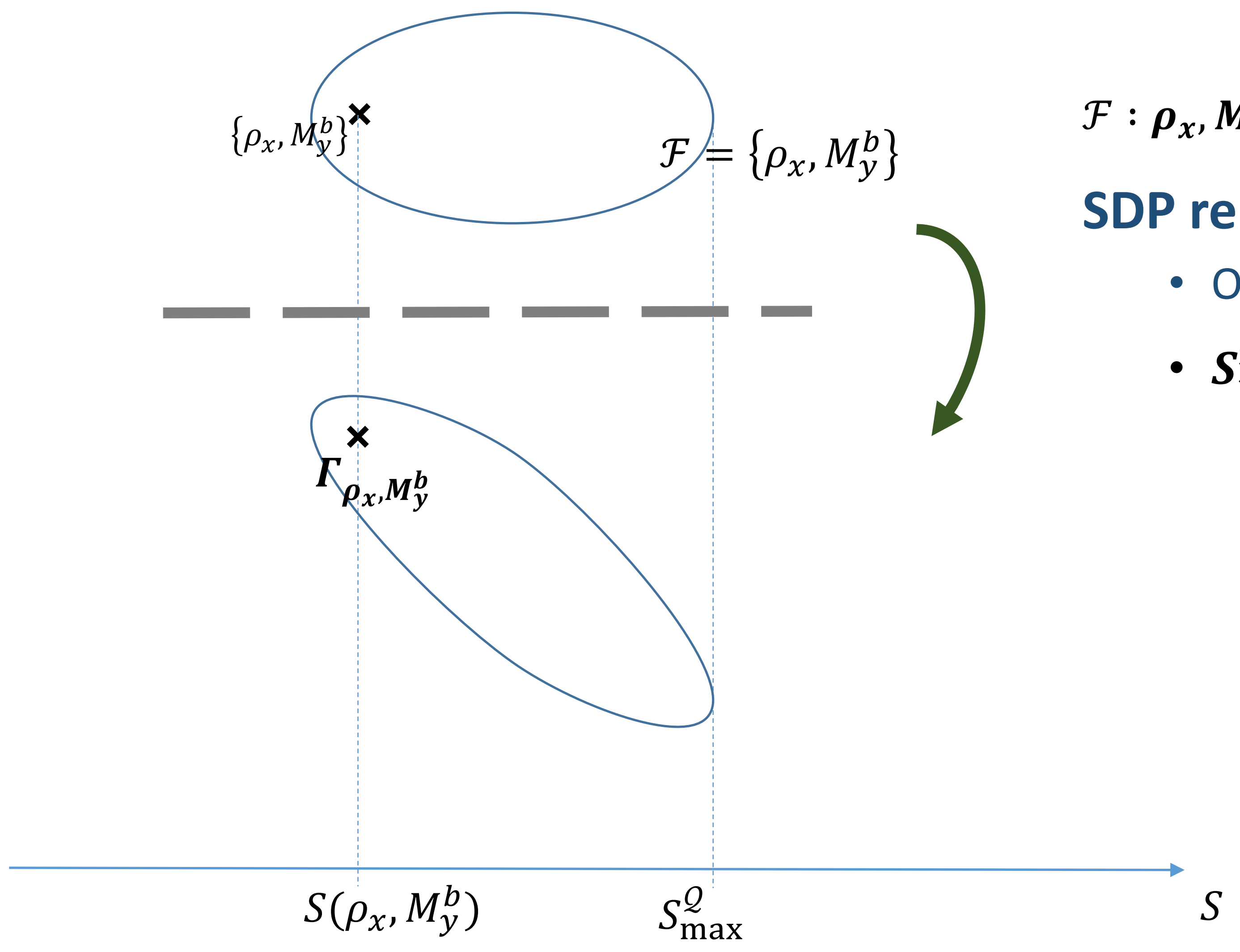
ρ_{111}	1	$\mathrm{Tr}(\rho_{111} \cdot \rho_{111})$	$\mathrm{Tr}(\rho_{111} \cdot \rho_{112})$	$\cdots$	$\mathrm{Tr}(\rho_{111} \cdot M_1^1)$	$\mathrm{Tr}(\rho_{111} \cdot M_1^2)$	$\cdots$
ρ_{112}	1	$\mathrm{Tr}(\rho_{112} \cdot \rho_{111})$	$\mathrm{Tr}(\rho_{112} \cdot \rho_{112})$	$\cdots$	$\mathrm{Tr}(\rho_{112} \cdot M_1^1)$	$\mathrm{Tr}(\rho_{112} \cdot M_1^2)$	$\cdots$
…	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
M_1^1	$\mathrm{Tr}(M_1^1)$	$\mathrm{Tr}(M_1^1 \cdot \rho_{111})$	$\mathrm{Tr}(M_1^1 \cdot \rho_{112})$	$\cdots$	$\mathrm{Tr}(M_1^1 \cdot M_1^1)$	$\mathrm{Tr}(M_1^1 \cdot M_1^2)$	$\cdots$
M_1^2	$\mathrm{Tr}(M_1^2)$	$\mathrm{Tr}(M_1^2 \cdot \rho_{111})$	$\mathrm{Tr}(M_1^2 \cdot \rho_{112})$	$\cdots$	$\mathrm{Tr}(M_1^2 \cdot M_1^1)$	$\mathrm{Tr}(M_1^2 \cdot M_1^2)$	$\cdots$
…	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

- $\forall \Gamma$, 'pseudo score' $S(\Gamma)$
- $S(\{\rho_x, M_y^b\}) = S\left(\Gamma_{\rho_x, M_y^b}\right)$

M. Navascués, S. Pironio, and A. Acín, Phys. Rev. Lett. 98, 010401 (2007)

Generalised (n, d)-RAC game



$$S^Q_{max} = \max_{M_y^b, \rho_x \in \mathcal{F}} S(\{\rho_x, M_y^b\})$$

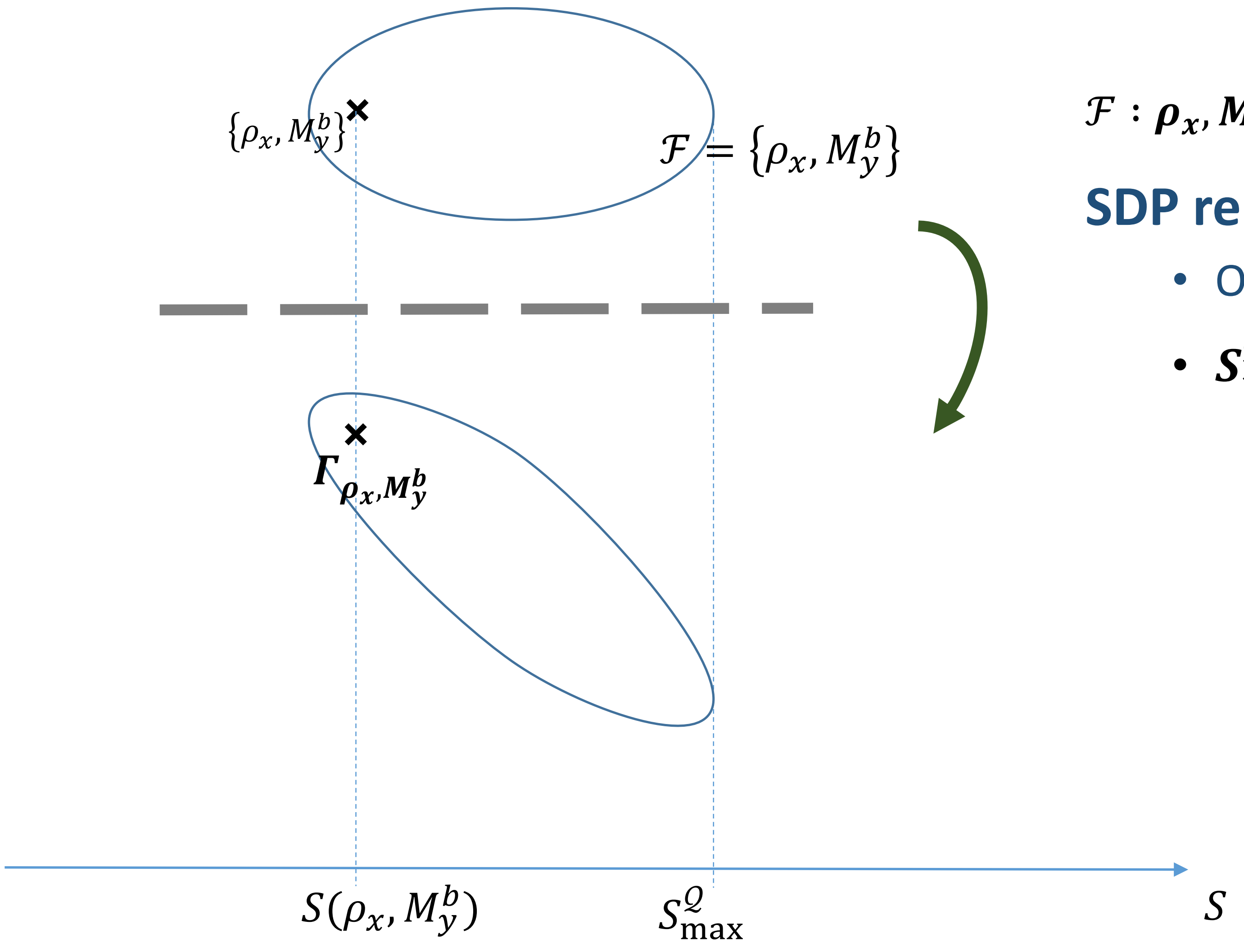
$\mathcal{F} : \rho_x, M_y^b \succcurlyeq 0$, **dimension d**, $\mathrm{Tr}(\rho_x) = 1, \forall y, \sum_b M_y^b = I_d$

SDP relaxation

- Order l moment matrix $\{\rho_x, M_y^b\} \mapsto \Gamma_{\rho_x, M_y^b}$
- $S^Q_{max} = \max_{M_y^b, \rho_x \in \mathcal{F}} S(\{\rho_x, M_y^b\})$

M. Navascués, and T. Vértesi, Phys. Rev. Lett. 115:020501 (2015)

Generalised (n, d)-RAC game



$$S_{max}^Q = \max_{M_y^b, \rho_x \in \mathcal{F}} S(\{\rho_x, M_y^b\})$$

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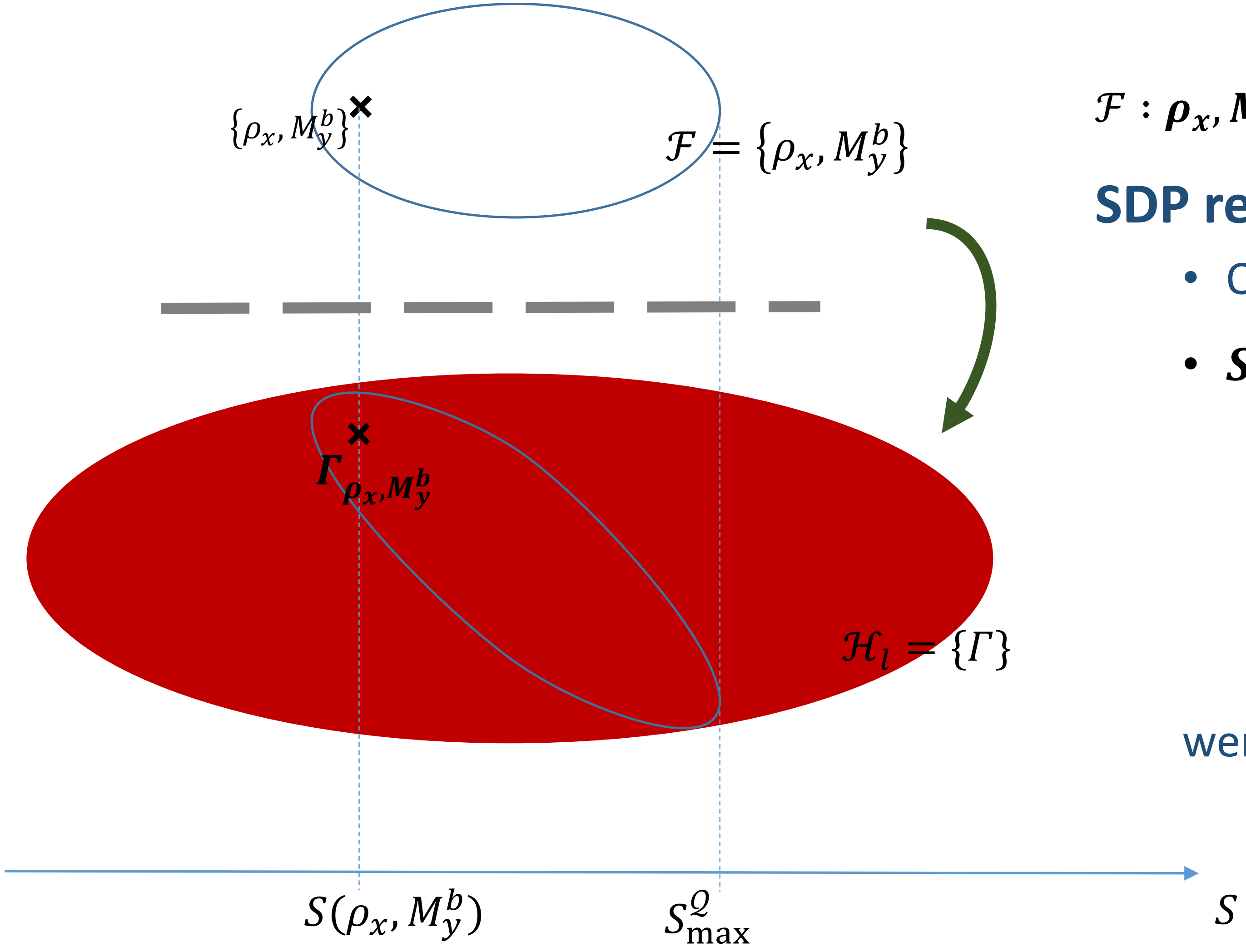
- Order l moment matrix $\{\rho_x, M_y^b\} \mapsto \Gamma_{\rho_x, M_y^b}$
- $$S_{max}^Q = \max_{M_y^b, \rho_x \in \mathcal{F}} S(\{\rho_x, M_y^b\}) = \max_{\substack{\Gamma_{M_y^b, \rho_x} \\ s.t. M_y^b, \rho_x \in \mathcal{F}}} S\left(\Gamma_{M_y^b, \rho_x}\right)$$

M. Navascués, and T. Vértesi, Phys. Rev. Lett. 115:020501 (2015)

Generalised $(\boldsymbol{n}, \boldsymbol{d})$-RAC game



$$S_{max}^{Q} = \max_{M_y^b, \rho_x \in \mathcal{F}} S(\{\rho_x, M_y^b\})$$

$\mathcal{F} : \rho_x, M_y^b \succcurlyeq 0,$ **dimension d,** $\mathrm{Tr}(\rho_x) = 1, \forall y, \sum_b M_y^b = I_d$

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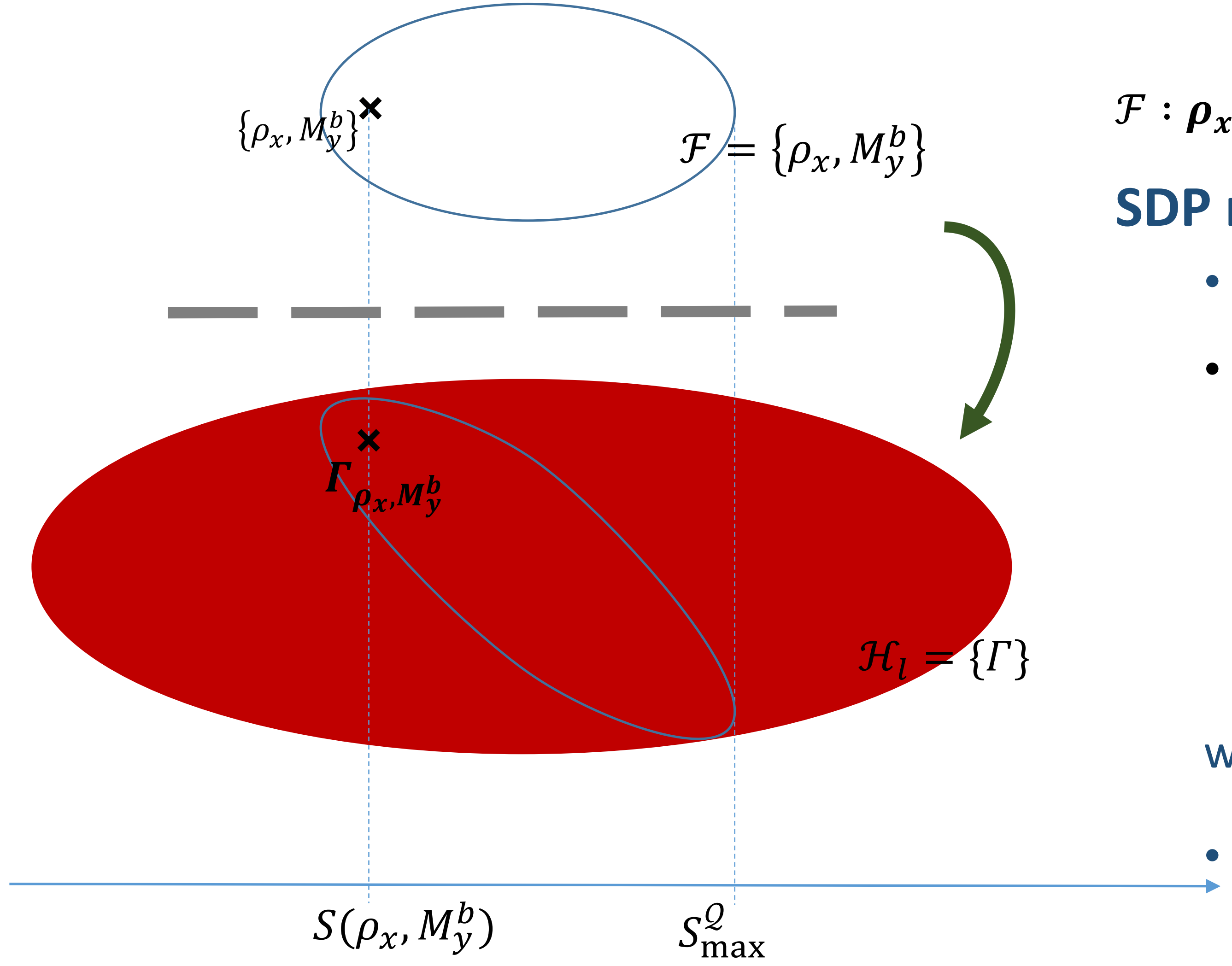
were $\mathcal{H}_l = \left\{\Gamma \succcurlyeq 0 | \Gamma \in \mathrm{Span}\left(\Gamma_{M_y^b, \rho_x}\right)\right\}$

M. Navascués, and T. Vértesi, Phys. Rev. Lett. 115:020501 (2015)

Generalised (n, d)-RAC game



$$S_{max}^{Q} = \max_{M_y^b, \rho_x \in \mathcal{F}} S(\{\rho_x, M_y^b\})$$

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SDP relaxation

- Order l moment matrix $\{\rho_x, M_y^b\} \mapsto \Gamma_{\rho_x, M_y^b}$
- $$S_{max}^{Q} = \max_{M_y^b, \rho_x \in \mathcal{F}} S(\{\rho_x, M_y^b\})$$
  $$= \max_{\substack{\Gamma_{M_y^b, \rho_x} \\ s.t. M_y^b, \rho_x \in \mathcal{F}}} S\left(\Gamma_{M_y^b, \rho_x}\right)$$
  $$\leq \max_{\Gamma \in \mathcal{H}_l} S(\Gamma)$$ ← SDP

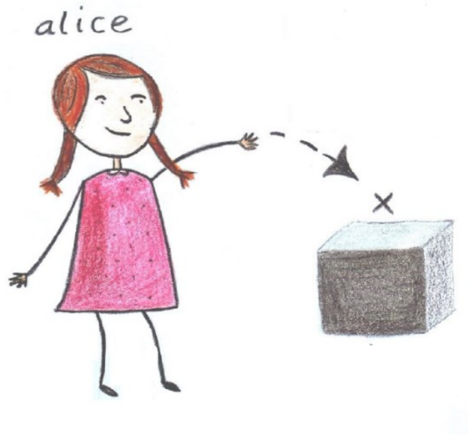
were $\mathcal{H}_l = \left\{\Gamma \succcurlyeq 0 | \Gamma \in \mathrm{Span}\left(\Gamma_{M_y^b, \rho_x}\right)\right\}$

- **How to construct $\mathcal{H}_l$?**

S
  - Sampling

M. Navascués, and T. Vértesi, Phys. Rev. Lett. 115:020501 (2015)

Quantum polynomial optimisation problems for dimension $\boldsymbol{d}$ variables, with symmetries

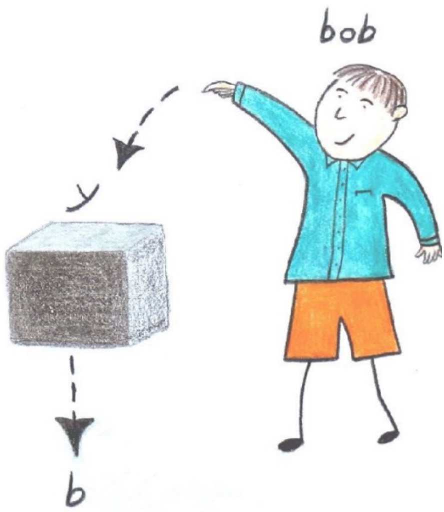


I. The Quantum Random Access code

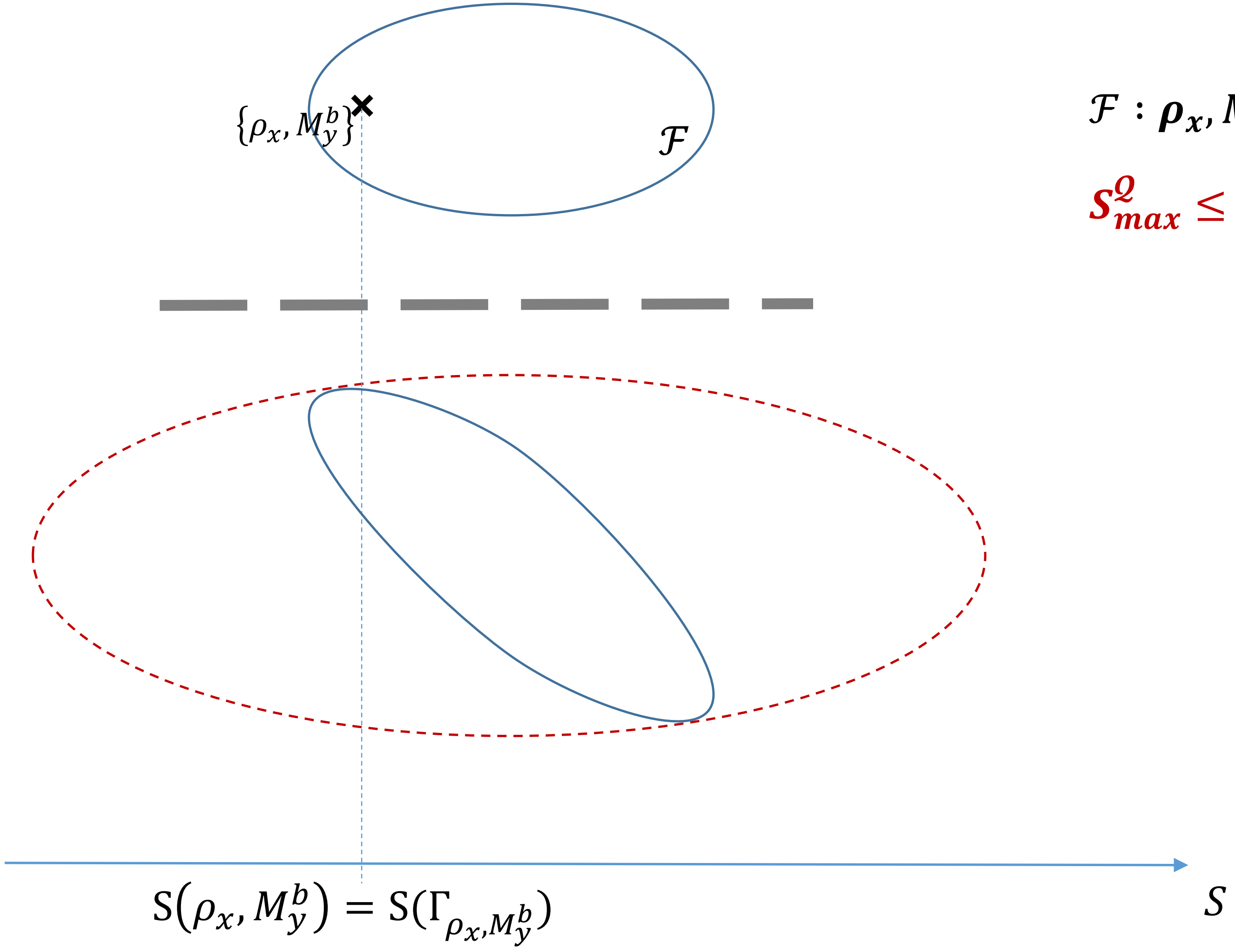
II. The NV relaxation

III. Sampling

IV. Symmetrisation of the NV hierarchy



Sampling



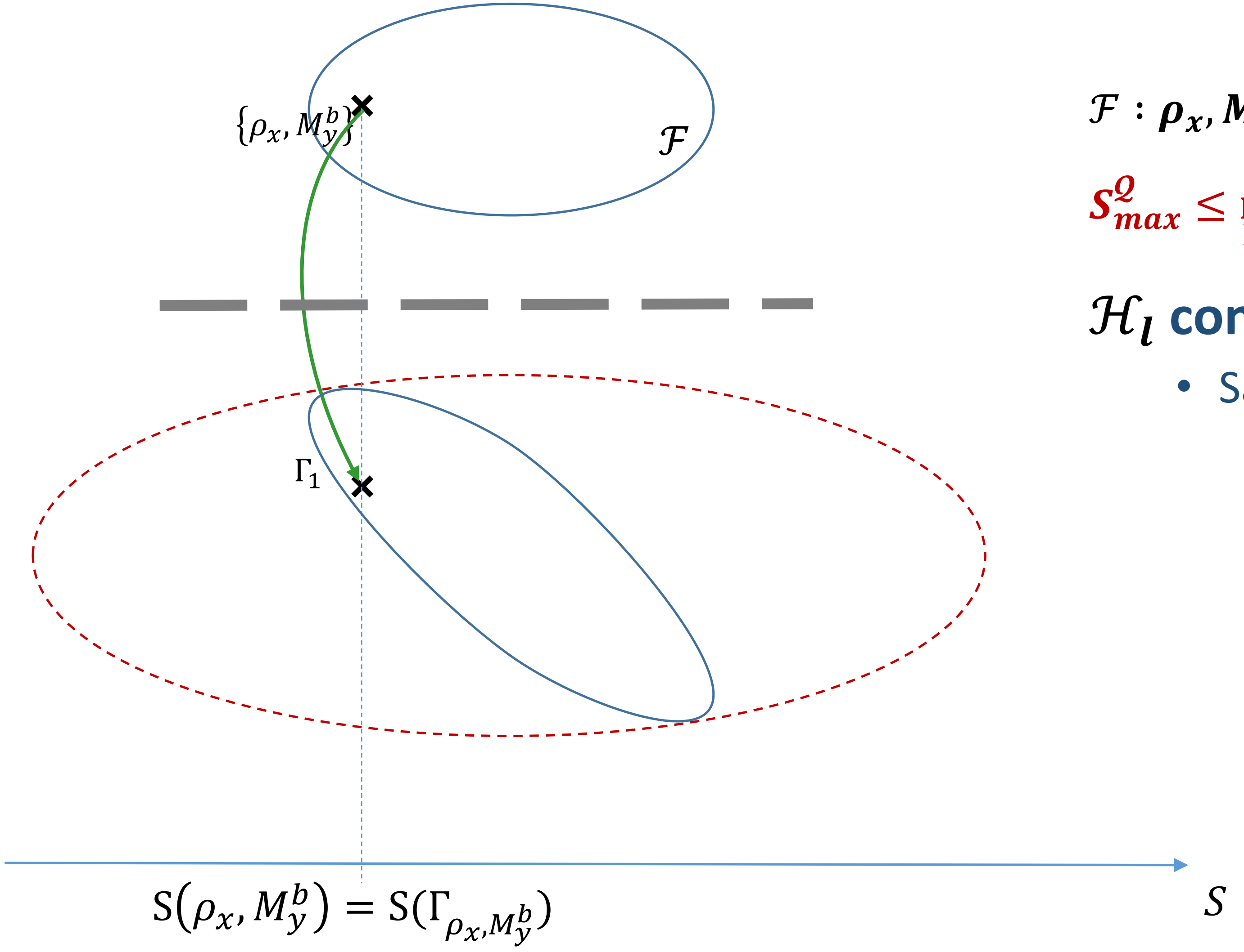
$$\boldsymbol{S^Q_{max} = \max_{M_y^b, \rho_x \in \mathcal{F}} S(\{\rho_x, M_y^b\})}$$

$\mathcal{F}$: $\boldsymbol{\rho_x, M_y^b \succcurlyeq 0}$, **dimension d**, $\mathbf{Tr}(\boldsymbol{\rho_x}) = \mathbf{1}, \forall \boldsymbol{y}, \sum_b \boldsymbol{M_y^b} = \boldsymbol{I_d}$

$\boldsymbol{S^Q_{max} \leq \max_{\Gamma \in \mathcal{H}_l} S(\Gamma)}$, where $\boldsymbol{\mathcal{H}_l} = \left\{\boldsymbol{\Gamma \succcurlyeq 0 | \Gamma} \in \mathrm{Span}\left(\boldsymbol{\Gamma_{M_y^b, \rho_x}}\right)\right\}$

M. Navascués, and T. Vértesi, Phys. Rev. Lett. 115:020501 (2015)

Sampling



$$S^{Q}_{max} = \max_{M_y^b, \rho_x \in \mathcal{F}} S(\{\rho_x, M_y^b\})$$

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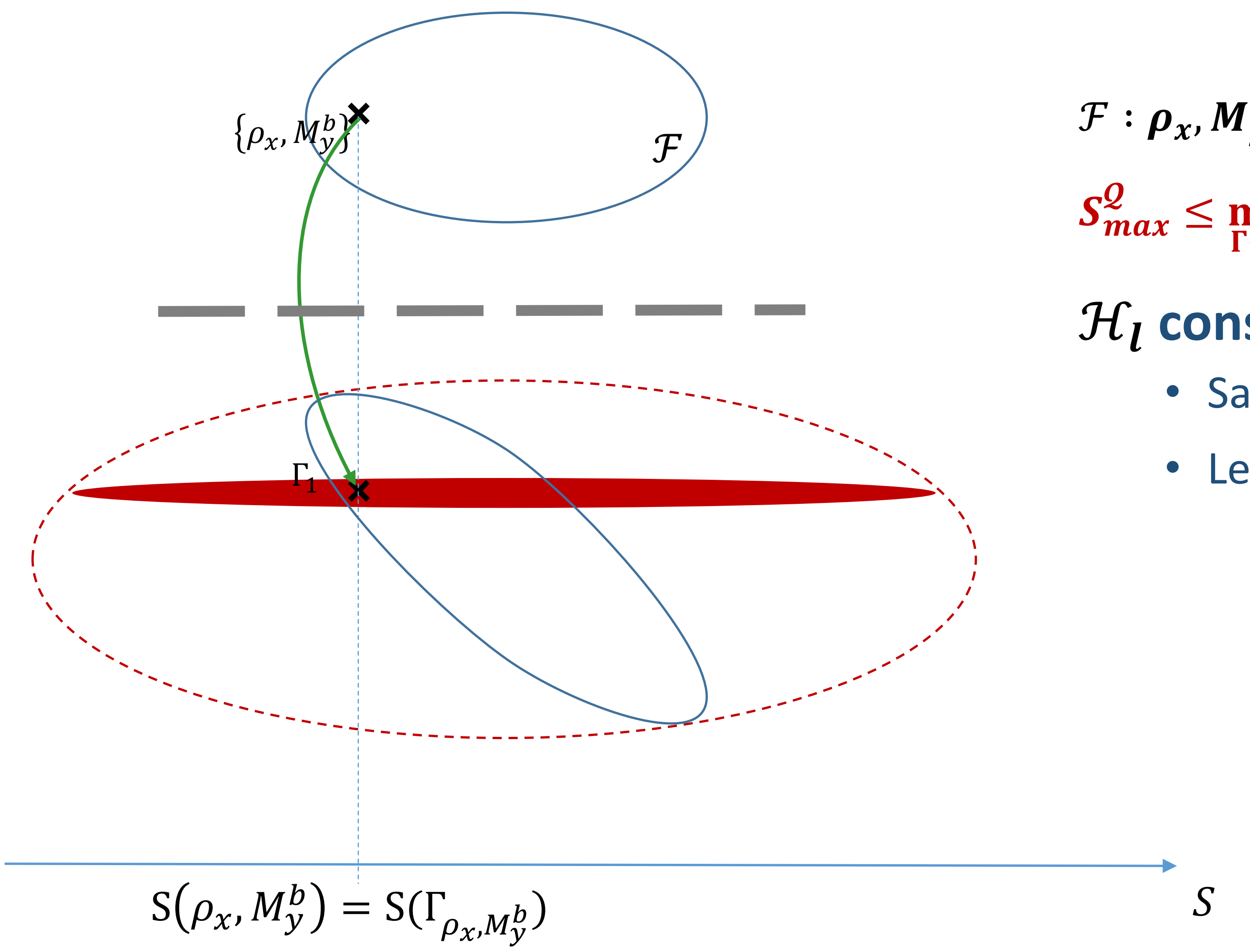
$\mathcal{H}_l$ construction

- Sample $\rho_x, M_y^b \in \mathcal{F}$, compute $\Gamma_1 = \Gamma_{\rho_x, M_y^b}$

M. Navascués, and T. Vértesi, Phys. Rev. Lett. 115:020501 (2015)

Sampling



$$S_{max}^{Q} = \max_{M_y^b, \rho_x \in \mathcal{F}} S(\{\rho_x, M_y^b\})$$

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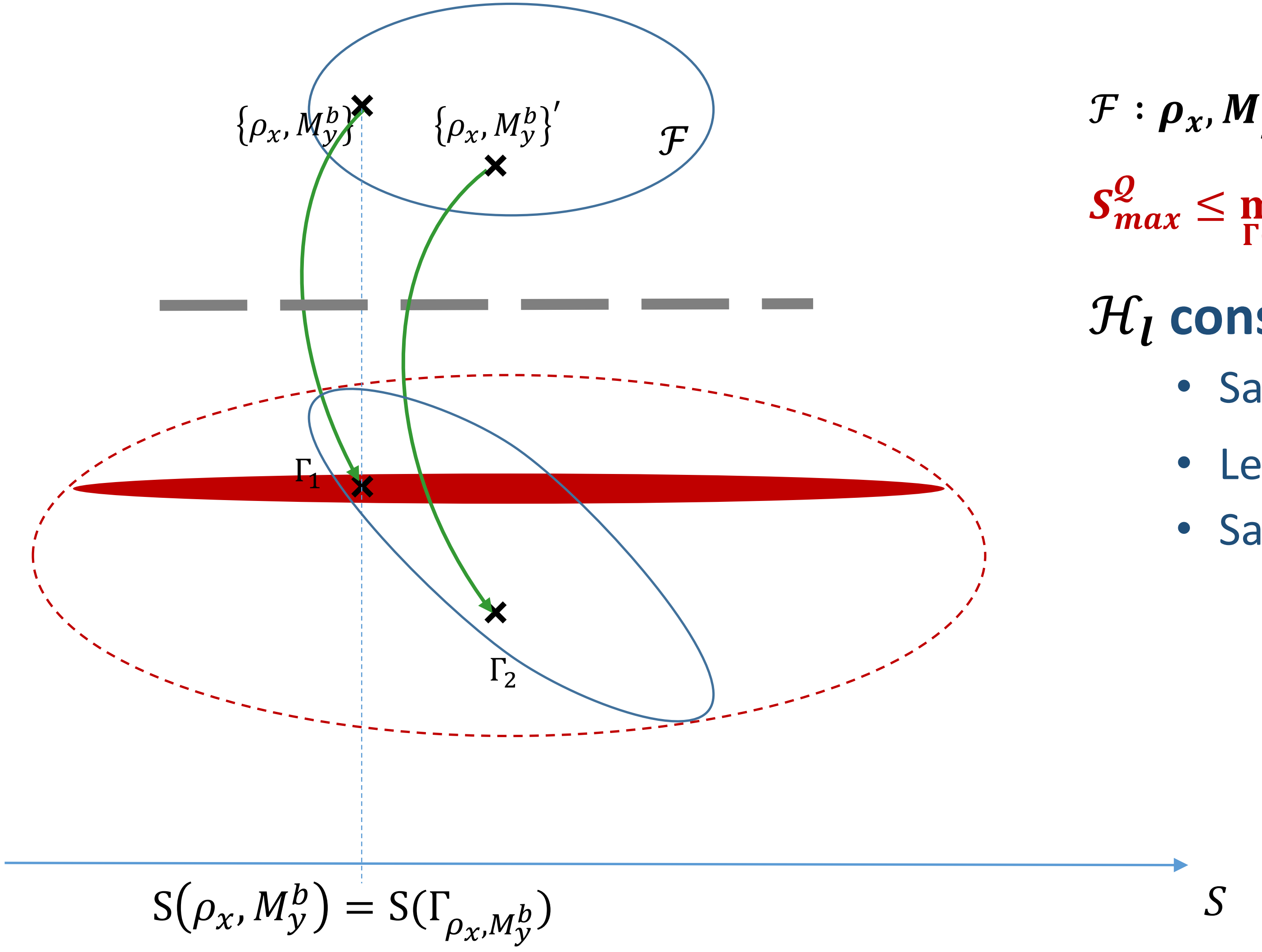
M. Navascués, and T. Vértesi, Phys. Rev. Lett. 115:020501 (2015)

Sampling



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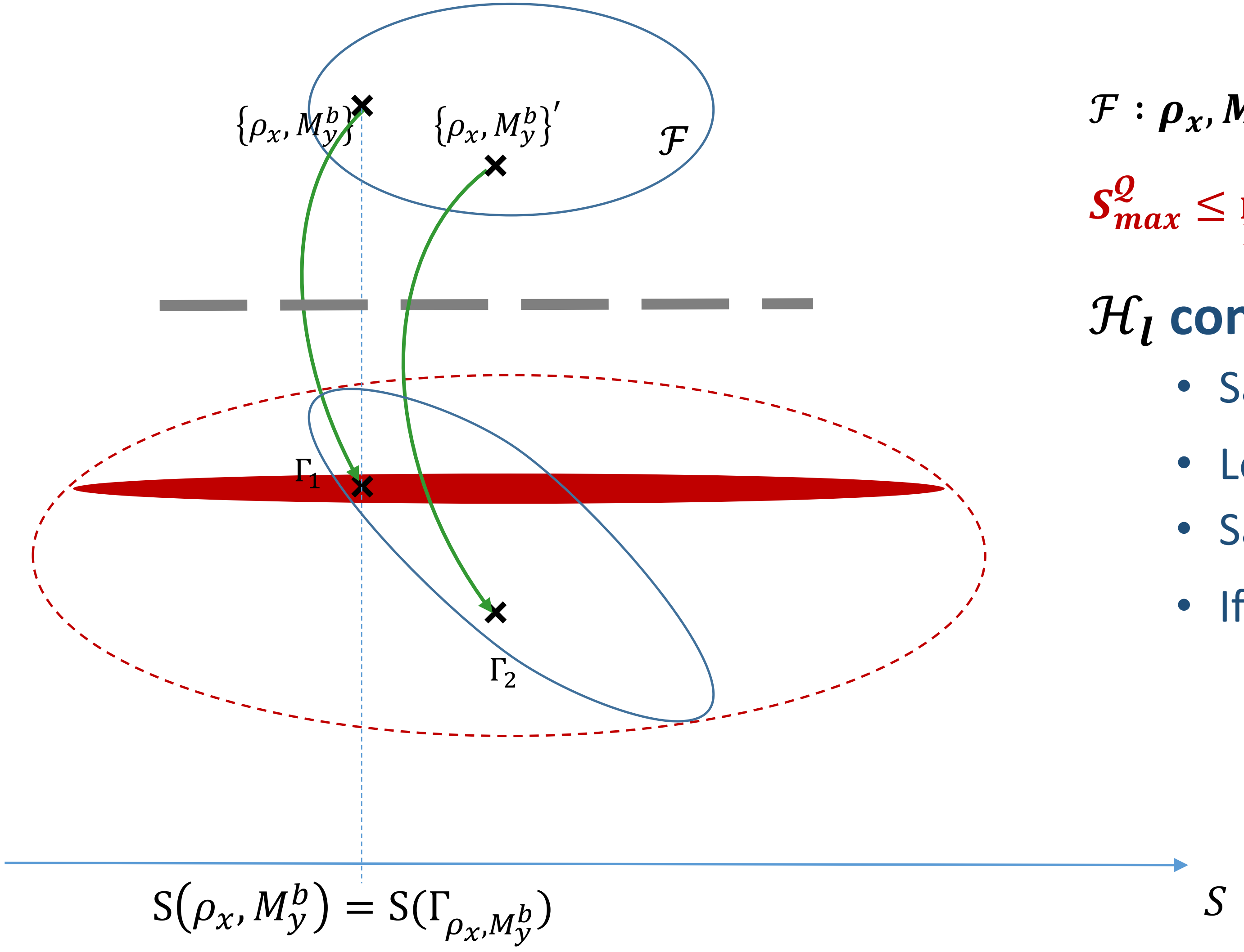
M. Navascués, and T. Vértesi, Phys. Rev. Lett. 115:020501 (2015)

Sampling



$$S_{max}^{Q} = \max_{M_y^b, \rho_x \in \mathcal{F}} S(\{\rho_x, M_y^b\})$$

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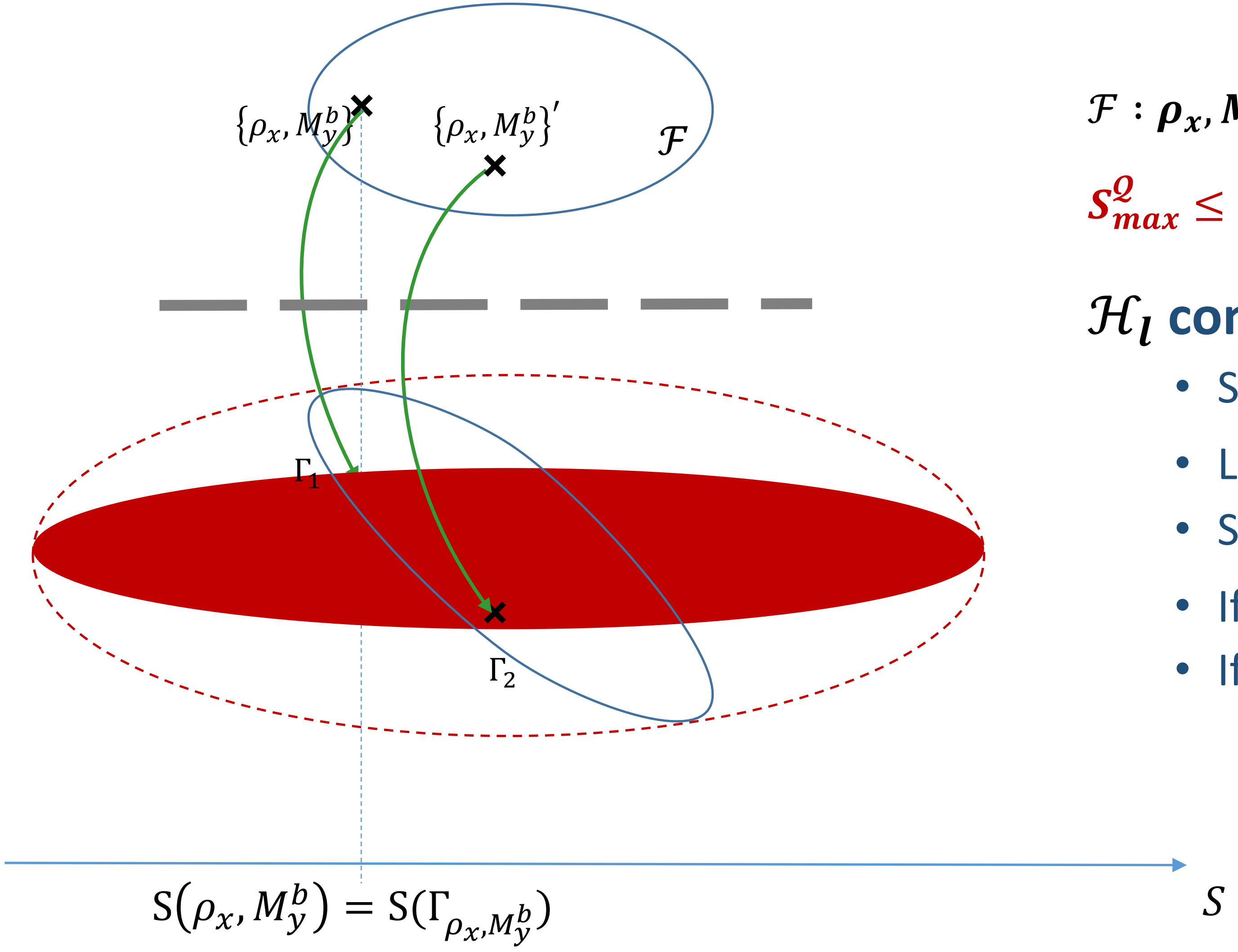
- If $\Gamma_2 \in \mathcal{L}$: STOP

M. Navascués, and T. Vértesi, Phys. Rev. Lett. 115:020501 (2015)

Sampling



$$S_{max}^{Q} = \max_{M_y^b, \rho_x \in \mathcal{F}} S(\{\rho_x, M_y^b\})$$

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$S_{max}^{Q} \leq \max_{\Gamma \in \mathcal{H}_l} S(\Gamma)$, where $\mathcal{H}_l = \left\{\Gamma \succcurlyeq 0 | \Gamma \in \text{Span}\left(\Gamma_{M_y^b, \rho_x}\right)\right\}$

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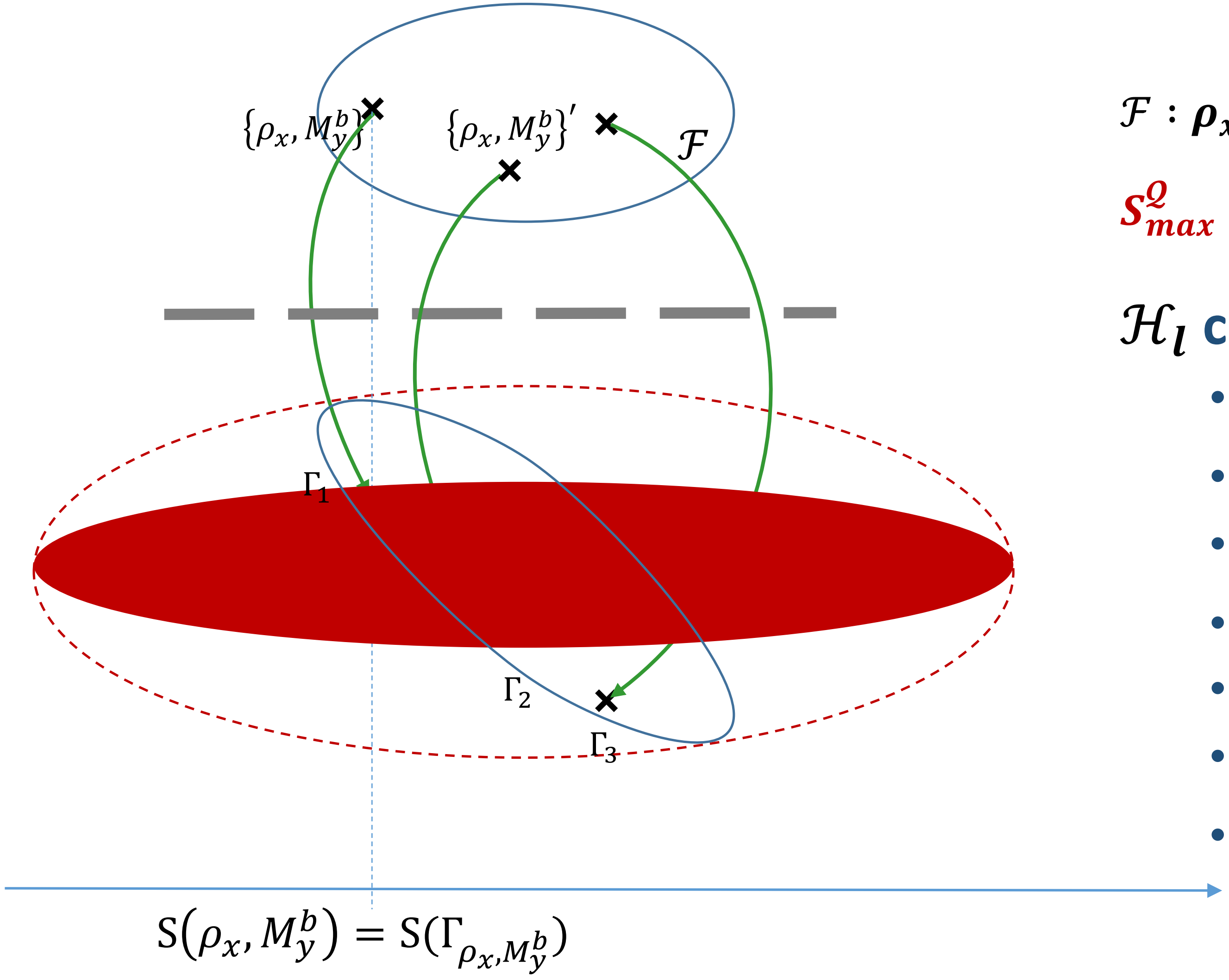
- Sample $\rho_x, M_y^b \in \mathcal{F}$, compute $\Gamma_1 = \Gamma_{\rho_x, M_y^b}$
- Let $\mathcal{L} = \mathrm{Span}(\Gamma_1)$
- Sample $\rho_x', M_y^{b\prime} \in \mathcal{F}$, compute $\Gamma_2 = \Gamma_{\rho_x', M_y^{b\prime}}$
- If $\Gamma_2 \in \mathcal{L}$: STOP
- If $\Gamma_2 \notin \mathcal{L}$, let $\mathcal{L} = \mathrm{Span}(\Gamma_1, \Gamma_2)$

M. Navascués, and T. Vértesi, Phys. Rev. Lett. 115:020501 (2015)

Sampling



$$S_{max}^{Q} = \max_{M_y^b, \rho_x \in \mathcal{F}} S(\{\rho_x, M_y^b\})$$

$\mathcal{F} : \rho_x, M_y^b \succcurlyeq 0$, **dimension d**, $\mathrm{Tr}(\rho_x) = 1, \forall y, \sum_b M_y^b = I_d$

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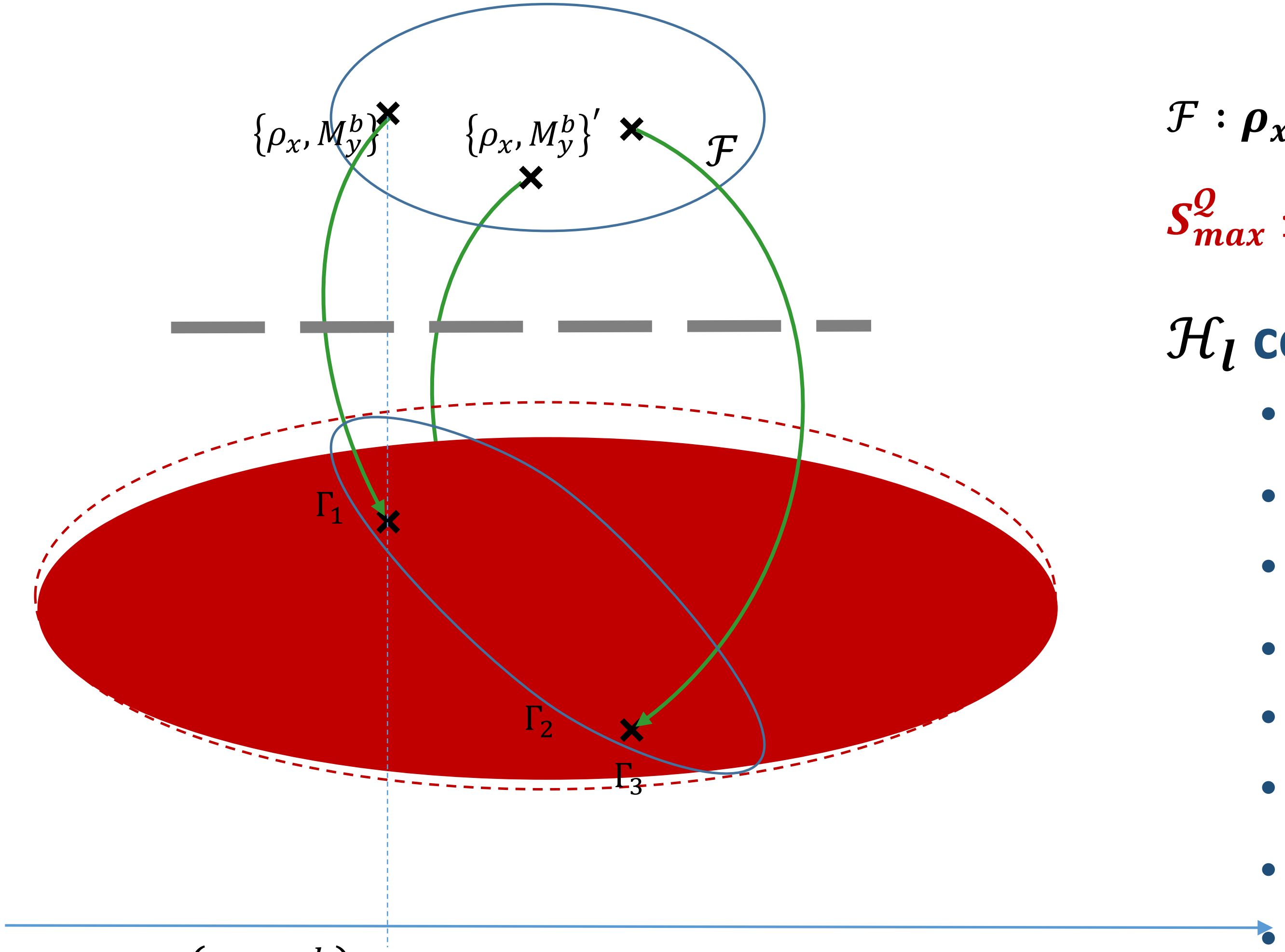
$\mathcal{H}_l$ construction

- Sample $\rho_x, M_y^b \in \mathcal{F}$, compute $\Gamma_1 = \Gamma_{\rho_x, M_y^b}$
- Let $\mathcal{L} = \mathbf{Span}(\Gamma_1)$
- Sample $\rho_x', M_y^{b\prime} \in \mathcal{F}$, compute $\Gamma_2 = \Gamma_{\rho_x', M_y^{b}{}'}$
- If $\Gamma_2 \in \mathcal{L}$: STOP
- If $\Gamma_2 \notin \mathcal{L}$, let $\mathcal{L} = \mathbf{Span}(\Gamma_1, \Gamma_2)$
- Sample $\rho_x'', M_y^{b\prime\prime} \in \mathcal{F}$, compute $\Gamma_3 = \Gamma_{\rho_x'', M_y^{b}{}''}$
- If $\Gamma_3 \in \mathcal{L}$: STOP

M. Navascués, and T. Vértesi, Phys. Rev. Lett. 115:020501 (2015)

Sampling



$S(\rho_x, M_y^b) = S(\Gamma_{\rho_x, M_y^b})$

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- Sample $\rho_x, M_y^b \in \mathcal{F}$, compute $\Gamma_1 = \Gamma_{\rho_x, M_y^b}$
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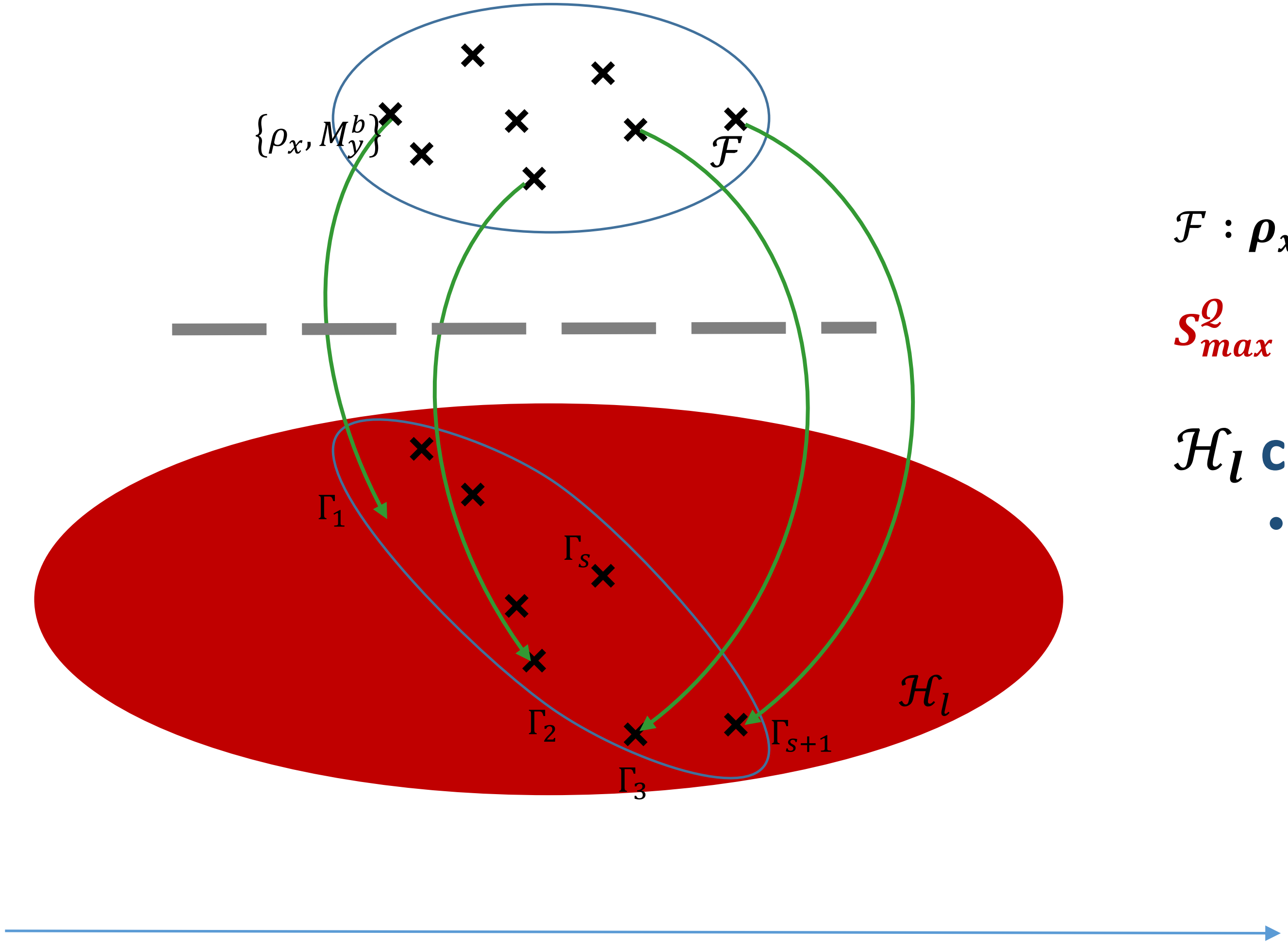
- If $\Gamma_3 \notin \mathcal{L}$, let $\mathcal{L} = \mathrm{Span}(\Gamma_1, \Gamma_2, \Gamma_3)$
- …

M. Navascués, and T. Vértesi, Phys. Rev. Lett. 115:020501 (2015)

Sampling



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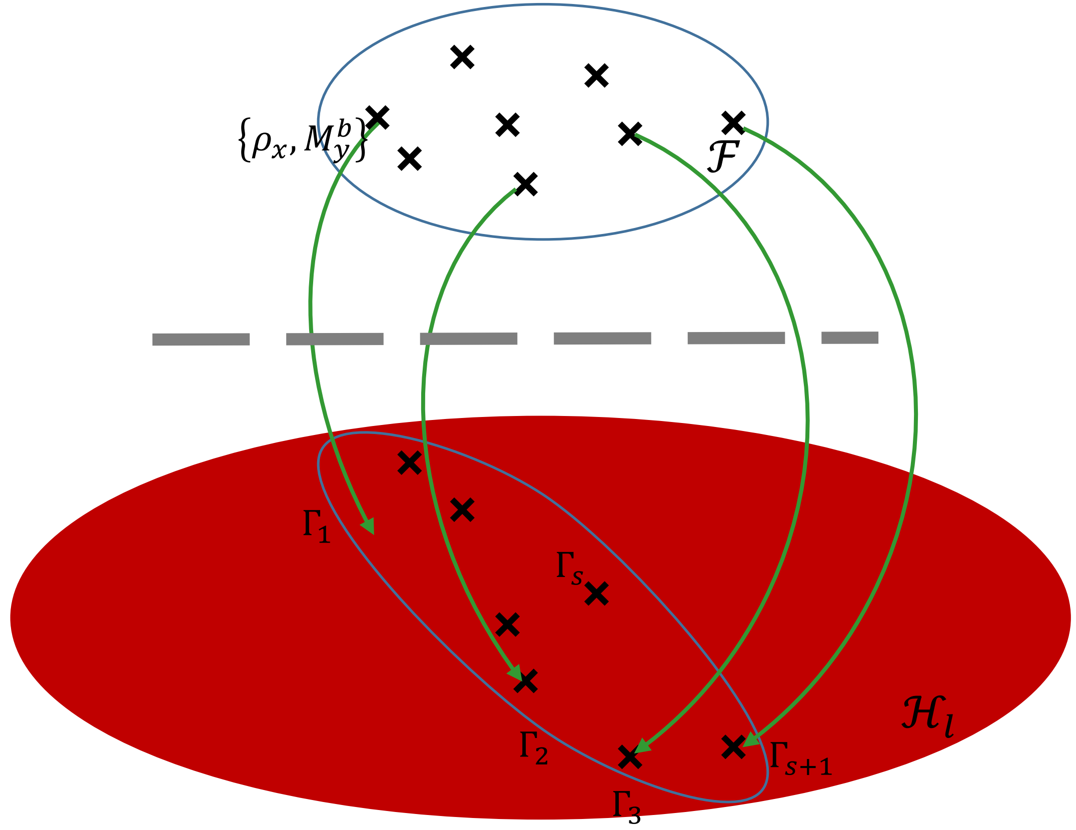
$\mathcal{H}_l$ construction

- After sampling $\Gamma_1 \ldots, \Gamma_{s+1}$:

  $\Gamma_{s+1} \in \mathrm{Span}(\Gamma_1, \ldots, \Gamma_s)$

M. Navascués, and T. Vértesi, Phys. Rev. Lett. 115:020501 (2015)

Sampling



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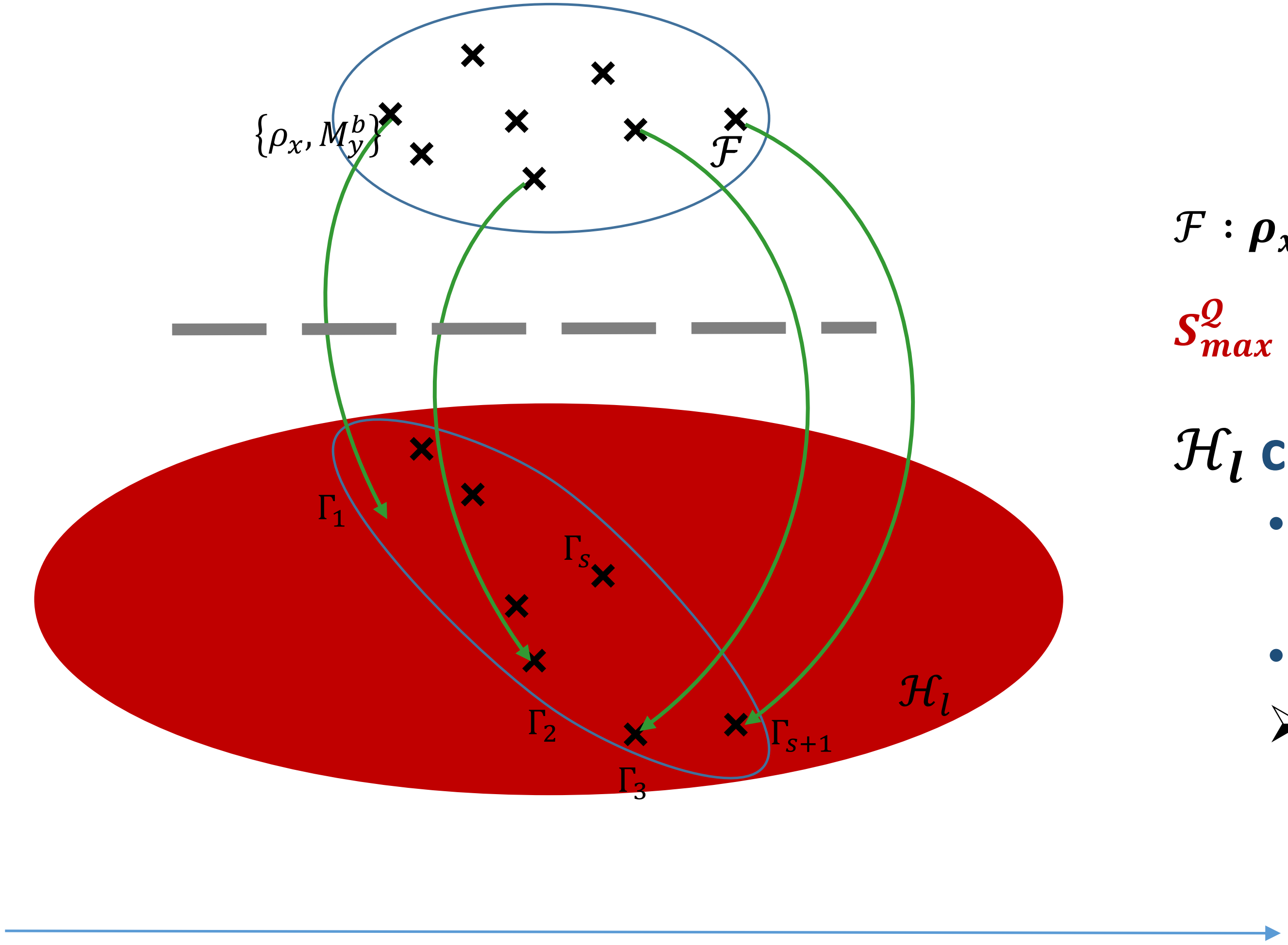
$\mathcal{H}_l$ construction

- After sampling $\Gamma_1 \ldots, \Gamma_{s+1}$:

  $\Gamma_{s+1} \in \mathrm{Span}(\Gamma_1, \ldots, \Gamma_s)$
- Must happen as $\dim(\Gamma) < \infty$

M. Navascués, and T. Vértesi, Phys. Rev. Lett. 115:020501 (2015)

Sampling



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M. Navascués, and T. Vértesi, Phys. Rev. Lett. 115:020501 (2015)

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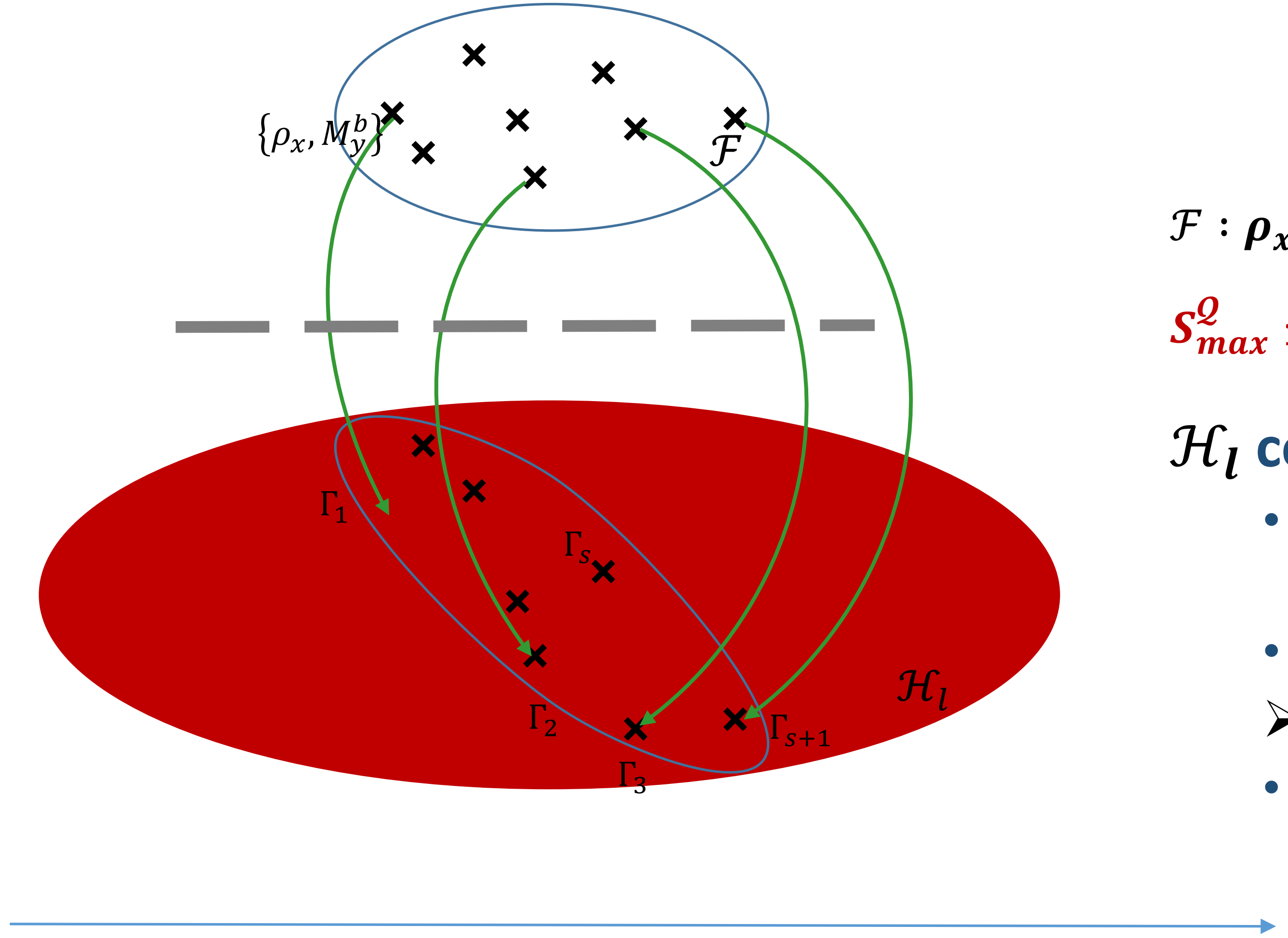

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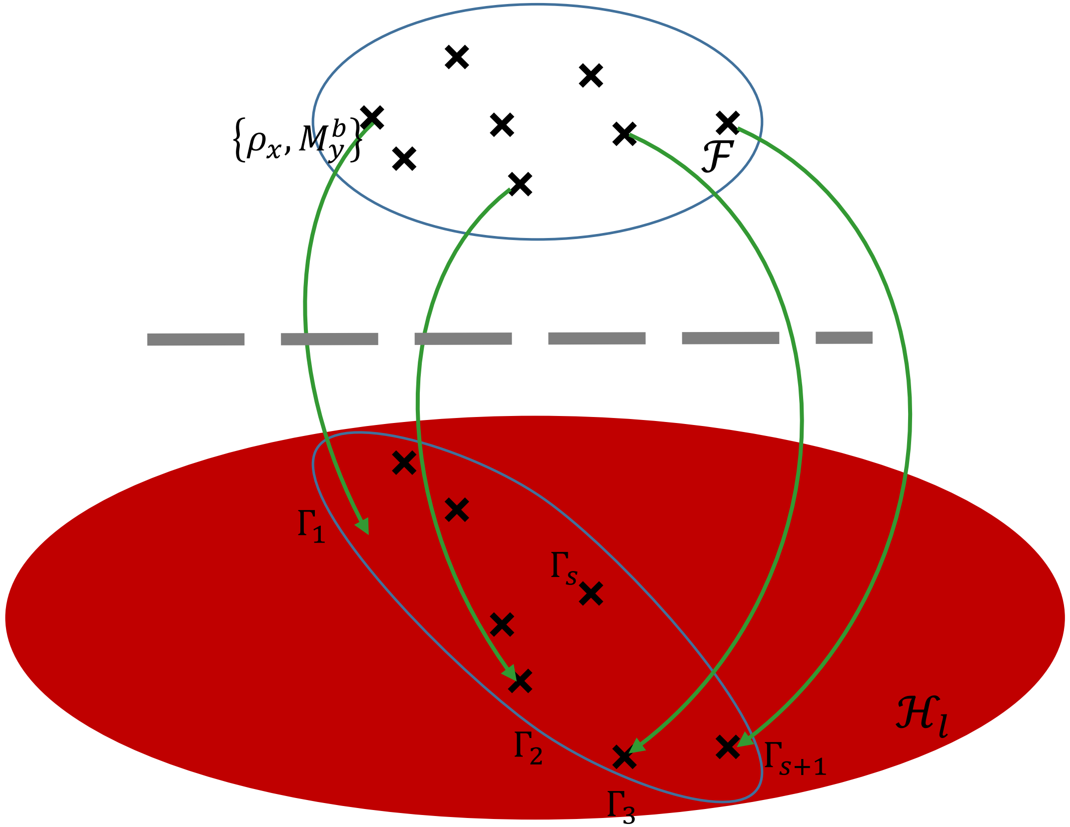
  $\Gamma_{s+1} \in \mathrm{Span}(\Gamma_1, \dots, \Gamma_s)$
- Must happen as $\dim(\Gamma) < \infty$
- ➢ $\mathcal{H}_l = \mathrm{Span}(\Gamma_1, \dots, \Gamma_s) \cap \{\Gamma \succcurlyeq 0\}$
- In practice not so obvious:
  - ➢ numerical errors

M. Navascués, and T. Vértesi, Phys. Rev. Lett. 115:020501 (2015)

Sampling



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  $\Gamma_{s+1} \in \mathrm{Span}(\Gamma_1, \dots, \Gamma_s)$
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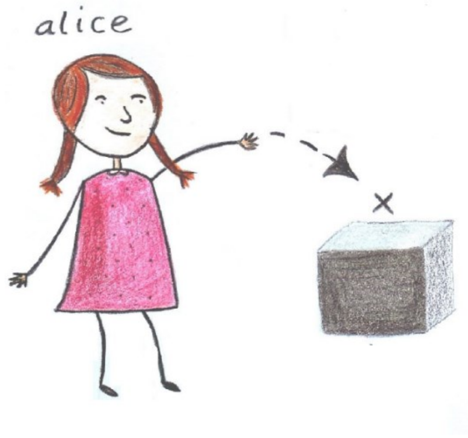
➢ $\mathcal{H}_l = \mathrm{Span}(\Gamma_1, \dots, \Gamma_s) \cap \{\Gamma \succcurlyeq 0\}$

- In practice not so obvious:
  - ➢ numerical errors
  - ➢ ‘unlucky’ sampling

M. Navascués, and T. Vértesi, Phys. Rev. Lett. 115:020501 (2015)

Quantum polynomial optimisation problems for dimension d variables, with symmetries

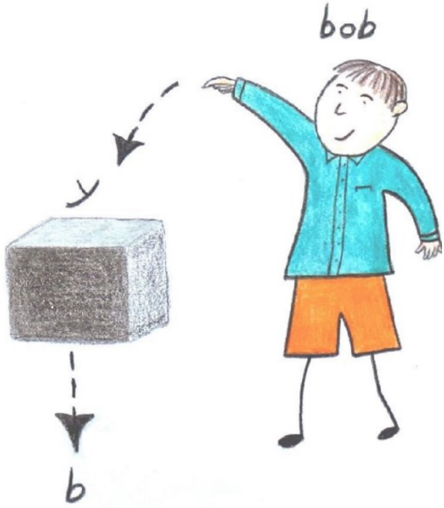


I. The Quantum Random Access code

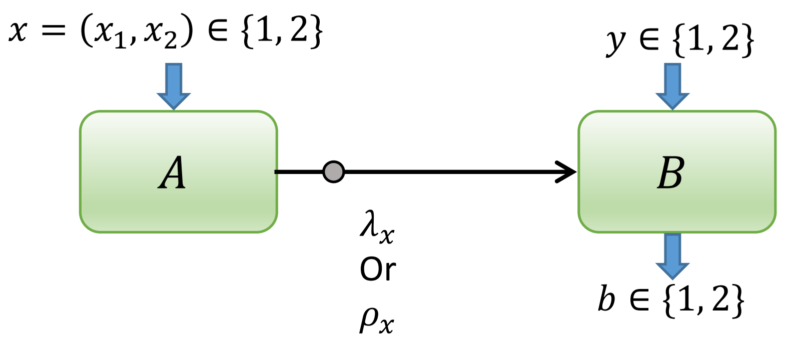
II. The NV relaxation

III. Sampling

IV. Symmetrisation of the NV hierarchy



NV Hierarchy Symetrisation



$$\boldsymbol{b = x_y}?$$

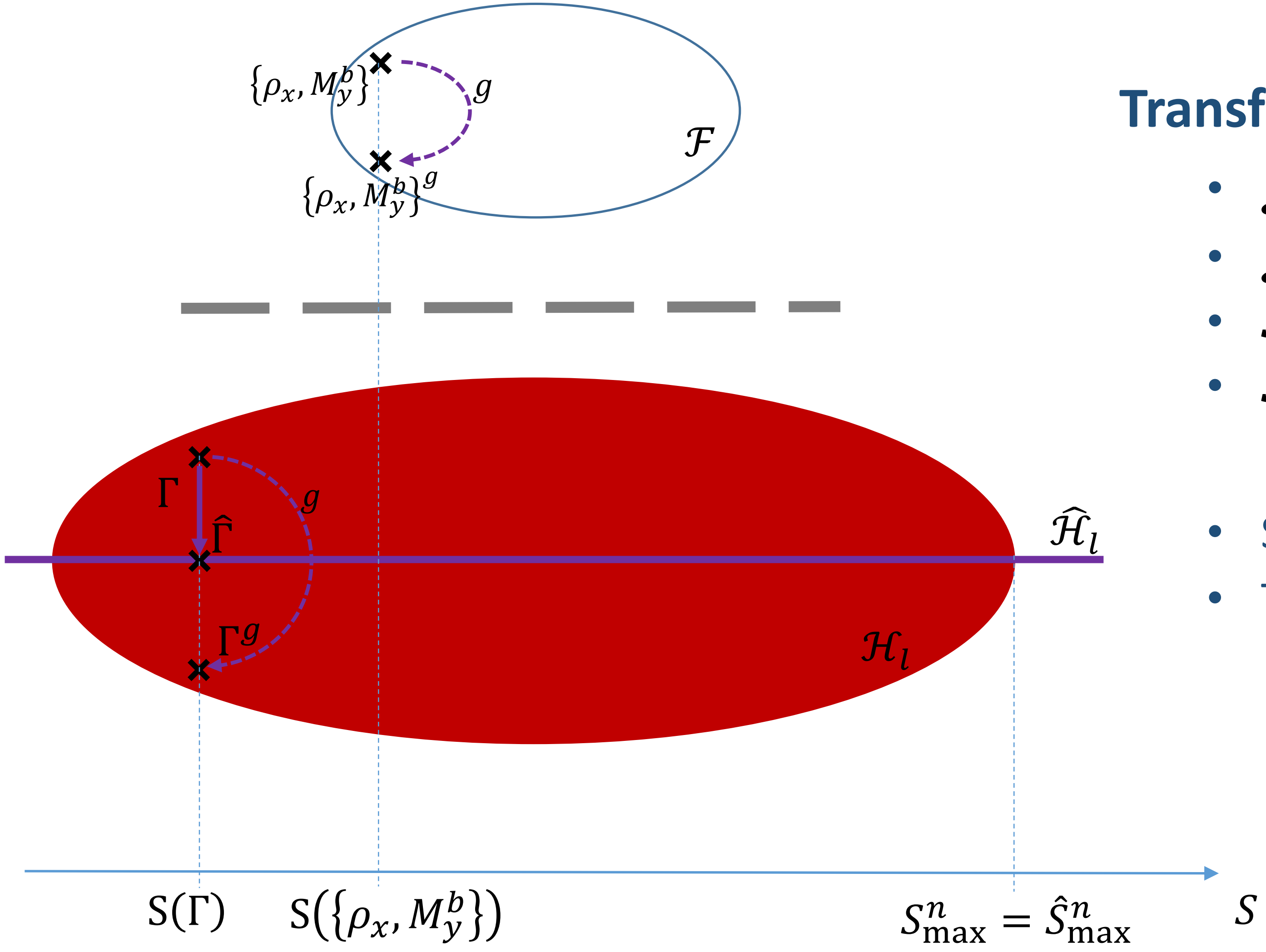
Group of relabelling of the game

- Game invariant by some transformations
  - $\boldsymbol{\sigma: x_1 \leftrightarrow x_2}$ and $\boldsymbol{y: 1 \leftrightarrow 2}$
  - $\boldsymbol{\tau: x_1: 1 \leftrightarrow 2}$ and $\boldsymbol{y = 1: (b: 1 \leftrightarrow 2)}$
  - …
- Form a group of symmetry $\boldsymbol{G}$

$$\boldsymbol{G = S_2 \wr S_2}$$

M.O. Renou, D. Rosset, A. Martin, and N. Gisin, JPA, 50(25):255301 (2017)

NV Hierarchy Symetrisation

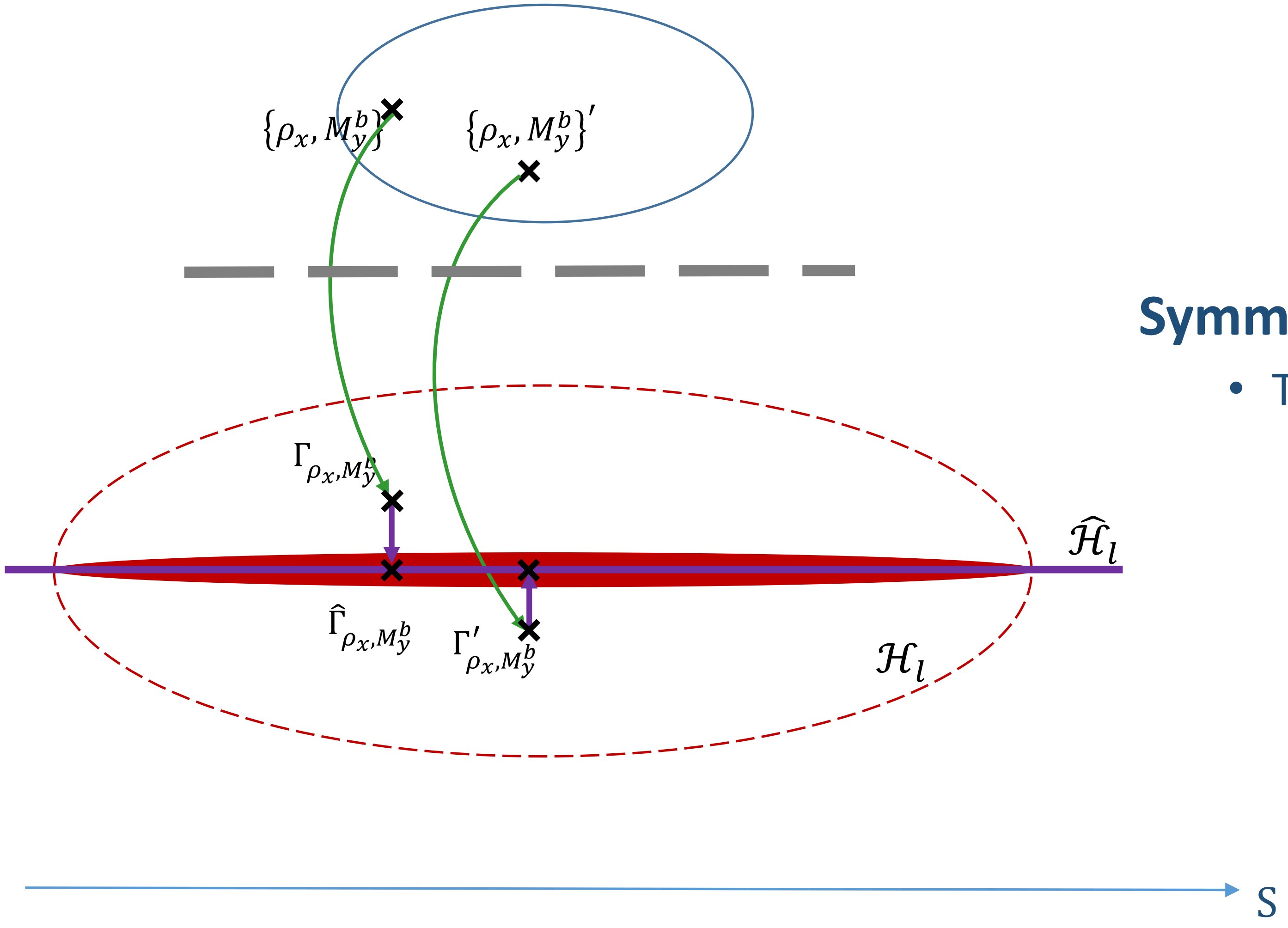


Transformed strategies

- $\boldsymbol{g} \in \boldsymbol{G}$ maps strategy $(\boldsymbol{\rho_x}, \boldsymbol{M_y^b})$ **to** $(\boldsymbol{\rho_x}, \boldsymbol{M_y^b})^g$
- $\boldsymbol{g} \in \boldsymbol{G}$ maps $\boldsymbol{\Gamma}$ to $\boldsymbol{\Gamma^g}$
- $\boldsymbol{S(\Gamma) = S(\Gamma^g)}$
- $\boldsymbol{S}$ is linear
  - Hence $\boldsymbol{S(\Gamma) = S(\hat{\Gamma}), \hat{\Gamma} = \frac{1}{|G|}\sum_g \Gamma^g}$
- Set $\boldsymbol{\hat{\mathcal{H}}_l = \{\hat{\Gamma}\}}$ much smaller than $\boldsymbol{\mathcal{H}_l}$
- The maximum is unchanged:
  - $\boldsymbol{S_{\max}^l = \max_{\hat{\Gamma} \in \hat{\mathcal{H}}_l} S(\hat{\Gamma})}$

K. Gatermann, and P.A. Parrilo, Journal of Pure and Applied Algebra 192, 95–128 (2004)

NV Hierarchy Symetrisation



Symmetrized sampling

- The sampled space $\hat{\mathcal{H}}_l$ is much smaller

NV Hierarchy Symetrisation

$$\Omega\hat{\Gamma}\Omega^{-1} = \begin{bmatrix} [\mathbb{1}_{d_1} \otimes \Gamma_1] & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \ddots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & [\mathbb{1}_{d_k} \otimes \Gamma_k] & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & [\mathbb{1}_{d_K} \otimes \Gamma_K] \end{bmatrix}$$

$$\boldsymbol{V} = (\boldsymbol{V}_1 \otimes \mathbb{C}^{m_1}) \oplus \cdots \oplus (\boldsymbol{V}_k \otimes \mathbb{C}^{m_k}) \oplus \cdots \oplus (\boldsymbol{V}_K \otimes \mathbb{C}^{m_K})$$

Symmetrized SDP

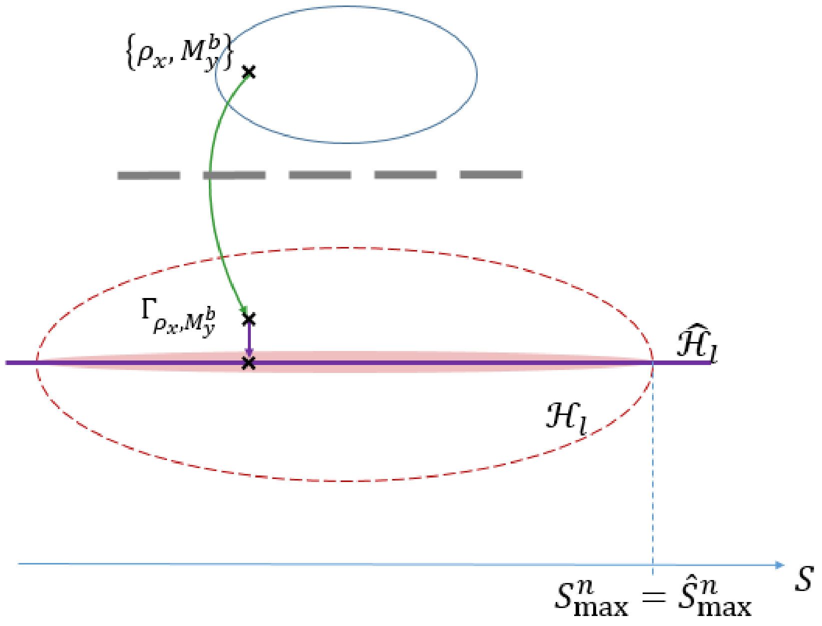
- $\widehat{\boldsymbol{\Gamma}}$ is invariant under $\boldsymbol{G}$

$$\hat{\boldsymbol{\Gamma}} = \boxplus_k \left(\mathrm{id}_{d_k} \otimes \boldsymbol{\Gamma}_\mathbf{k}\right)$$

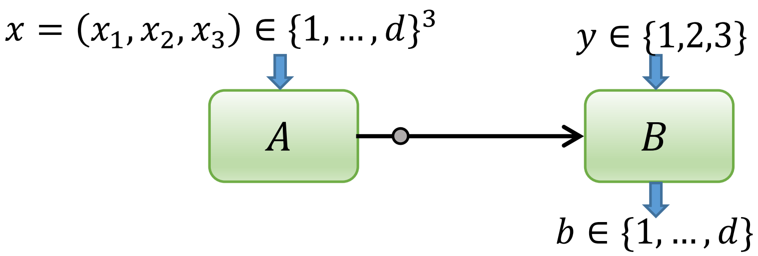
- The SDP $\boldsymbol{S}^n_{max} = \max_{\hat{\Gamma} \in \widehat{\mathcal{H}}_n} \boldsymbol{S}\left(\widehat{\boldsymbol{\Gamma}}\right)$ breaks into block SDPs

$$\left\{\max_{\Gamma_k \in \widehat{\mathcal{H}}_n^k} \boldsymbol{S}(\boldsymbol{\Gamma}_\mathbf{k})\right\}_k$$

NV Hierarchy Symetrisation



$$\Omega\hat{\Gamma}\Omega^{-1} = \begin{bmatrix} [\mathbb{1}_{d_1} \otimes \Gamma_1] & 0 & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & [\mathbb{1}_{d_k} \otimes \Gamma_k] & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & [\mathbb{1}_{d_K} \otimes \Gamma_K] \end{bmatrix}$$



$$\boldsymbol{b = x_y}?$$

	# Basis elements		SDP (+ blkdiag) time (sec)		
(n, d)	standard	sym	standard	sym	Result
(3,2)	224	28	11	2	0.7887
(3,3)	11380	82	$> 8.5 \times 10^4$	4	0.6989
(3,4)	-	82	-	15	0.6474
(3,5)	-	82	-	120	0.6131

Level of relaxation $\boldsymbol{l}$: $\mathbf{1 + AB}$. '-': unable to perform a computation.

A. Tavakoli, D. Rosset, and M.O. Renou, Phys. Rev. Lett. 121, 250507 (2018)

NV Hierarchy Symetrisation

srosset.github.io/qdimsum/

QDimSum

Q DIMSUM
Symmetric SDP relaxations for qudit systems

Symmetric SDP relaxations for qudits systems

View the Project on GitHub
denisrosset/qdimsum

Download ZIP File | Download TAR Ball | View On GitHub

QDimSum

This package was written by Denis Rosset, Armin Tavakoli and Marc-Olivier Renou.

It implements the algorithms described in

- A. Tavakoli, D. Rosset and M.-O. Renou, Enabling computation of correlation bounds for finite-dimensional quantum systems via symmetrisation, arXiv:1808.02412

and is based on the Navascués-Vértesi hierarchy described in

- M. Navascués, A. Feix, M. Araújo, and T. Vértesi, Characterizing finite-dimensional quantum behavior

We also mention the first use of symmetrisation applied to the Navascués-Vértesi hierarchy in the independent work:

- E. A. Aguilar, J. J. Borkała, P. Mironowicz, M. Pawłowski, Connections Between Mutually Unbiased Bases and Quantum Random Access Codes

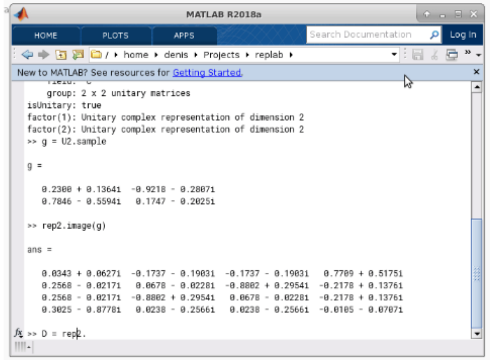
lab.github.io/replab/

s / Welcome to RepLAB!

Welcome to RepLAB!

Current version: 0.9.0 (GitHub / latest release ZIP / installation instructions).

RepLAB provides tools to study representations of finite groups and decompose them numerically. It is compatible with both MATLAB and Octave.



Decomposition of the $U \otimes U$ representation of the unitary group of dimension 2.

https://denisrosset.github.io/qdimsum/
https://replab.github.io/replab/

Open Questions

RAC game

- Why #basis elements saturates with $\boldsymbol{d}$?
  - A finite level relaxation fails to capture the complexity of increasing $\boldsymbol{d}$?
  - Hint that for fixed $\boldsymbol{n}$, increasing $\boldsymbol{d}$ does not add complexity?

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- Make it rigorous?
  - by quantifying the probability of numerical errors or ‘unlucky’ sampling?
  - **Question for mathematicians?**

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  - ➢ **Question for mathematicians?**

Open problem

- This could be useful for the ‘MUB’ open problem:

number of Mutually Unbiased Basis in dimension 6

  - ➢ Long standing mathematical open problem

P. Horodecki, L. Rudnicki, K. Życzkowski, Five open problems in theory of quantum information, arXiv:2002.03233 (2020)