Rational interpolation and model order reduction for data-driven controller design

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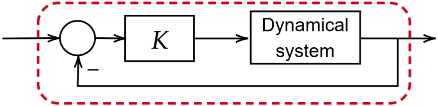
Department of Automatic Control, Lund University

8th European Congress of Mathematics

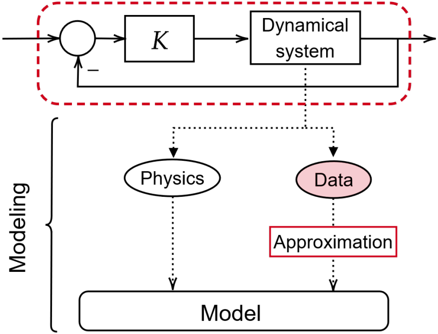
21/06/2021

Intersections between modeling and control



Objectives:
- Stability
- Performances

Intersections between modeling and control



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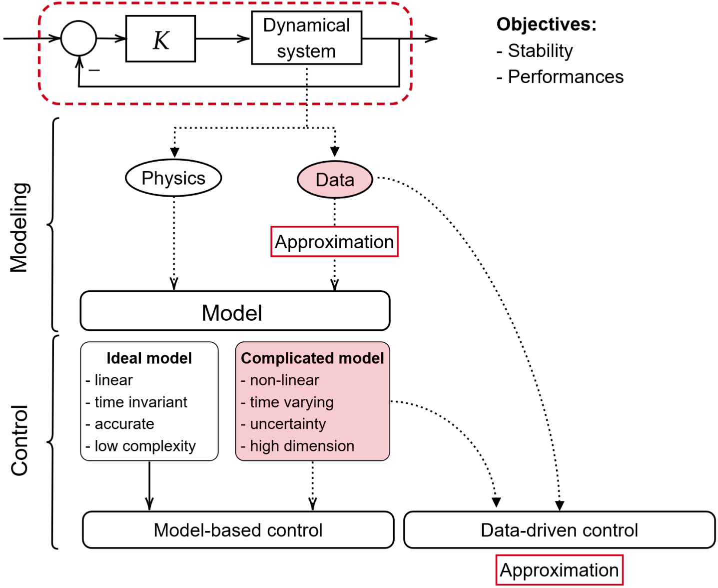

Objectives:

- Stability
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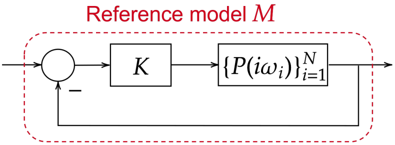
Overview

1. Introduction
   - Intersections between modeling and control
   - Loewner Data-Driven Control (L-DDC)
2. Application to infinite dimensional systems
   - Motivations
   - Case study: the continuous crystallizer
   - Data-driven control approach using the L-DDC framework
3. Conclusion

Loewner Data-Driven Control (L-DDC)

Considered problem

Given data from the system P, design K such that the resulting closed-loop is as close as possible to the reference model M



$$\frac{PK^\star}{1+PK^\star} = M$$

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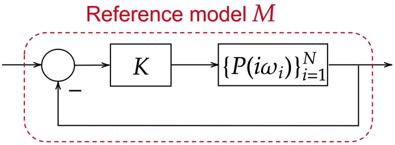

$$\frac{PK^\star}{1+PK^\star} = M$$

Approximate $K^\star$

Loewner interpolation

$$\forall i,\ K(i\omega_i) = K^\star(i\omega_i)$$

$$K^\star(i\omega_i) = \frac{M(i\omega_i)}{P(i\omega_i)(1-M(i\omega_i))}$$

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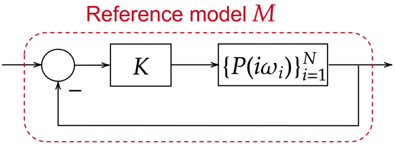

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performant
AND reachable
(Instability estimation)

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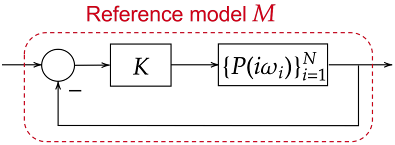

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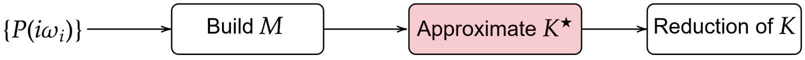

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Loewner interpolation

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$K^\star(i\omega_i) = \frac{M(i\omega_i)}{P(i\omega_i)(1-M(i\omega_i))}$

under internal stability constraint

Application to infinite dimensional systems

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   - Case study: the continuous crystallizer
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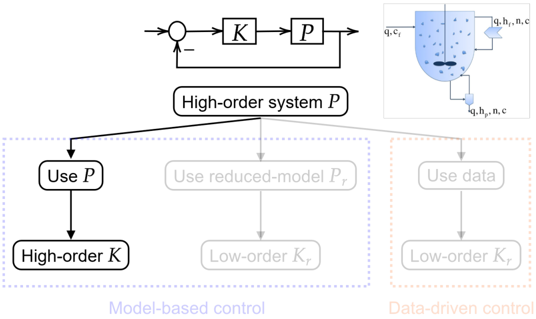
Motivations



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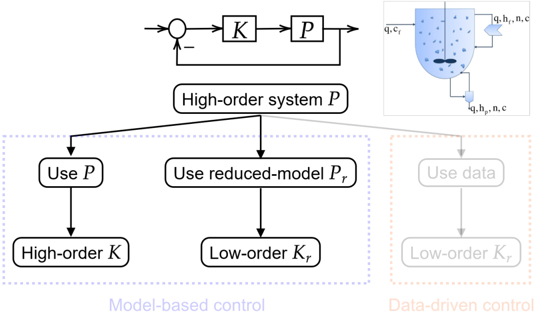
Motivations



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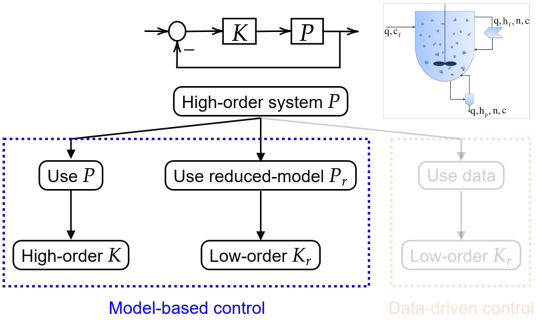

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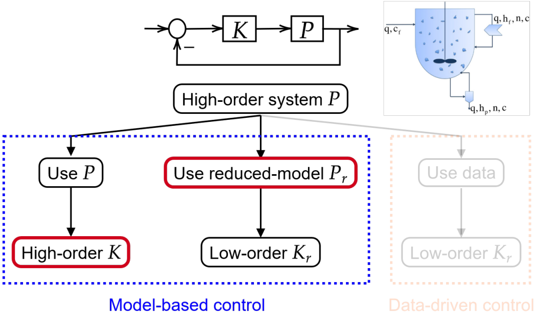

- Model Order Reduction is essential

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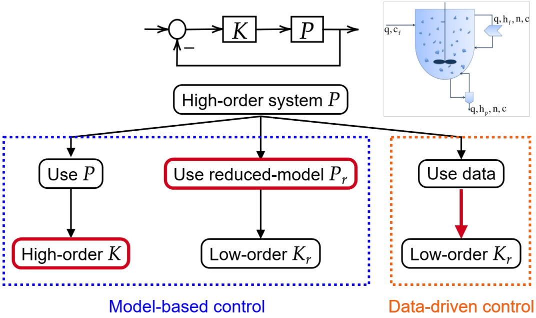

- Model Order Reduction is essential
- and it can also be used for data-driven control.

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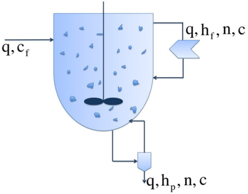
[3] *From reference model selection to controller validation: Application to Loewner Data-Driven Control*, Kergus, Olivi, Poussot-Vassal, Demourant, *IEEE Control Systems Letters*, 2019.

Case study: the continuous crystallizer



- **Goal:** stabilize the plant around $c_{ss} = 4.09 mol/L$

1. *A mathematical model for continuous crystallization*, Rachah, Noll, Espitalier, Baillon, *Mathematical Methods in the Applied Sciences*, 2016.

2. *$\mathcal{H}_\infty$-Control of a continuous crystallizer*, Vollmer, Raisch, *Control Engineering Practice*, 2001.

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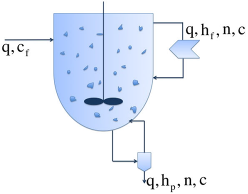

- **Goal:** stabilize the plant around $c_{ss} = 4.09 mol/L$
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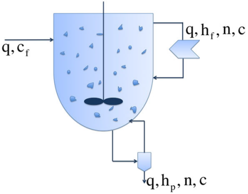
Case study: the continuous crystallizer



- **Goal:** stabilize the plant around $c_{ss} = 4.09 mol/L$
- Unstable system and sustained oscillations
- Linearization of the PDEs around c_{ss}

$$P(s) = \frac{\Delta c(s)}{\Delta c_f(s)} = \frac{p_{12}(s)}{p_{13}(s) + q_{12}(s)e^{-sk_f} + r_{12}(s)e^{-sk_p}}$$

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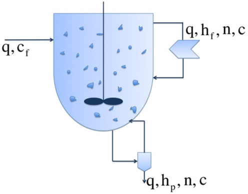

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$\rightarrow$ Frequency-domain data easily accessible
$N = 500$ frequencies, logspaced between 10^{-3} and 1 rad.s^{-1}

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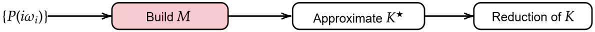
L-DDC Step 1: Building a reference model



$$\begin{cases} y_{z_i}^T P(z_i) = 0 \\ y_{p_j} P(p_j) = \infty \end{cases} \Rightarrow \boxed{\begin{cases} {y_{z_i}}^T M(z_i) = 0 \\ M(p_j) y_{p_j} = y_{p_j} \end{cases}}$$

Achievable performance of multivariable systems with unstable zeros and poles, Havre, Skogestad, *International Journal of Control*, 2001.

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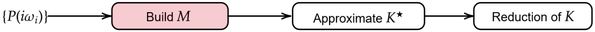

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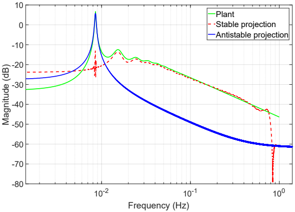
1. Determine the system's nature: stable/unstable, NMP or not

$$\begin{array}{ccccc} P(\imath\omega) & = & P_s(\imath\omega) & + & P_{as}(\imath\omega) \\ \mathcal{L}_2 & = & \mathcal{H}_2 & \oplus & \mathcal{H}_2^{\perp} \end{array}$$



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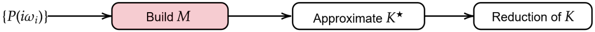

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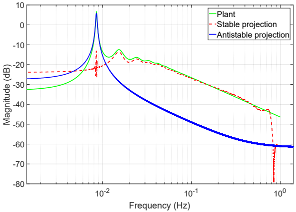
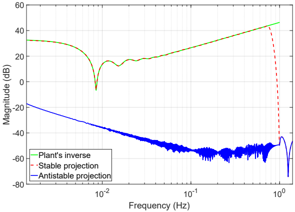
$$\mathrm{P}^{-1}(\imath\omega) = \mathrm{P}_s^{-1}(\imath\omega) + \mathrm{P}_{as}^{-1}(\imath\omega)$$





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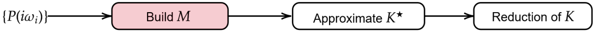

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1. Determine the system's nature: stable/unstable, NMP or not
2. If any, estimate the instabilities of the system

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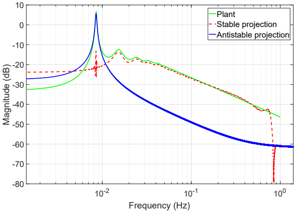
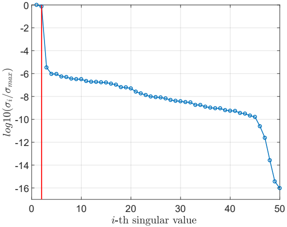
Use P_{as} for Principal Hankel Component analysis





Estimating unstable poles in simulations of microwave circuits, Cooman, Seyfert, Amari, *International Microwave Symposium*, 2018.

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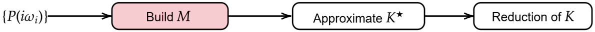

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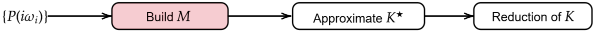
Estimated RHP poles	$1.07 \times 10^{-4} \pm 0.852 \times 10^{-2} \jmath$
RHP poles (direct search)	$0.99 \times 10^{-4} \pm 0.89 \times 10^{-2} \jmath$

L-DDC Step 1: Building a reference model



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3. Build an achievable reference model

$$M_{init}(s) = \frac{1}{1+\tau s}, \ \tau = 1s$$

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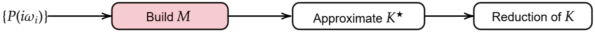

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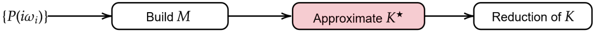
$$\boxed{M = 1 - (1 - M_{init}) B_p}$$

$$B_p(s) = \prod_{j=1}^{n_p} \frac{s - p_j}{s + p_j}$$

$$\forall j = 1 \dots n_p, \ B_p(p_j) = 0$$

$$\forall \omega, \ |B_p(\jmath\omega)| = 1$$

LDDC Step 2: Controller identification and reduction



Objective: obtain a rational model $\mathsf{K} = (E, A, B, C, D)$ such that:

$$\forall i = 1 \dots N, \mathsf{K}(\imath\omega_i) = \mathsf{K}^\star(\imath\omega_i) = \frac{M(\jmath\omega_i)}{P(\jmath\omega_i)(1 - M(\jmath\omega_i))}.$$

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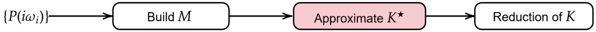

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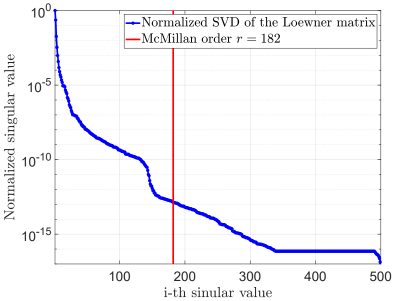
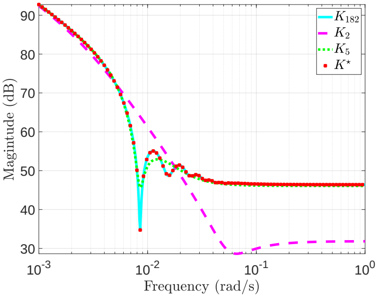


SVD of $\mathbb{L}$.



Obtained controllers.

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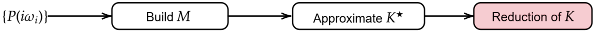
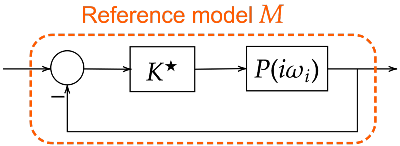
LDDC Step 3: Closed-loop stability analysis





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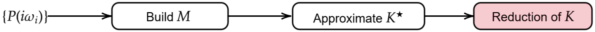

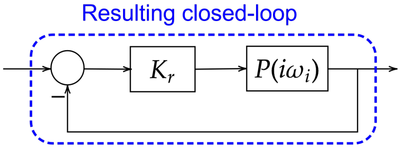

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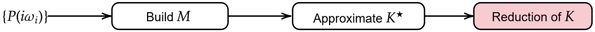
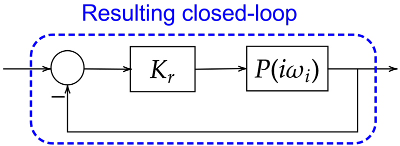
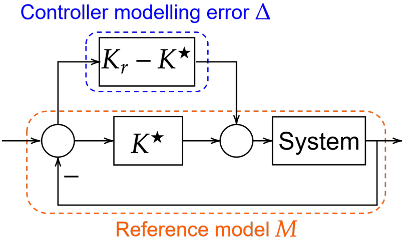






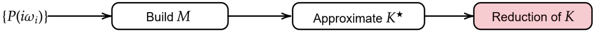
Application of the small-gain theorem

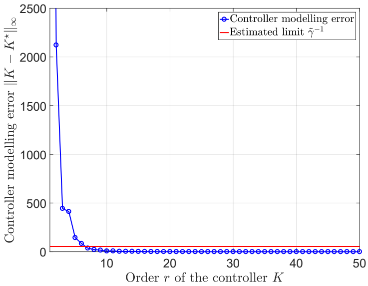
The closed-loop is well-posed and internally stable for all stable $\Delta = K - K^\star$ such that $\|\Delta\|_\infty \leq \beta$ if and only if $\|(1 - \mathrm{M})\mathrm{P}\|_\infty < \frac{1}{\beta}$

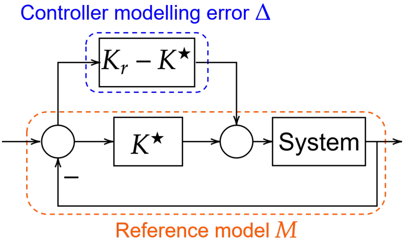
→ Limiting the controller modelling error allows to ensure closed-loop internal stability!

Data-driven controller validation, Van Heusden, Karimi, Bonvin, *IFAC Proceedings*, 2009.

LDDC Step 3: Closed-loop stability analysis







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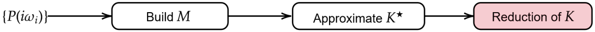
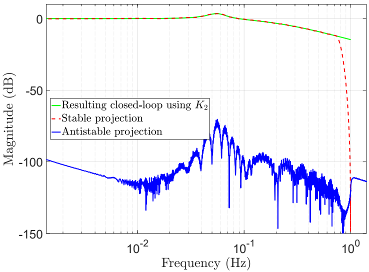
$\rightarrow$ Limiting the controller modelling error allows to ensure closed-loop internal stability!

Alternative closed-loop stability analysis



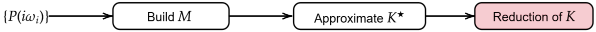
$$H(\jmath\omega_i) = \frac{P(\jmath\omega_i)K_r(\jmath\omega_i)}{1 + P(\jmath\omega_i)K_r(\jmath\omega_i)}$$

1st option



1. *Model-free closed-loop stability analysis: A linear functional approach*, Cooman, Seyfert, Olivi, Chevillard, Baratchart, *IEEE Transactions on Microwave Theory and Techniques*, 2018

Alternative closed-loop stability analysis



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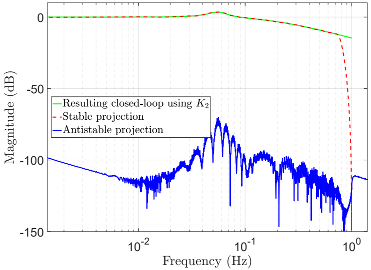

2nd option

1. Loewner interpolation: $\hat{H}(\jmath\omega_i) = H(\jmath\omega_i)$

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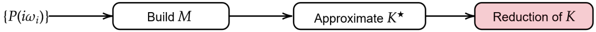
1. *Model-free closed-loop stability analysis: A linear functional approach*, Cooman, Seyfert, Olivi, Chevillard, Baratchart, *IEEE Transactions on Microwave Theory and Techniques*, 2018
2. *Interpolation-based infinite dimensional model control design and stability analysis*, Poussot-Vassal, Kergus, Vuillemin, *chapter to appear.*

Alternative closed-loop stability analysis



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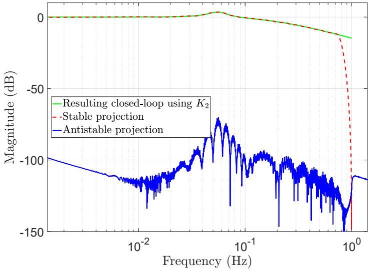

2nd option

1. Loewner interpolation: $\hat{H}(\jmath\omega_i) = H(\jmath\omega_i)$
2. Stable projection on $\mathcal{RH}_\infty$: $\hat{H}_s = \underset{H \in \mathbb{S}^+_{n,n_i,n_o}}{\arg\min} \|H - \hat{H}\|_\infty$

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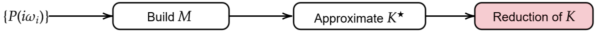
3. *On the closest stable descriptor system in the respective spaces $\mathcal{RH}_2$ and $\mathcal{RH}_\infty$*, Köhler, *Linear Algebra and its Applications*, 2014.

Alternative closed-loop stability analysis



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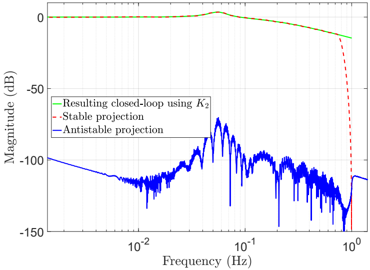
1st option



2nd option

1. Loewner interpolation: $\hat{H}(\jmath\omega_i) = H(\jmath\omega_i)$
2. Stable projection on $\mathcal{RH}_\infty$: $\hat{H}_s = \underset{H \in \mathbb{S}^+_{n,n_i,n_o}}{\arg\min} \|H - \hat{H}\|_\infty$
3. Stability index $S = \|\hat{H}_s - \hat{H}\|_\infty$

$$S = 4.3511 \cdot 10^{-6}$$

1. *Model-free closed-loop stability analysis: A linear functional approach*, Cooman, Seyfert, Olivi, Chevillard, Baratchart, *IEEE Transactions on Microwave Theory and Techniques*, 2018
2. *Interpolation-based infinite dimensional model control design and stability analysis*, Poussot-Vassal, Kergus, Vuillemin, *chapter to appear.*
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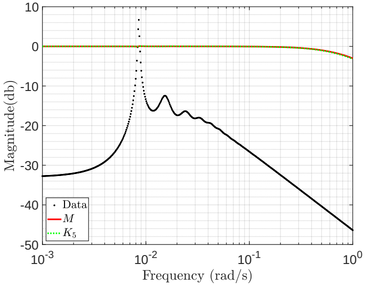
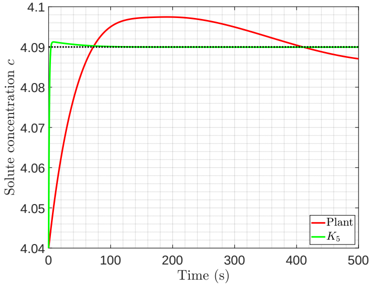
Results

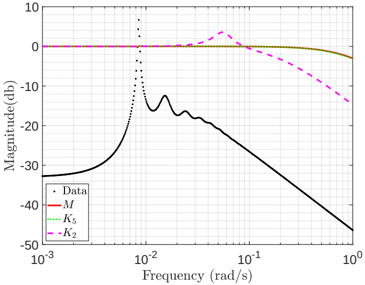


Closed-loop transfer functions.

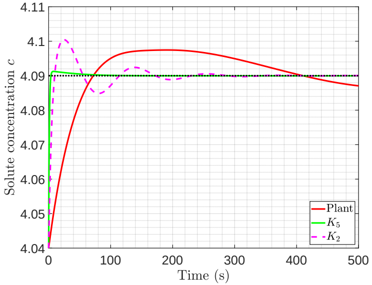


Time-domain simulation.

Results



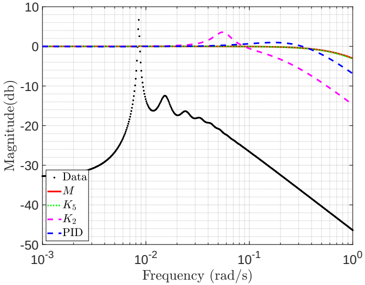
Closed-loop transfer functions.



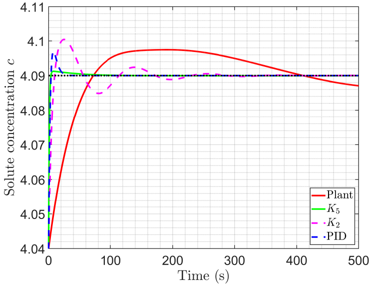
Time-domain simulation.

⇒ Impact of the **complexity-accuracy trade-off**

Results



Closed-loop transfer functions.



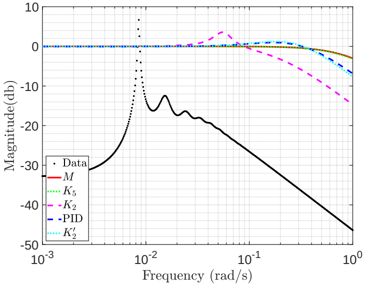
Time-domain simulation.

⇒ Impact of the **complexity-accuracy trade-off**

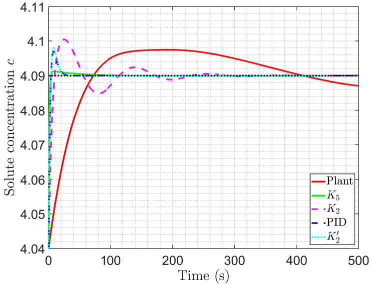
- Comparison with a robust PID[1]

1. *Advanced PID Control*, Åström, Hägglund, 2006

Results



Closed-loop transfer functions.



Time-domain simulation.

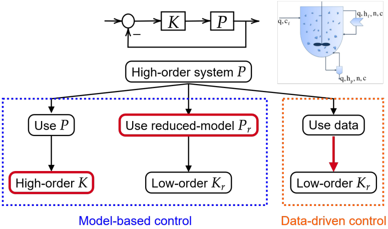
⇒ Impact of the **complexity-accuracy trade-off**

- Comparison with a robust PID[1]
- Use the closed-loop obtained with the PID as new reference model

⇒ Impact of the choice of the **specifications**

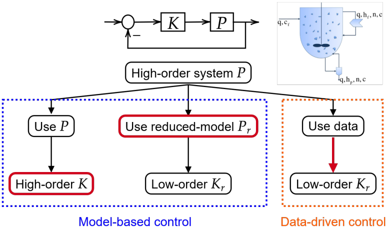
1. *Advanced PID Control*, Åström, Hägglund, 2006

Conclusion



The Loewner framework can be used as a central tool for the control of infinite dimensional transfer functions.

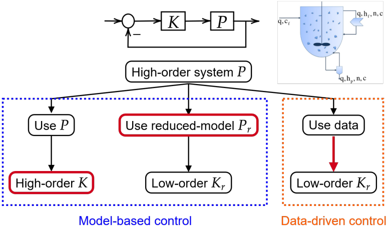
Conclusion



The Loewner framework can be used as a central tool for the control of infinite dimensional transfer functions.

	Model-based design	L-DDC
Method	more steps	direct
Controller structure	fixed order/poles	linear
Specifications	flexible (robust)	not flexible (only stability)
Stability guarantees	for P_r	conservative or not embedded

Conclusion



The Loewner framework can be used as a central tool for the control of infinite dimensional transfer functions.

Extension to other types of systems?

	Model-based design	L-DDC
Method	more steps	direct
Controller structure	fixed order/poles	linear
Specifications	flexible (robust)	not flexible (only stability)
Stability guarantees	for P_r	conservative or not embedded