Convexity properties of the total isoperimetric profile

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The isoperimetric profile

Given an open, bounded set $\Omega \subset \mathbb{R}^n$, the isoperimetric profile $\mathcal{J}$ is

$$\mathcal{J} : V \mapsto \inf\{P(E) \,:\, E \subset \Omega\,,\, |E| = V\}$$

where $|\cdot|$ is the usual n-dimensional Lebesgue measure $\mathcal{L}^n(\cdot)$ and $P(\cdot)$ the variational perimeter (also known as Caccioppoli/De Giorgi perimeter).

Theorem (Leonardi, S. - forthcoming)

Let $\Omega \subset \mathbb{R}^2$ be in the class $\mathcal{A}$ ($\supset \mathcal{K}^2$), and let $R := \mathrm{inr}(\Omega)$. Then,

$$\mathcal{J}(V) \text{ is concave on } [0, \pi R^2]\,,$$
$$\mathcal{J}(V) \text{ is convex on } [\pi R^2, |\Omega|]\,.$$

Moreover, $\mathcal{J}^2(V)$ is globally convex.

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Total and relative isoperimetric profiles

An important result [Kuwert (2003)] states that the square of the isoperimetric profile is concave (for $\Omega \in \mathcal{K}^2$). We claim it is convex.
What's the deal? Fact is, there are two different isoperimetric profiles.

The relative one

$$\mathcal{J}_{\mathrm{rel}}(V) = \inf_{\substack{E \subset \Omega \\ |E| = V}} \{ P(E; \Omega) \}$$

- $\mathcal{J}_{\mathrm{rel}}^2$ is concave;
- ∂E_m touches $\partial \Omega$ orthogonally;

The total one

$$\mathcal{J}_{\mathrm{tot}}(V) = \inf_{\substack{E \subset \Omega \\ |E| = V}} \{ P(E) \}$$

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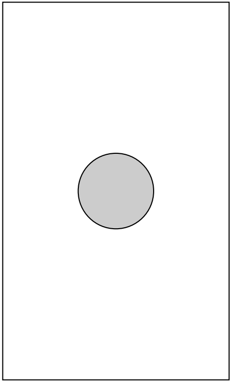

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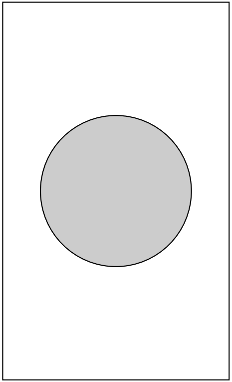

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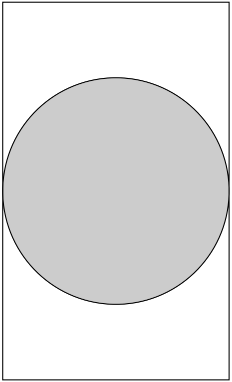

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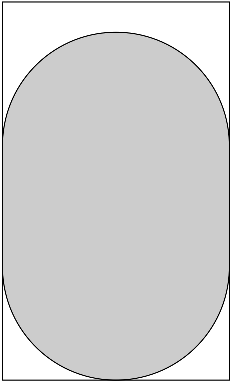

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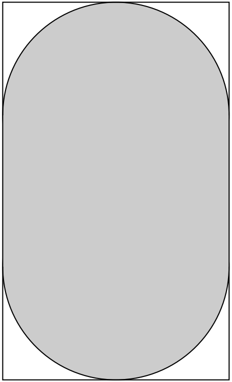

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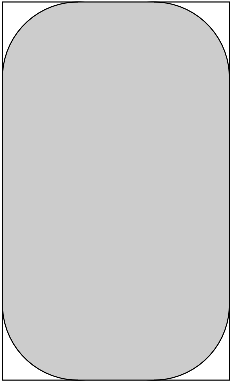

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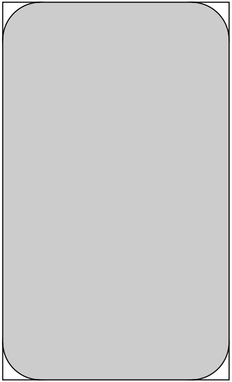

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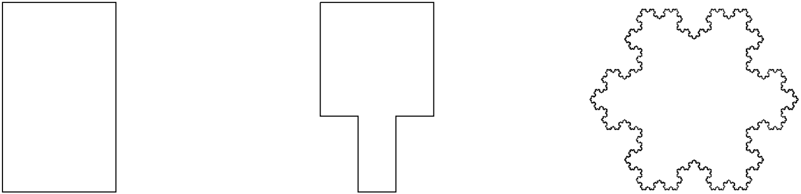
The class $\mathcal{A}$

We prove a similar structure for sets satisfying weaker hypotheses. We say that Ω belongs to the class $\mathcal{A}$ if it

- is a Jordan domain and $\partial\Omega$ has zero 2-Lebesgue measure;
- has no necks of radius r for all $r \le R$.



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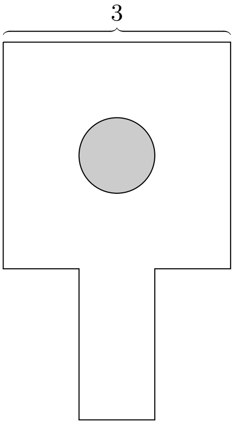

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- what can happen at scale R for convex sets, now can happen at any scale $r \leq R$.

This happens for those $r < R$ such that

$$\{x \in \Omega \,:\, \mathrm{dist}(x, \partial\Omega) \geq r\} \neq \overline{\mathrm{int}(\{x \in \Omega \,:\, \mathrm{dist}(x, \partial\Omega) \geq r\})}$$

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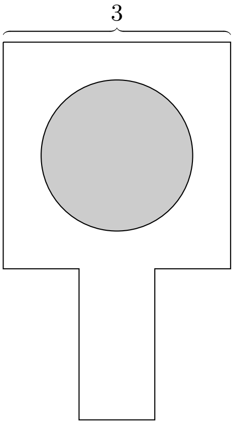

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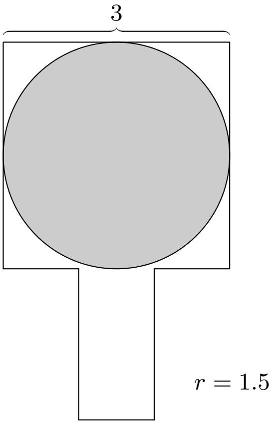

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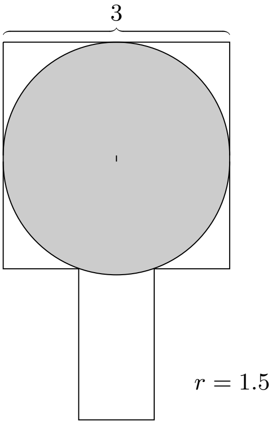

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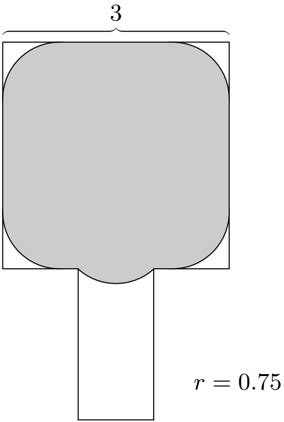

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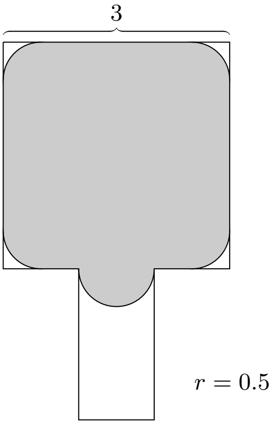

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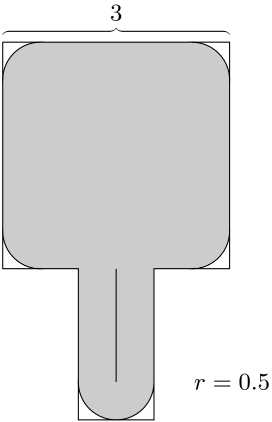

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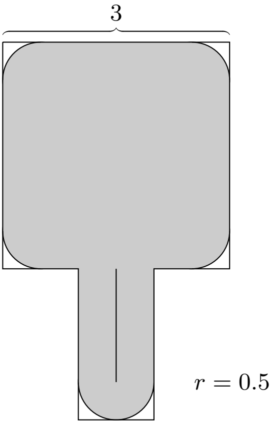

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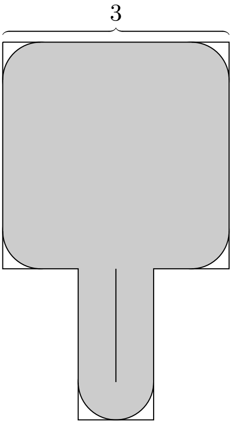

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Uniqueness

One can easily state a uniqueness condition in terms of the sets

$$\Omega_r := \{x \in \Omega \,:\, \mathrm{dist}(x, \partial\Omega) \geq r\}\,.$$

Theorem (Leonardi, S. - 2020)

Let $\Omega \in \mathcal{A}$, and $r < R$. The set difference $\Omega_r \setminus \overline{\mathrm{int}(\Omega_r)}$ consists of a finite collection Γ_r of $\mathrm{C}^{1,1}$ curves.

Uniqueness of isoperimetric sets

Let $\Omega \in \mathcal{A}$ be such that

- Ω_R is a singleton,
- for all $r < R$, there is at most 1 curve in Γ_r.

Then for all $V \geq \pi R^2$, there exists a unique isoperimetric set.

How do we prove it: the PMC functional

Isoperimetric sets for $V \leq \pi R^2$ are trivial. For greater volumes, we consider the auxiliary functional

$$\mathcal{F}_r[E] := P(E) - \frac{1}{r}|E|\,.$$

It is immediate to prove that if $\overline{E}$ minimizes $\mathcal{F}_r$ in a class large enough of subsets of Ω, then $\overline{E}$ is isoperimetric in Ω.

$\min \mathcal{F}_r$	competitors	isoperimetric sets
$h^{-1}(\Omega) > r$	$\{E \subset \Omega\}$	$\lvert C^M(\Omega)\rvert < V \leq \lvert\Omega\rvert$
$h^{-1}(\Omega) \leq r \leq R$	$\{E \subset \Omega \,:\, \lvert E\rvert \geq \pi r^2\}$	$\pi R^2 \leq V \leq \lvert C^M(\Omega)\rvert$

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The duality

Theorem (Leonardi, S. - forthcoming)

Let $\Omega \in \mathcal{A}$, and let $V \geq \pi R^2$. A set E is isoperimetric for volume V if and only if there exists $r \leq R$ such that E minimizes $\mathcal{F}_r$ in the appropriate class.

This is a duality in the sense of the Legendre transform. Convexity follows.

- $\mathcal{J}$ is convex on $V \geq \pi R^2$;
- $\mathcal{J}^2$ is globally convex;
- when Ω_R is a singleton and $\Omega_r = \overline{\mathrm{int}(\Omega_r)}$ for all $r < R$
  - $\mathcal{J}$ is strictly convex on $V \geq \pi R^2$;
  - the map $V \mapsto r^{-1}$ is a bijection.

Higher dimension

Theorem (Alter, Caselles, Chambolle - 2005)

Let $\Omega \subset \mathbb{R}^n$ be convex, and of class $\mathrm{C}^{1,1}$, and let $V \geq |C(\Omega)|$. A set E is isoperimetric for volume V if and only if there exists $r \leq h(\Omega)^{-1}$ such that E minimizes $\mathcal{F}_r$. Moreover, this set is unique and convex.

Extend to $\mathcal{F}_r$ for $h(\Omega)^{-1} \leq r \leq R$ within the adjusted class of competitors.

Possible outcomes

- completely fill the gap of volumes between $\omega_n R^n$ and $|C(\Omega)|$;
- if there is continuity in the volume of minimizers (wrt r), duality still works and yields convexity properties;
- proving minimizers are convex, settling a conjecture standing from 25 years.

The evolutionary counterpart: a conjecture

Let $\Omega \in \mathcal{K}^2$, be k-rotationally symmetric wrt p (+ more). Consider $\{E_V\}_V$

$$E_V := \begin{cases} \text{the unique isoperimetric set centered on } p\,, & \text{if } V < \pi R^2\,, \\ \text{the unique isoperimetric set } k\text{-rot. sym. wrt } p\,, & \text{if } V \geq \pi R^2\,. \end{cases}$$

Is there a geometric evolution describing this family?

Conjecture: given $S \in \{E_V\}$, if we let it evolve through

$$v_t(x) := \begin{cases} -\kappa(x)\,, & x \in \Omega\,, \\ 0\,, & x \in \partial\Omega\,, \end{cases}$$

we would end up with a movie showing all $E \in \{E_V\}$ with $|E| \geq |S|$.

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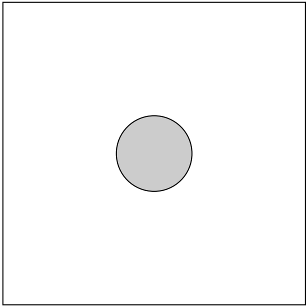

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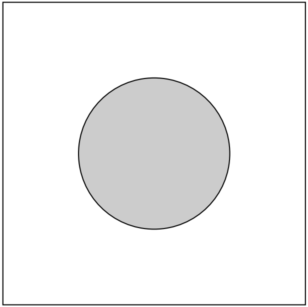

Conjecture: given $S \in \{E_V\}$, if we let it evolve through

$$v_t(x) := \begin{cases} -\kappa(x)\,, & x \in \Omega\,, \\ 0\,, & x \in \partial\Omega\,, \end{cases}$$

we would end up with a movie showing all $E \in \{E_V\}$ with $|E| \geq |S|$.

The evolutionary counterpart: a conjecture

Let $\Omega \in \mathcal{K}^2$, be k-rotationally symmetric wrt p (+ more). Consider $\{E_V\}_V$

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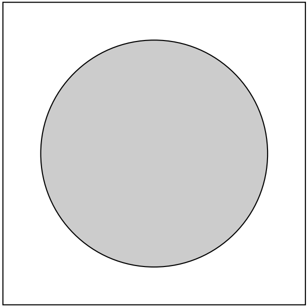

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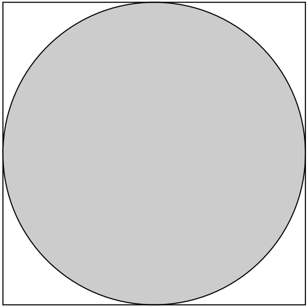

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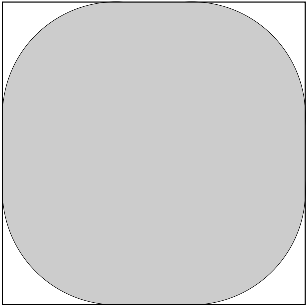

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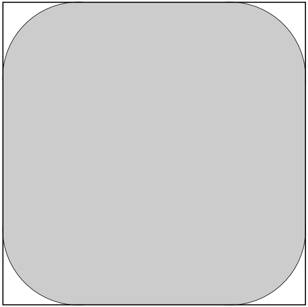

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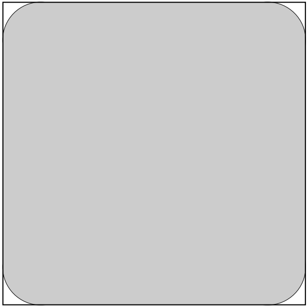

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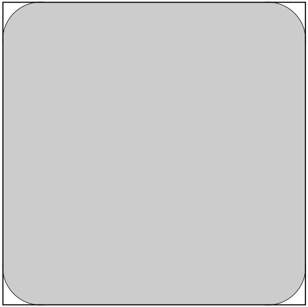

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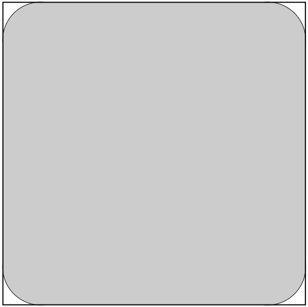

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Thanks for your attention!