

(EYLENBERG-MOORE, KLEISLI)
(AND DESCENT FACTORIZATIONS)

FERNANDO LUCATELLI NUNES

UTRECHT UNIVERSITY

8th European Congress of Mathematics

Portoroz, Slovenia

22nd JUNE / 2021

- Fundamental aspects

low/higher dimensional categorical structures

(Universal Algebra
Monads / Monadicity
Lawvere Theories
•••)

- Computer Sciences

(Programming Languages
Software correctness
Algebraic Effects
•••)

- Fundamental aspects
 - low/higher dimensional categorical structures

→ Monads / Monadicity

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- low/higher dimensional categorical structures

→ Monads / Monadicity

* Lucatelli Nunes,

Semantic Factorization and Descent

arXiv, 02/2019

(To appear in TAC)

- Aim of the talk:

- Sketch one of the consequences in the context of CAT.

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- Aim of the talk:

- Sketch one of the consequences in the context of CAT.

Categories, functors and natural transformations.

* Lucatelli Nunes,

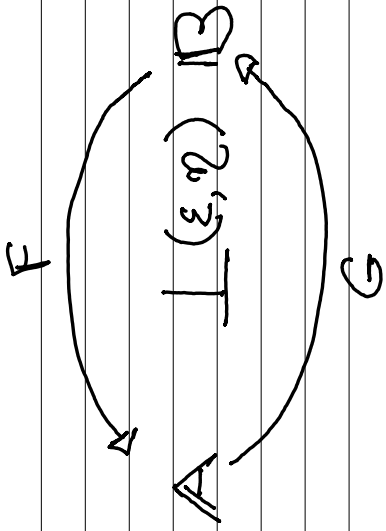
Semantic Factorization and Descent

orXiv, 02/2019

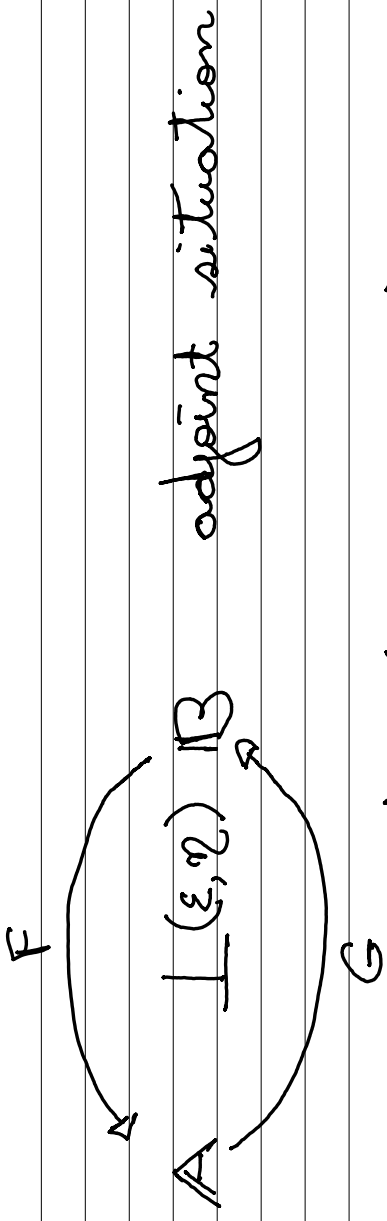
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•

$$A \xrightarrow{G} B \text{ functor}$$



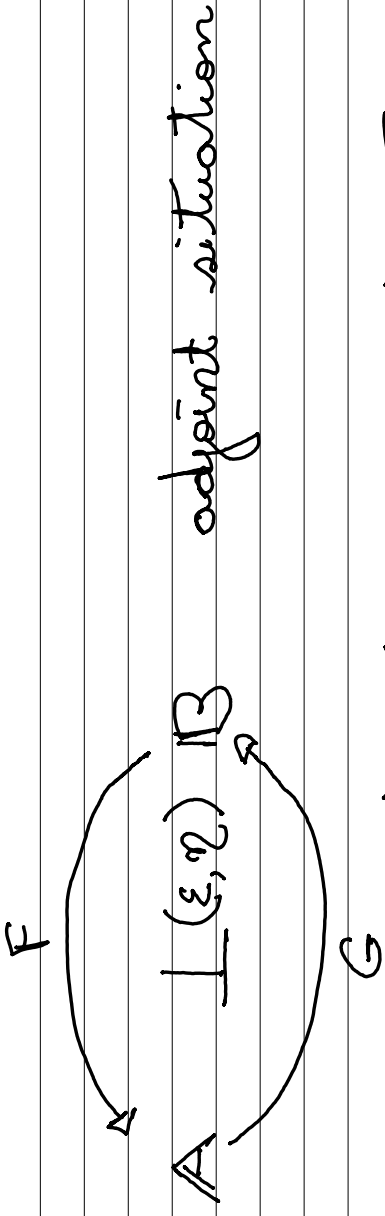
adjoint situation



\rightarrow monad $(GF, G\varepsilon F, \eta)$

\rightarrow comonad $(FG, F\eta G, \varepsilon)$

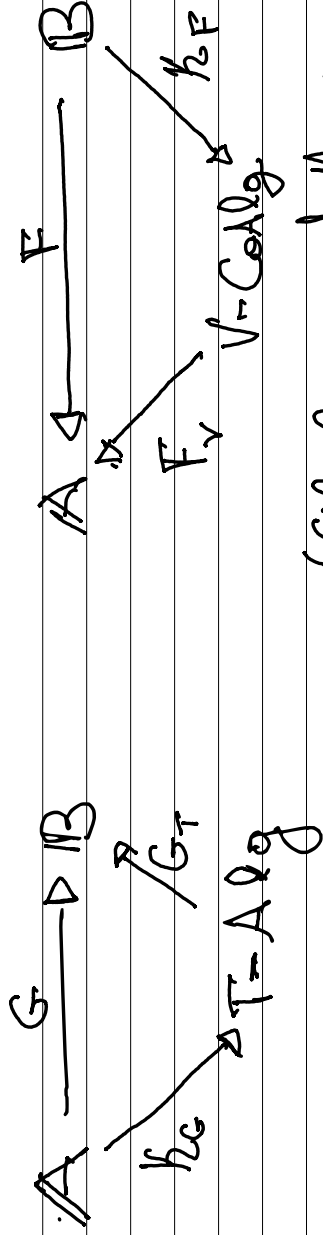
(P.S. Huber, 6f)



→ monad $(GF, GεF, η) = T$

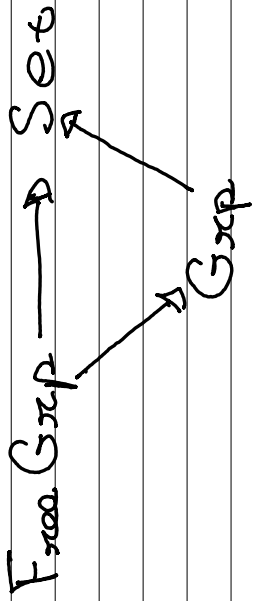
→ comonad $(FG, FηG, ε) = V$

• Eilenberg - Moore Factorizations :

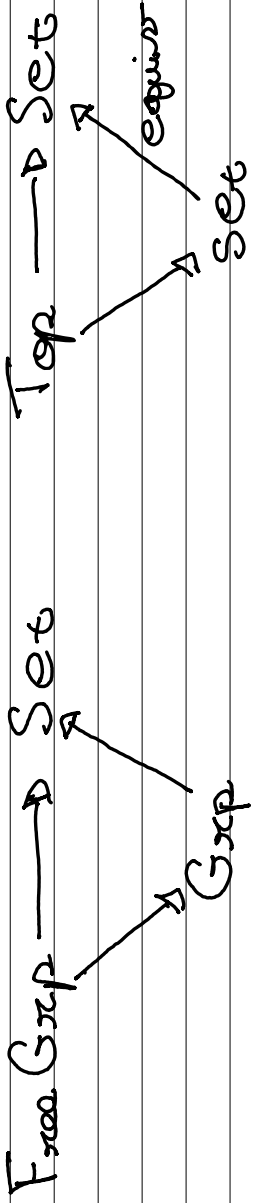


(Eilenberg and Moore, 1965)

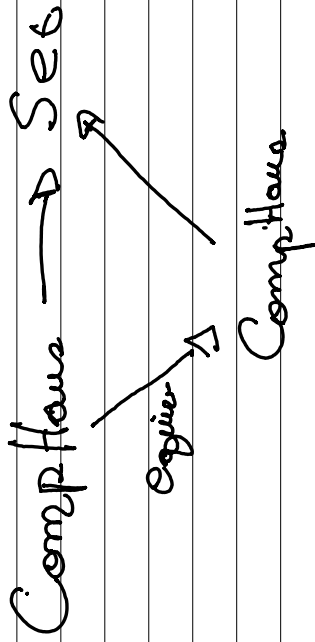
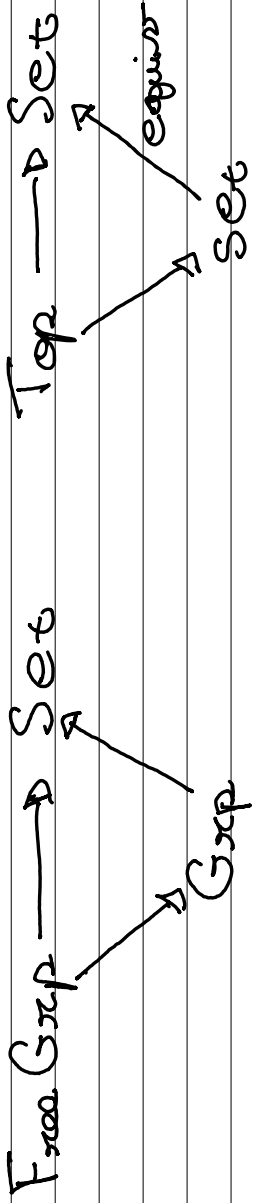
- Examples (Eilenberg - Moore Factorizations)



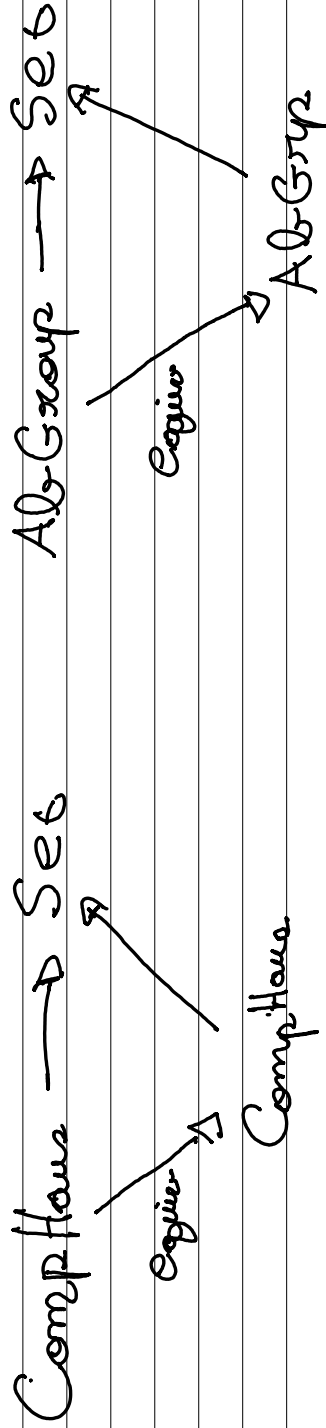
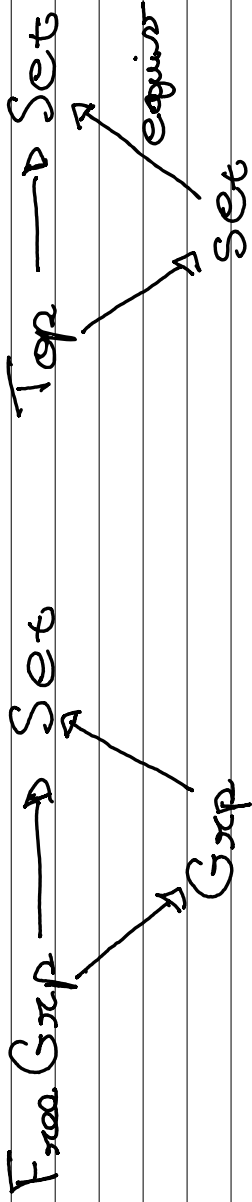
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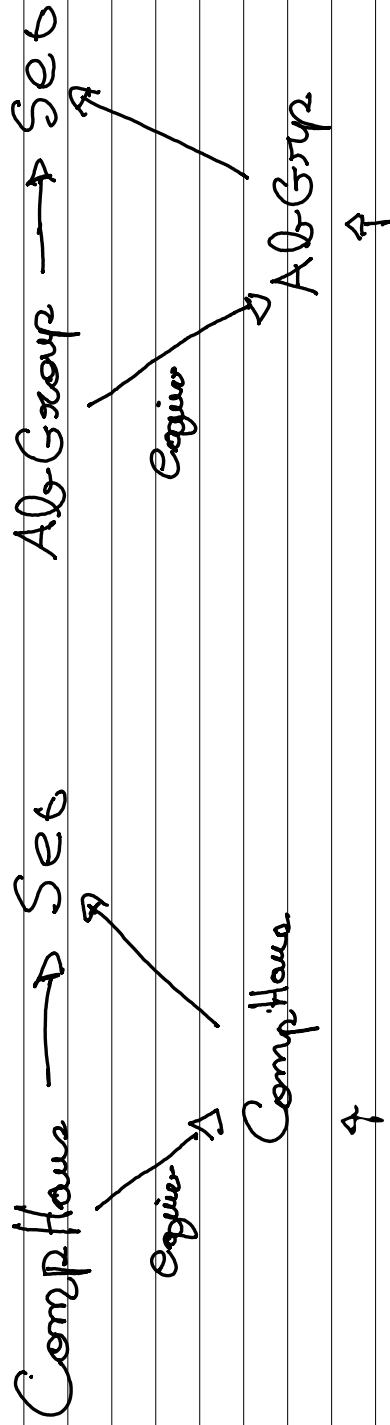
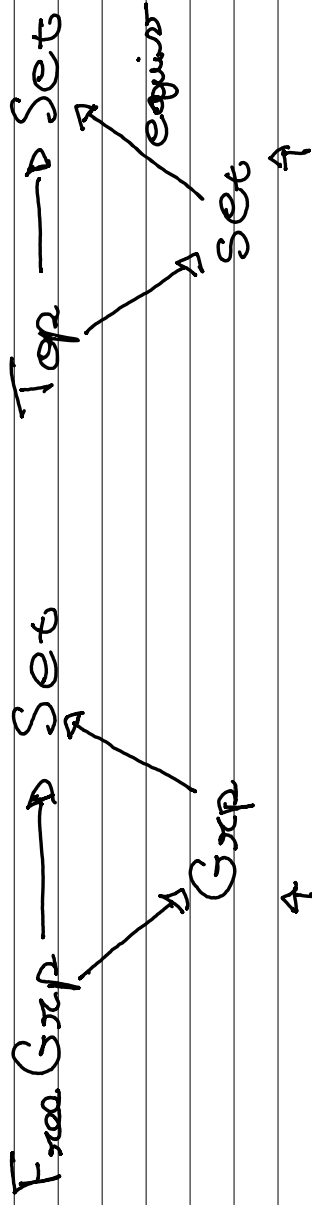
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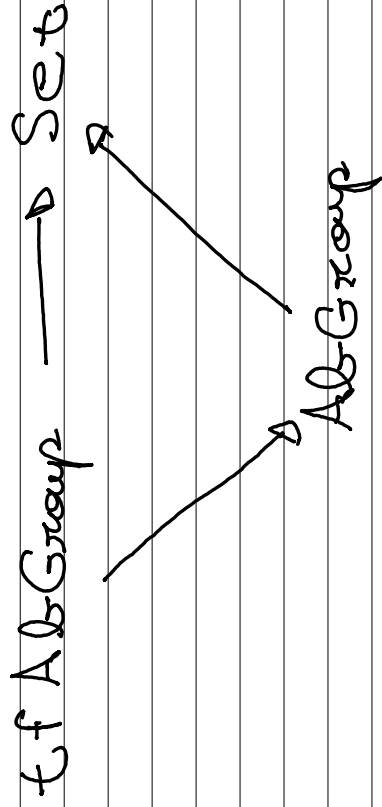
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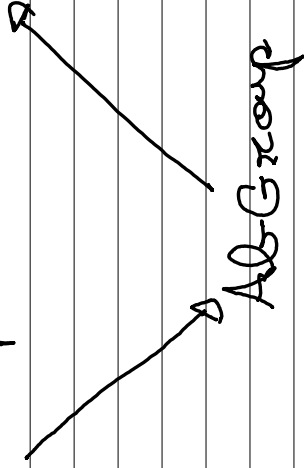


- Further Examples (Eilenberg - Moore Factorizations)



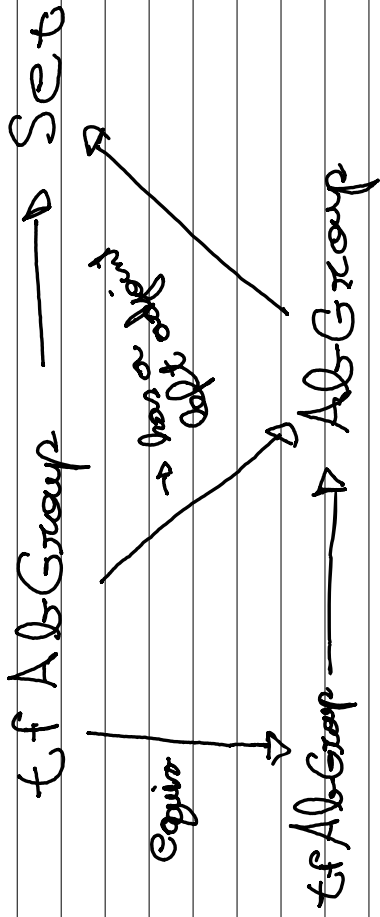
• Further Examples (Eilenberg - Moore Factorizations)

$\text{AbGroup} \rightarrow \text{Set}$

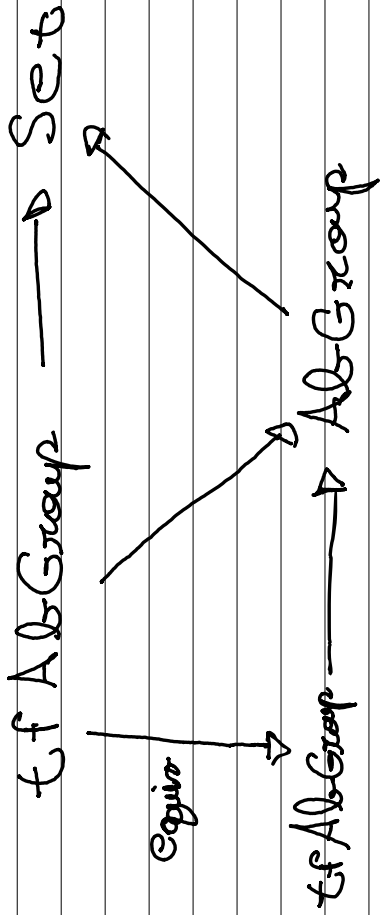


\rightarrow not monadic

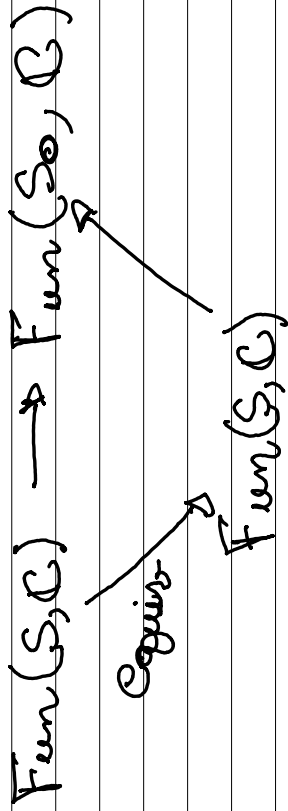
• Further Examples (Eilenberg - Moore Fortsetzungen)



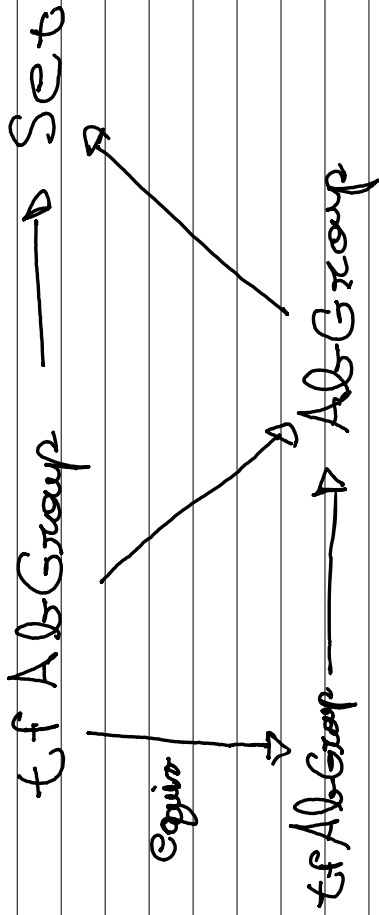
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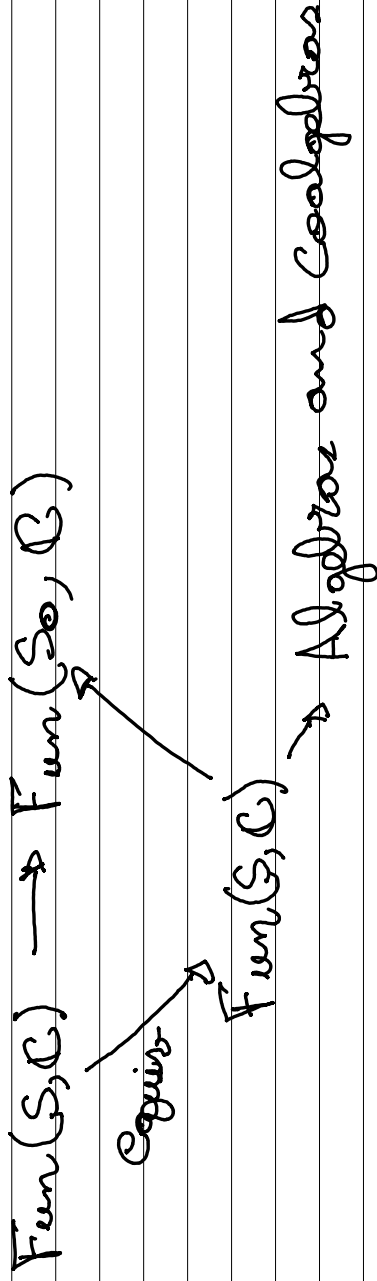
Last example: $S \rightarrow$ small category
 $C \rightarrow$ cocomplete and complete



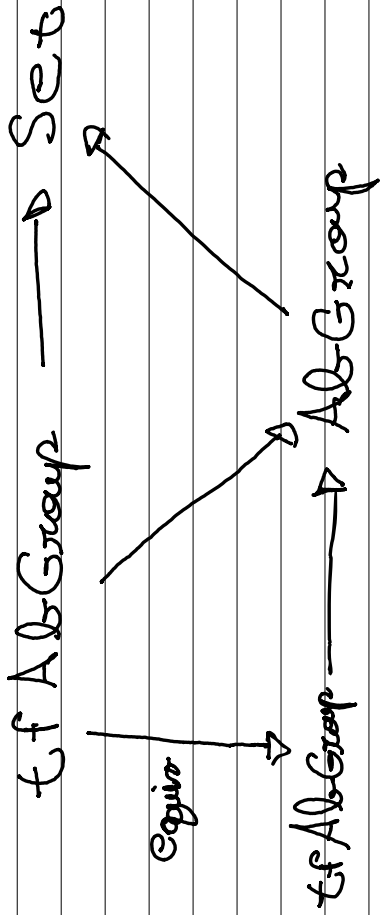
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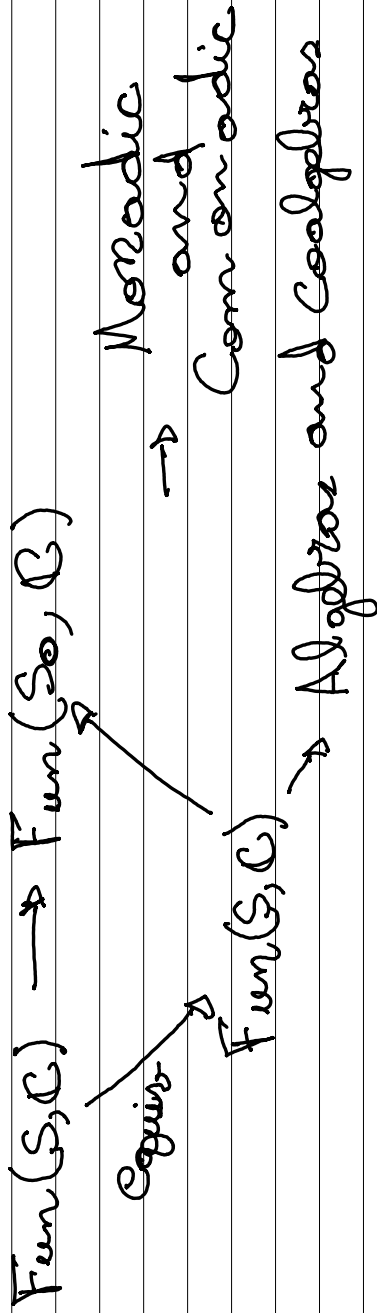
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- Further Examples (Eilenberg - Moore Factorizations)



Last example: $S \rightarrow$ small category
 $C \rightarrow$ cocomplete and complete



- Getting back to the general setting:

$$A \xrightarrow{G} B$$

- General setting:

$$A \xrightarrow{G} B$$

does NOT have Eilenberg -
Moore factorization in
general.

• General setting:

$$A \xrightarrow{G} B$$

does NOT have Eilenberg -
Moore factorization in
general.

(We need at least to assume
that G has codensity monad)

- General setting: cobordism pairs pushout of G

$A \xrightarrow{G} B$

$\xrightarrow{\quad} B \amalg_G B$

$\xrightarrow{\quad} G$

along G in CAT

- General setting: cokernel pair of G \triangleright pushout of G along G

$$A \xrightarrow{G} B \rightrightarrows B \amalg_G B$$

$$\downarrow \quad \downarrow$$

$$E \quad \downarrow$$

$$\downarrow \text{ in Cat}$$

- General setting: cokernel pair of G pushout of G along G

$$A \xrightarrow{G} B \rightrightarrows B \amalg_G B$$

$$\downarrow \quad \searrow$$

$$Eg$$

\downarrow in CAT

But we won't talk about this factorization!

• General setting: Instead, we take into consideration the 2-dimensional structure of CAT.

$$A \xrightarrow{G} B \rightrightarrows B \xrightarrow{G} B \rightrightarrows B \xrightarrow{G} B$$

2-dimensional
cokernel
diagram

- General setting:

$$A \xrightarrow{G} B \rightrightarrows B \xrightarrow{G} B \rightrightarrows B \xrightarrow{G} B$$

opcosmos category
of G along G .

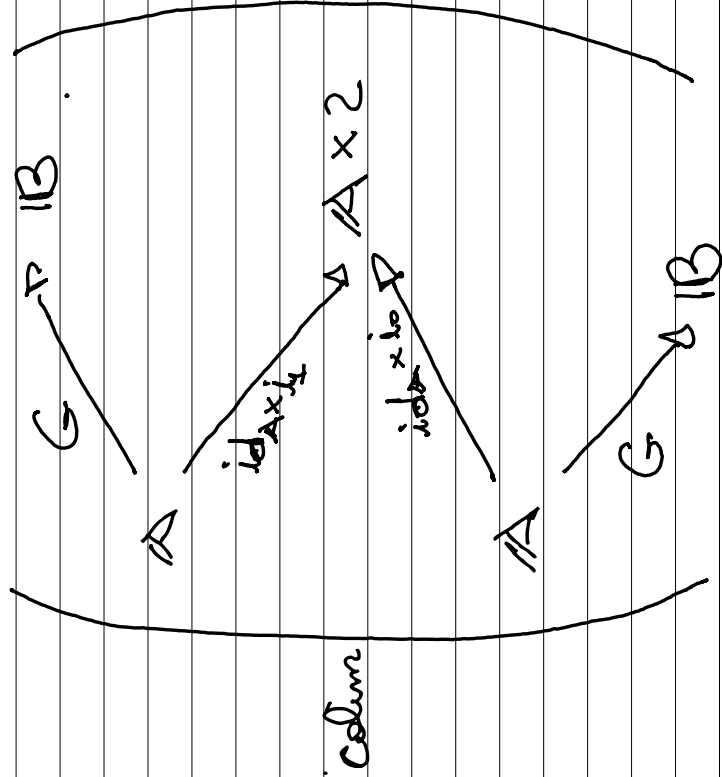
• General setting:

$$A \xrightarrow{G} B \xrightarrow{G} B \xrightarrow{G} B \xrightarrow{G} B \xrightarrow{G} B \xrightarrow{G} B \xrightarrow{G} B \xrightarrow{G} B$$

under

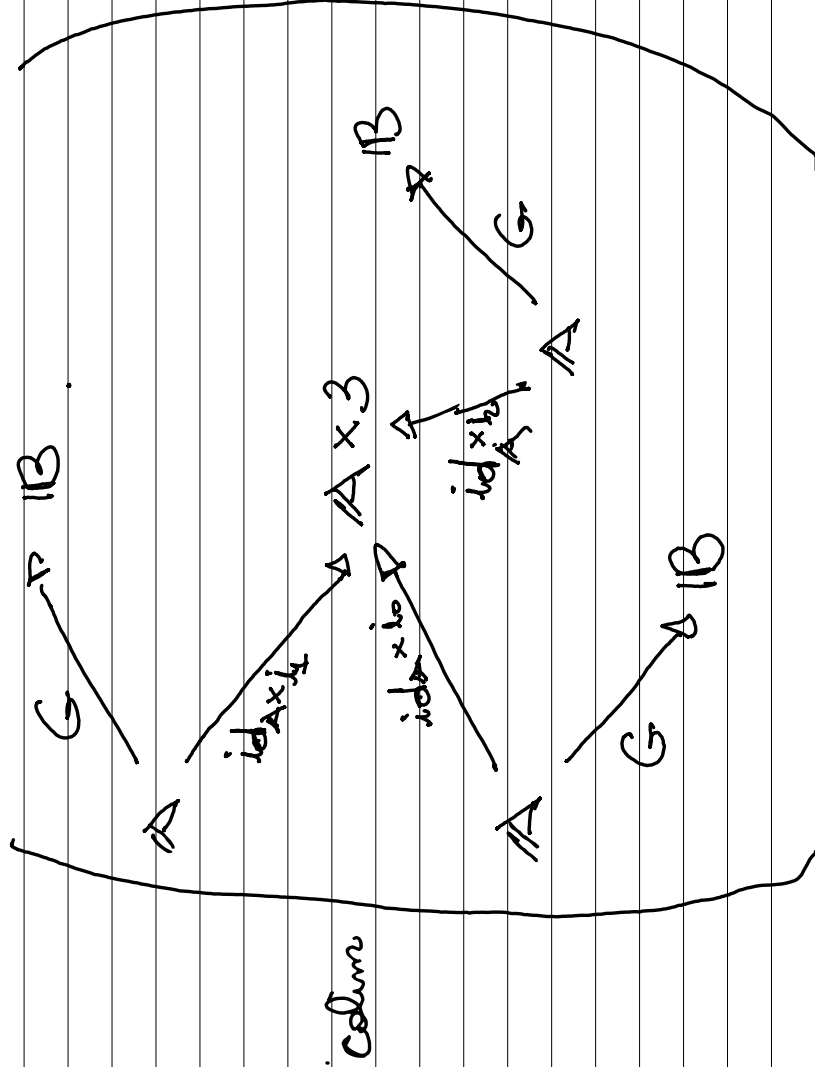
opcosmo category
of G along G .

$$\approx B \xrightarrow{G} B$$



• General setting:

$$A \xrightarrow{G} B \xrightarrow{G} B \xrightarrow{G} B \xrightarrow{G} B \xrightarrow{G} B$$



$$\cong B \xrightarrow{G} B \xrightarrow{G} B$$

$$2 \uparrow$$

$$3 = 1 \uparrow$$

$$0$$

• General setting: 2-dimensional colored diagram of G

$$A \xrightarrow{G} B \xrightarrow{B \uparrow G} B \xrightarrow{B \uparrow G} B \xrightarrow{B \uparrow G} B$$

• General setting:

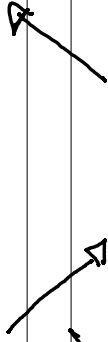
$$A \xrightarrow{G} B \rightrightarrows B \xrightarrow{G} B \rightrightarrows B \xrightarrow{G} B$$

Descent

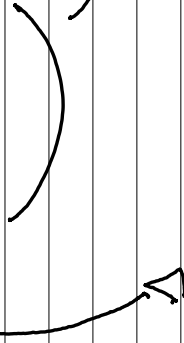
→ 2-dimensional
limit / category of
descent data

• General setting: \mathcal{H}_G

$$A \xrightarrow{G} B \rightrightarrows B \xrightarrow{G} B \rightrightarrows B \xrightarrow{G} B \rightrightarrows B \xrightarrow{G} B$$



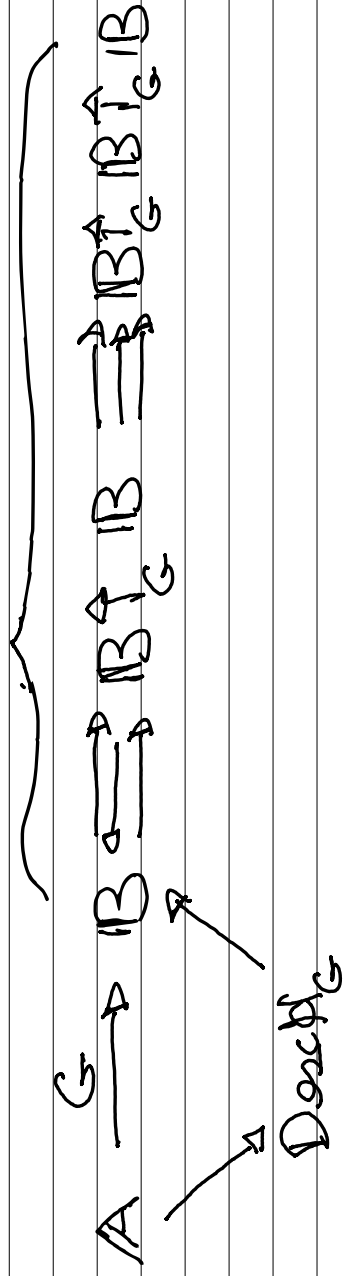
Descent



→ 2-dimensional
limit / category of
low descent

Universal
Properties

• General setting:



"Semantic descent factorization" of G

• General setting:

$$A \xrightarrow{G} B \xrightarrow{D} B \xrightarrow{G} B \xrightarrow{B} B \xrightarrow{G} B \xrightarrow{B} B$$

Desc H_G

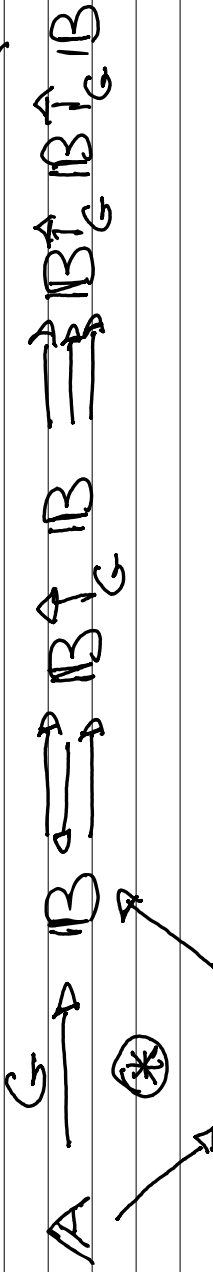
"Semantic descent factorization" of G

We should emphasize that

EVERY functor G has

the semantic descent factorization!

• General setting: \mathcal{H}_G



"Semantic descent factorization" of G

• THEOREM (LUATELLI NUNES, 2019)

G right adjoint $\implies D \circledast$ coincides w. $A \xrightarrow{G} B$

Eilenberg
more \rightarrow

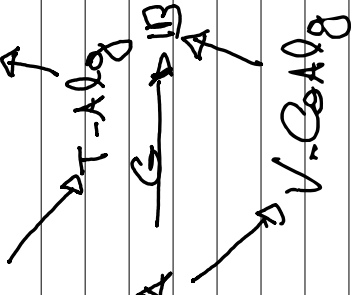
G left adjoint $\implies D \circledast$ coincides w. $A \xrightarrow{G} B$

\rightarrow V-Coalg

⊗ Semantic descent factorization

THEOREM (LURATI 2019)

G right adjoint $\iff D \otimes$ coincides w. $A \xrightarrow{G} B$



G left adjoint $\iff D \otimes$ coincides w. $A \xrightarrow{G} B$

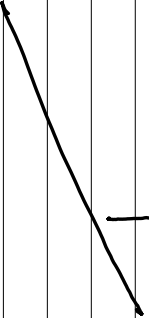
Message!

Semantic descent factorization

(of any functor)

is a (natural) generalization of the

Eilenberg-Moore Factorizations



Thank you

for your attention!