Spatial-temporal modelling of temperature for pricing temperature index insurance

Che Mohd Imran Che Taib

Joint work with Mukminah Darus

Faculty of Ocean Engineering Technology and Informatics,
Universiti Malaysia Terengganu, 21030 Kuala Nerus, Terengganu, Malaysia

8th European Congress of Mathematics, 20 - 26 June 2021, Portorož, Slovenia

Introduction: Weather Risk and Weather Derivatives

- Weather risk plays a major role in weather-dependent industry such as agriculture, tourism and energy.
- Hedging such risk using the weather derivatives showing a success story since 1997 when the Chicago Mercantile Exchange (CME) began trading weather derivatives on the exchange.
- Weather contracts are settled agains an objectively measurable index heating-degree-days (HDD), cooling-degree-days (CDD) or cumulative average temperature (CAT),

$$\mathrm{HDD}(\tau_1,\tau_2) = \sum_{t=\tau_1}^{\tau_2} \max\left(c - T(t), 0\right),$$

$$\mathrm{CDD}(\tau_1,\tau_2) = \sum_{t=\tau_1}^{\tau_2} \max\left(T(t) - c, 0\right),$$

$$\mathrm{CAT}(\tau_1,\tau_2) = \sum_{t=\tau_1}^{\tau_2} T(t).$$

UNIVERSITI MALAYSIA TERENGGANU
UMT

Introduction: From weather derivatives to weather index insurance

- In many developing countries, the weather contracts are not available which makes hedging using weather derivatives is not possible. Weather index insurance has gradually become an interesting hedging tool and designed for households in developing countries.

- Premium of the insurance contract for CDD-type is given as (Taib and Benth [11])

$$P(t) = e^{-r(\tau_1 - t)} \mathbb{E}_Q \left[X(\tau_1, \tau_2) \mid \mathcal{F}_t \right], \tag{1}$$

  where $X(\tau_1, \tau_2) = k \times \mathrm{CDD}(\tau_1, \tau_2)$.

- The insurance contract is solely depends on the index observed at certain weather station. However, the farmers usually live in the suburban which is far from the place where the index is measured. The spatial dependency structure of temperature is important to assess the correlation risk since the local weather variations are not fully covered by the stations where contracts are traded.

Introduction

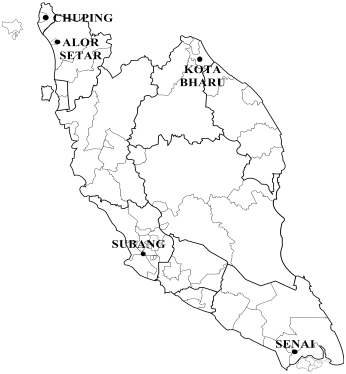

Figure: Map of peninsular Malaysia with five weather stations.

- The most problematic issue in designing temperature insurance contract is the prediction of temperature at any location far from the weather station. One of the methods to estimate weather at certain location is by using kriging.

Continuous model (Barth et al. [3])

- Denote $C(\mathcal{A})$ as space of real-valued continuous functions on some Borel subset $\mathcal{A} \subset \mathbb{R}^d$ and $L^2(\mathcal{A})$ is the space of square-integrable functions on $\mathcal{A}$ with respect to the Lebesgue measure. In addition, the space of continuous function on $\mathbb{R}_+ \times \mathcal{A}$ is denoted by $C^{1,0}(\mathbb{R}_+ \times \mathcal{A})$, which are continuously differentiable in the first variable. Suppose $\mathcal{D}$ is a compact domain in $\mathbb{R}^2$ with piecewise smooth boundary $\partial\mathcal{D}$, equipped with Euclidean metric and $C(\mathbb{R}_+ \times \mathcal{D})$ be the space of continuous function on $\mathbb{R}_+ \times \mathcal{D}$.
- Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, P)$ be a complete filtered probability space which satisfies the usual conditions. For $t \in \mathbb{R}_+$ and $x \in \mathcal{D}$, the spatial-temporal model for temperature $T(t,x)$ is given as

$$
\begin{aligned}
dT(t,x) = \left(\frac{\partial}{\partial t}\Lambda(t,x) - \alpha(t,x)(T(t,x) - \Lambda(t,x)) \right) dt \\
+ \sigma(t,x) dW(t,x) \,,
\end{aligned}
\tag{2}
$$

where $\alpha \in C(\mathbb{R}_+ \times \mathcal{D})$ and $\sigma \in C(\mathbb{R}_+ \times \mathcal{D})$ are the space-time speed of mean reversion and space-time volatility respectively.

Continuous model (Barth et al. [3])

- The $\Lambda \in C^{1,0}(\mathbb{R}_+ \times \mathcal{D})$ describes the seasonal mean function of the temperature in $\mathcal{D}$ and $W(t,x)$ is a centered Gaussian random field with covariance,

$$\mathrm{Cov}\left(W(t,x), W(s,y)\right) = \min(s,t)q(x,y), \; s,t \in \mathbb{R}_+, \, x,y \in \mathcal{D}. \tag{3}$$

- The covariance is modelled by a symmetric and strictly positive definite function $q \in C(\mathcal{D} \times \mathcal{D})$. It can also be represented as integral kernel of an operator $\mathcal{Q}$ which acting on $L^2(\mathcal{D})$. Following assumption on $\mathcal{D}$ and q, the $\mathcal{Q}$ is a symmetric Hilbert-Schmidt operator on $L^2(\mathcal{D})$ with a positive spectrum. We may show $\mathcal{Q}$ in terms of Mercer expansion by

$$\mathcal{Q} = \sum_{i=1}^{\infty} \lambda_i e_i \otimes e_i,$$

where $\{\lambda_i\}_{i\in\mathbb{N}}$ and $\{e_i\}_{i\in\mathbb{N}}$ are sequences of eigenvalues of $\mathcal{Q}$ and the associated set of eigenfunctions respectively.

Continuous model (Barth et al. [3])

- The eigenfunctions is $\mathcal{Q}e_i = \lambda_i e_i$ for $i \in \mathbb{N}$. Hence, the $W(t)$, for every $t \in \mathbb{R}_+$ given by

$$W(t) = \sum_{i=1}^{\infty} \sqrt{\lambda_i}\beta_i(t)e_i,$$

where $\{\beta_i\}_{i\in\mathbb{N}}$ is known as independent sequence of standard one-dimensional Brownian motion,

$$\beta(t) = \frac{1}{\sqrt{\lambda_i}}\left(W(t), e_i\right)_{L^2(\mathcal{D})}, i \in \mathbb{N}.$$

- The explicit solution of $T(\tau, x)$ given $T(t, x)$ for $\tau \geq t$ is

$$\begin{aligned} T(\tau, x) =& \Lambda(t, x) + (T(t, x) - \Lambda(t, x))e^{-\alpha(t,x)(\tau - t)} \\ &+ \int_t^{\tau} \sigma(s, x)e^{-\alpha(t,x)(\tau - t)}dW(s, x). \end{aligned} \tag{4}$$

Continuous model (Barth et al. [3])

- For a single location, the spatio-temporal model in Eq. (2) with constant mean reversion rate turn to

$$dT(t) = \frac{\partial}{\partial t}\Lambda(t) - \alpha\left(T(t) - \Lambda(t)\right)dt + \sigma(t)dW(t) \tag{5}$$

and its corresponding solution is

$$\begin{aligned} T(\tau) =& \Lambda(t) + \left(T(t) - \Lambda(t)\right)\mathrm{e}^{-\alpha(\tau - t)} \\ &+ \int_t^\tau \sigma(s)\mathrm{e}^{-\alpha(\tau - s)}dW(s). \end{aligned} \tag{6}$$

Discrete model (Šaltytė-Benth et al. [10])

- The model contains two main components namely the seasonal mean $\mu(t,x)$ which explains the trend and residual $\varepsilon(t,x)$ modelling the random fluctuations around the trend in both space and time respectively. In a simple form, the spatial-temporal model can be written as
$$T\left(t,x\right)=\mu(t,x)+\varepsilon\left(t,x\right). \tag{7}$$
where
$$\mu(t,x)=\Lambda\left(t,x\right)+\alpha\left(x\right)\left(T\left(t-1,x\right)-\Lambda\left(t-1,x\right)\right).$$
- The seasonal mean function at any given time $t\in[0,\infty)$ and location $x\in\mathcal{D}$ is determined by a deterministic seasonal function $\Lambda(t,x)$ of space and time and deseasonalized temperatures weighted by non-random mean reversion parameter $\alpha(x)$.

Discrete model (Šaltytė-Benth et al. [10])

- The residual component $\varepsilon(t,x)$ is simply a factorization of non-random seasonal function $\sigma(t,x)$ and stationary Gaussian spatial-temporal random field with zero mean $B(t,x)$,

$$\varepsilon(t,x) = \sigma(t,x)B(t,x).$$

- For any time $t \in [0,\infty)$ and $x \in \mathcal{D} \subset \mathbb{R}^2$, we assume that $\sigma(t,x)$ satisfies the following condition

$$\sigma(t,x) = \sigma(t+365,x).$$

- For all $x, y \in \mathcal{D}$, $B(t,\cdot)$ is assumed to be independent in time with spatial correlation function

$$\operatorname{corr}\{B(t,x), B(t,y)\} = q(x,y).$$

Here, the random field $\varepsilon(t,x)$ are uncorrelated in time but correlated in space.

Discrete model (Šaltytė-Benth et al. [10])

- Given $\Sigma_x(\theta)$ as diagonal variance matrix at the location $x \in \mathcal{D}$, then the covariance function of $\varepsilon(t,x)$ can be expressed as

$$\mathrm{Cov}(B(t,x), B(t,y); \theta) = q(x,y)\Sigma_x(\theta_t)\Sigma_y(\theta_t). \tag{8}$$

  The notation $\theta = (\theta_x; \theta_t)^{\mathsf{T}} \in \Theta$ refers to the $k \times 1$ parameter vector, where Θ is an open subset of R^k and T is transposing.

- We now discuss temporal model for certain temperature field at a single location (for example the weather station where temperatures data are recorded). The temperature at single location $i = 1, ..., n$ can be expressed as

$$T_i(t) = \mu_i(t) + \epsilon_i(t), \tag{9}$$

  where $\mu_i(t)$ and $\epsilon_i(t)$ are the mean and residual process at time $t = 1, ..., \tau$ for spatial location $s_i \in \mathcal{D}$ respectively.

Discrete model (Šaltytė-Benth et al. [10])

- The mean is represented as

$$\mu_i(t) = \Lambda_i(t) + \alpha_i \left(T_i \left(t - 1 \right) - \Lambda_i \left(t - 1 \right) \right), \tag{10}$$

and the seasonal function is expressed as

$$\Lambda_i(t) = a_0^i + a_1^i t + a_2^i \sin \left(\frac{2\pi(t - a_3^i)}{365} \right), \tag{11}$$

with constant a_0^i and a_1^i are describing the average level of temperature and slope of linear trend respectively. The other two parameters a_2^i and a_3^i represent amplitude of the mean and phase angle respectively.

Discrete model (Šaltytė-Benth et al. [10])

- Finally, the residual process $\varepsilon(t)$ is given as

$$\varepsilon_i(t) = \sigma_i(t) B_i(t) \tag{12}$$

where $\sigma_i(t)$ is a seasonal dependent standard deviation function and $B_i(t)$ is a zero-mean independent Gaussian random process.

- The seasonal variance function

$$\sigma_i^2(t) = c_0^i + \sum_{k=1}^{4} \left[c_k^i \cos\left(\frac{2k\pi t}{365}\right) + c_{k+1}^i \sin\left(\frac{2k\pi t}{365}\right) \right] \tag{13}$$

is used to explain the empirical seasonal variance observed in the data.

Model Fitting

- In Table 1, we summarize descriptive statistics of DATs. The temperatures are not deviate much for all stations with the lowest temperatures are between 22.80 to 23.20 and the highest temperatures are in the interval $[29.90, 31.70]$. A small standard deviation in all stations indicates the low volatility of temperatures variation.

Table: Discriptive statistics of DATs

Stations	Min	Max	Q1	Med	Q3	Mean	Std.
Alor Setar	23.20	31.40	27.00	27.70	28.50	27.70	1.14
Chuping	23.20	31.70	26.70	27.40	28.10	27.41	1.16
Kota Bharu	22.80	30.90	26.60	27.30	28.10	27.32	1.15
Senai	22.90	29.90	25.80	26.50	27.20	26.50	1.04
Subang	23.10	31.50	27.10	27.90	28.70	27.88	1.15

Pricing of temperature index insurance

- For a period of $[\tau_1, \tau_2]$, the policyholder at certain location x will receive indemnity equivalent to

$$X(\tau_1, \tau_2, x) = k \times \sum_{u=\tau_1}^{\tau_2} \max(T(\tau, x) - c, 0). \tag{14}$$

- Price of the temperature index insurance can be represented as

$$P(t, \tau_1, \tau_2, x) = \exp(-r(\tau_2 - t))\mathbb{E}[X(\tau_1, \tau_2, x) \,|\, \mathcal{F}_t]\,. \tag{15}$$

- The pricing procedure includes the prediction of temperature at certain location x, $T(t_0, x_0)$ using universal Kriging, and simulate the temperature for certain time $t \in [\tau_1, \tau_2]$ using (**??**). By setting $r = 0.05$, $c = 28$ and $k =$ RM50, we proceed with finding the expected claim size $\mathbb{E}[X|\mathcal{F}_t]$ and discount it to obtain the present value.

Pricing of temperature index insurance

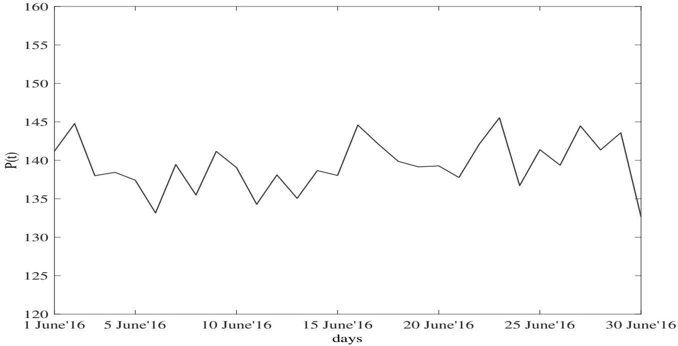

Figure: Evolution of the insurance price for contract month July 2016.

Conclusion

- The spatio-temporal temperature models able to predict the temperature at any location and time by using the information revealed at the closest stations as reference. The prediction enables us to calculate the premium of the insurance contract based on temperature index as proposed in weather derivatives.
- The advantage of considering the index based pricing is avoiding moral hazard problem. However, the index may not represents the real damage experienced by farmers. There is situation where farmers get paid without losing and there also farmers who did not get indemnity eventhough the crop damages.
- For the insurance company, the risk can be hedged using weather derivatives traded at, for example CME. However, there is no weather derivatives contract traded in developing countries. They may in fact use the so called geographical hedging for that reason, that is using other place which has a correlation to the country of origin.

Barnett, B. J., Barrett, C. B. and Skees, J. R. (2008). Poverty traps and index based risk transfer products. *World Development* **36**, pp. 1766–1785.

Barnett, B. J. and Mahul, O. (2007). Weather index insurance for agriculture and rural areas in lower income countries. *American Journal of Agricultural Economics* **89**, pp. 1241–1247.

Barth, A., Benth, F. E. and Potthoff, J. (2011). Hedging of Spatial Temperature Risk with Market-Traded Futures. *Applied Mathematical Finance*, **18**(2), pp. 93–117.

Benth, F. E., Kallsen, J. and Meyer-Brandis, T. (2007). A non-Gaussian Ornstein-Uhlenbeck process for electricity spot pice modelling and derivatives pricing. *Appl. Math. Finance*, **14**(2), pp. 153–169.

Benth, F. E., Klüppelberg, C., Müller, G. and Vos, L. (2011). Futures pricing in electricity markets based on stable CARMA spot models, *Submitted manuscript*, arXiv:1201.1151.

Universiti Malaysia Terengganu UMT

Benth, F. E. and Šaltytė Benth J. (2013), Modelling and Pricing in Financial Markets for Weather Derivatives, *World Scientific*, Vol. 17.

Benth, F. E., Šaltytė Benth J. and Koekebakker, S. (2008). *Stochastic Modelling of Electricity and Related Markets*. World Scientific.

Collier, B., Skees, J. R. and Barnett, B. J. (2009). Weather index insurance and climate change: opportunities and challenges in lower income countries. *Geneva Papers on Risk and Insurance*, Issues and Practice **34**, pp. 401–424.

Gundogdu, K. S. and Guney, I., Spatial Analyses of Groundwater Levels Using Universal Kriging (2007), *Journal of Earth System Science*, **116**(1), pp. 49–55.

Šaltytė Benth J., Benth, F. E. and Jalinkas, P. (2007). A Spatial-temporal Model for Temperature with Seasonal Variance. *Journal of Applied Statistics*, **34**(7), pp. 823–841.

- Taib, C.M.I.C. and Benth, F. E. (2012). Pricing of temperature index insurance. *Rev. of Dev. Finance*, 2, pp. 2231.
- Kambhammettu, B. V. N. P., Allena, P. and King, J. P. (2011), Application and Evaluation of Universal Kriging for Optimal Contouring of Groundwater Levels, *J. Earth Syst. Sci*, **120**, pp. 413–422.
- Mandallaz, D. (2000), Estimation of the Spatial Covariance in Universal Kriging: Application to Forest Inventory, *Environmental and Ecological Statistics*, 7, pp. 263–284.