Spanning bipartite subgraphs of triangulations of a surface

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Our papers
(Atsuhiro Nakamoto, Kenta Noguchi, Kenta Ozeki)

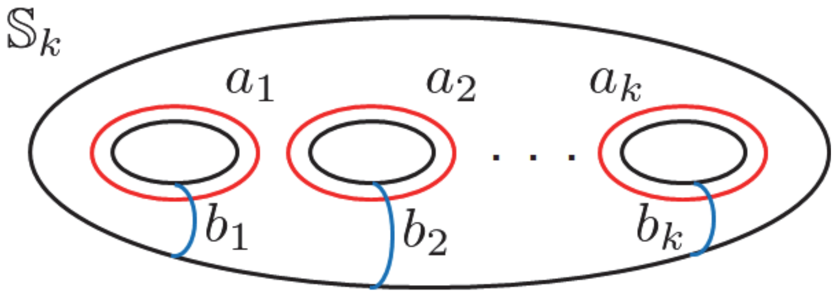
[2] Extension to even triangulations,
SIAM J. Discrete Math. **29** (2015), 2075-2087.

[3] Spanning bipartite quadrangulations of even triangulations,
J. Graph Theory **90** (2019), 267-287.

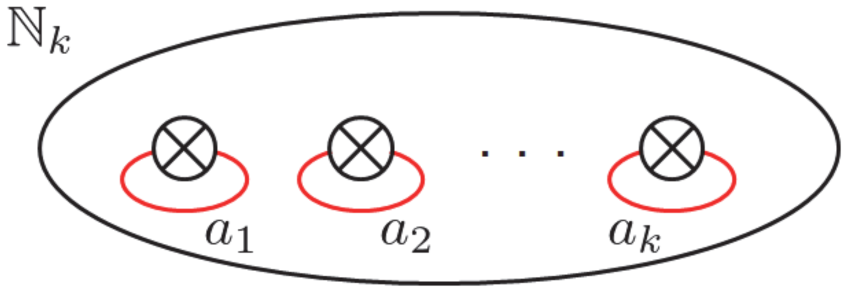
[4] Extension to 3-colorable triangulations,
SIAM J. Discrete Math. **33** (2019), 1390-1414.

Surfaces:
compact connected 2-manifolds without boundary

$\mathbb{S}_k$: orientable surface of genus k

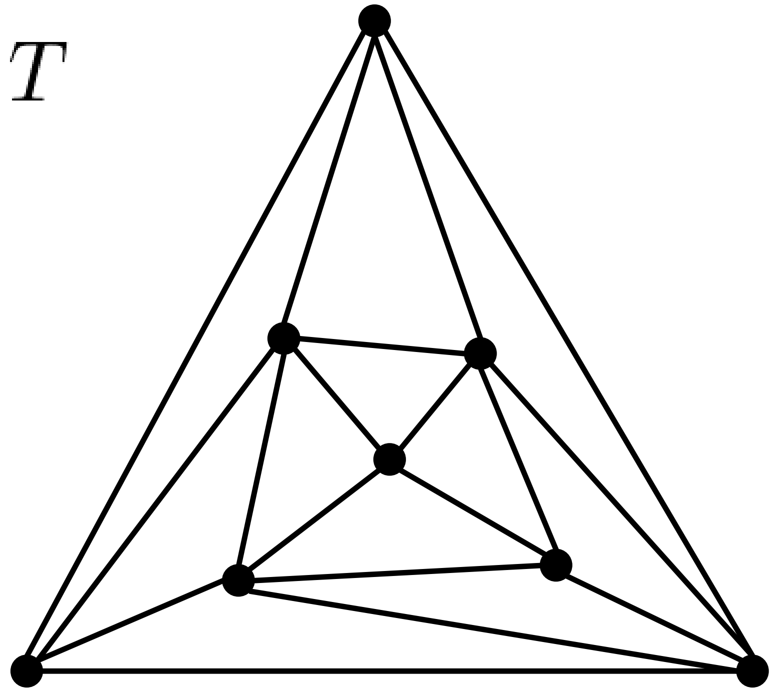


$\mathbb{N}_k$: nonorientable surface of crosscap number k

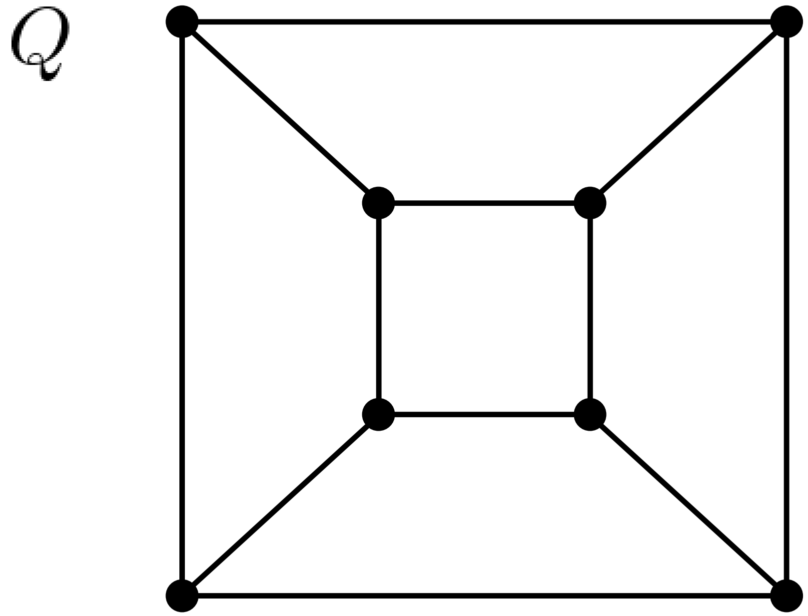


Graphs on surfaces

Triangulation (tri.)



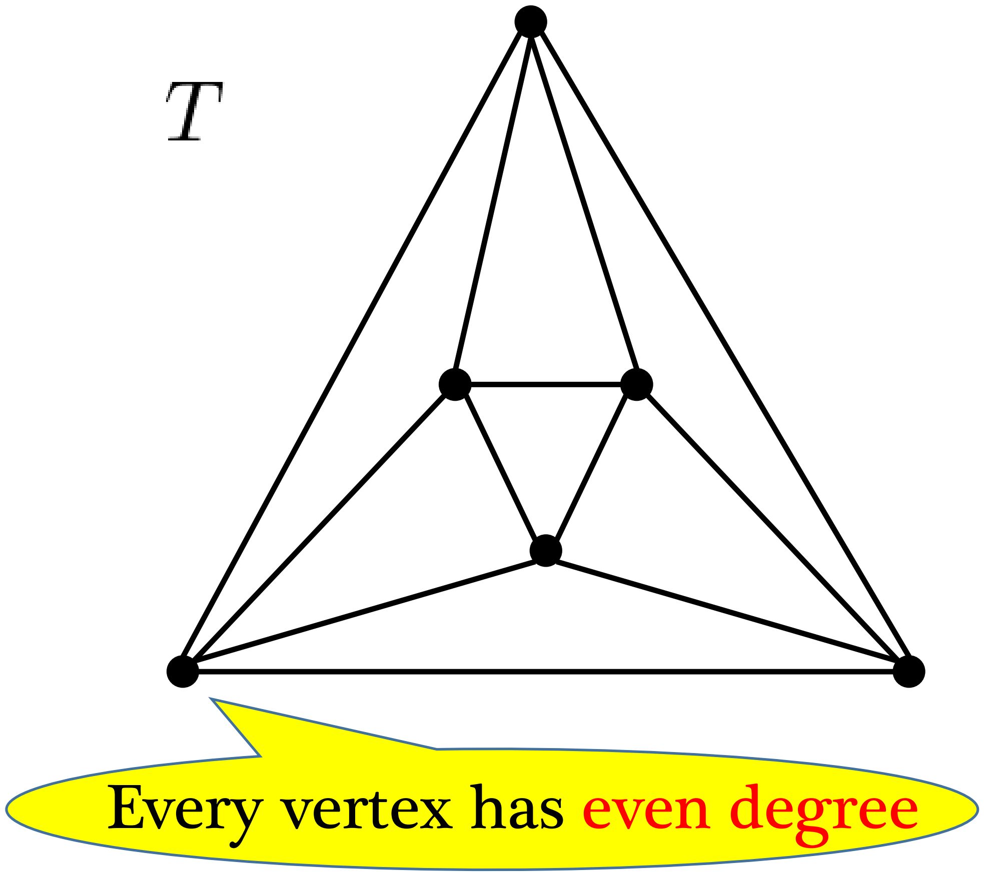
Quadrangulation (quad.)

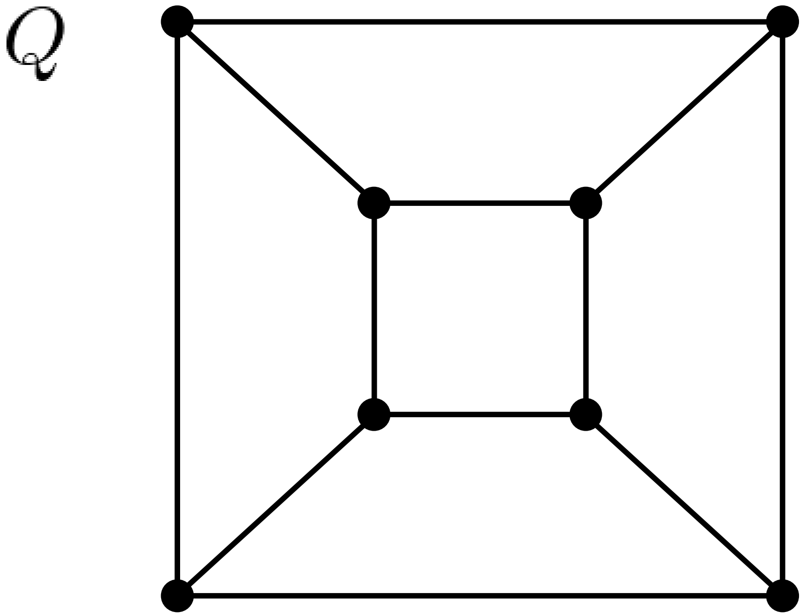


Graphs on surfaces

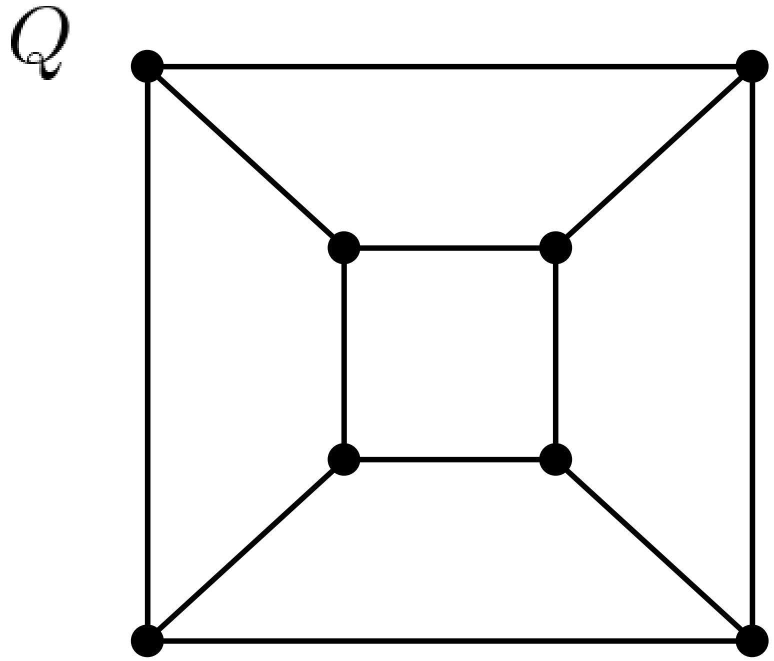
Eulerian (even) triangulation

Quadrangulation



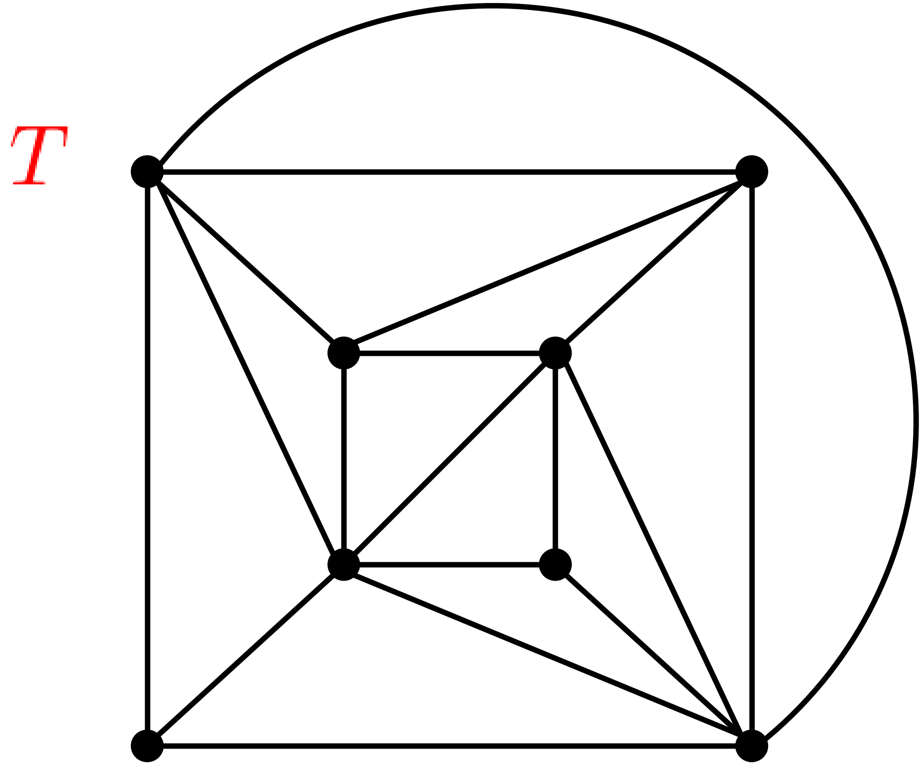


Quad. $\leftrightarrow$ Tri.



For a given quad., we can extend it to a triangulation by adding a diagonal in each face.

Quad. $\leftrightarrow$ Tri.



For a given triangulation, we often find a quad. as a spanning subgraph.

Additional requirements

For a given quad., can we extend it to

- Eulerian tri.?
- 3-colorable tri.?
- 4-connected tri.?

For a given tri., does it have

- bipartite quad.?
- 3-connected quad.?

Additional requirements

For a given quad., can we extend it to

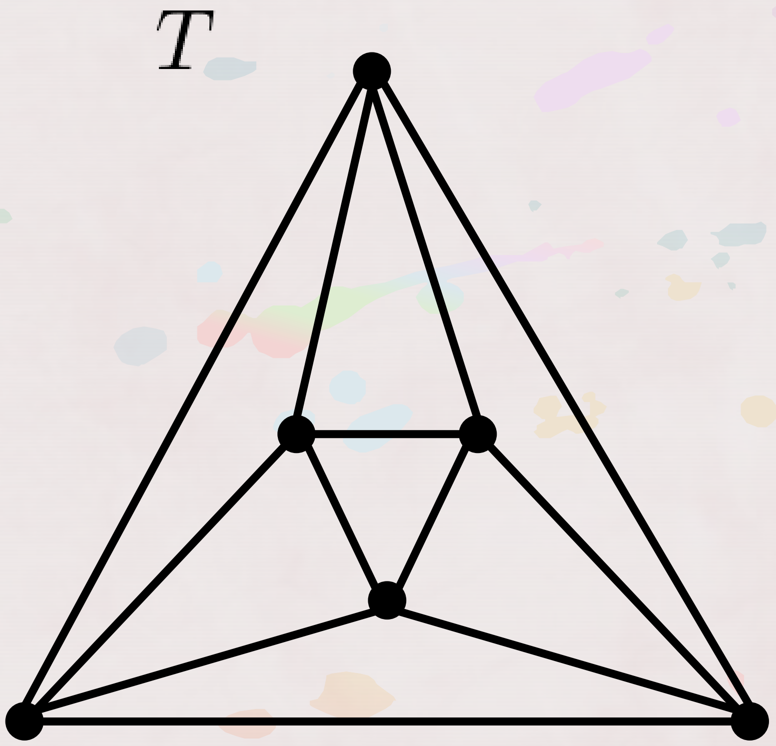
[2]	• Eulerian tri.?	Yes
[4]	• 3-colorable tri.?	∃ iff condition
	• 4-connected tri.?	Yes if it is simple

For a given tri., does it have

[3]	• bipartite quad.?	∃ iff condition for toroidal Eulerian tri.
	• 3-connected quad.?	Yes if it is 5-connected

[3]

A spanning quad. subgraph



[3]

Spanning quad.

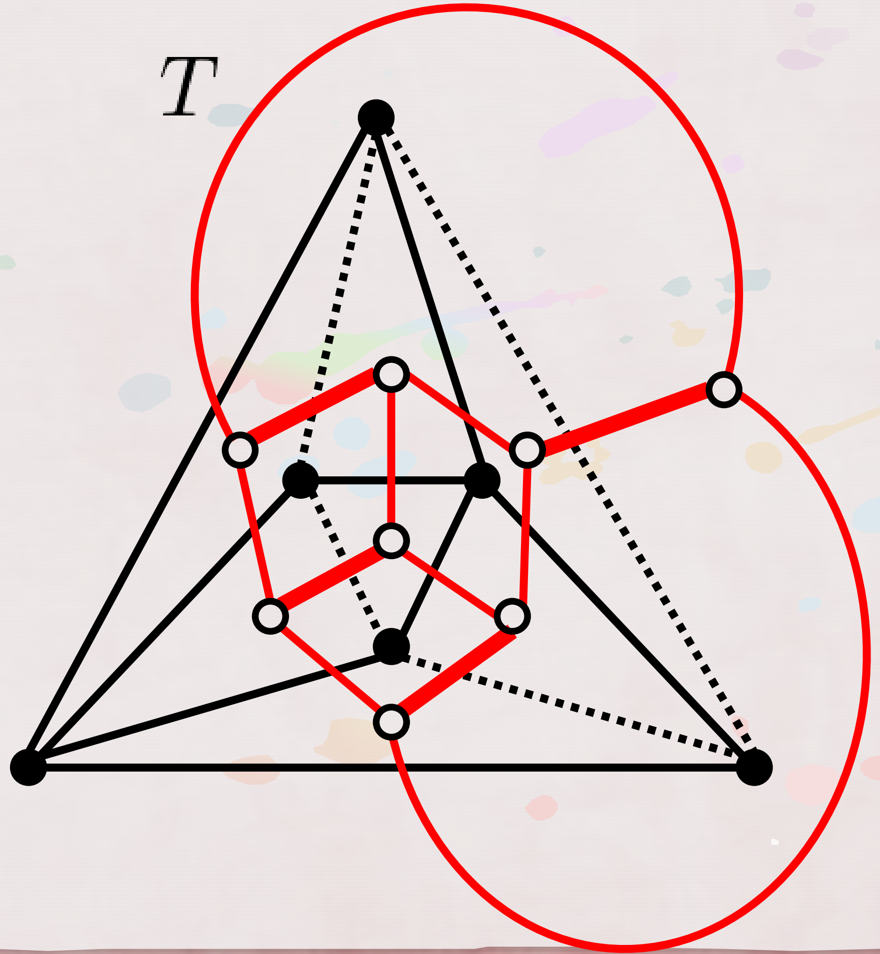
Proposition A

Let T be a loopless triangulation on a surface.

Then T has a spanning quad.

[3]

Proof



[3]

Problem

Let T be a loopless triangulation of a surface.
Does T have a spanning bipartite quad.?

Remark

- Every plane quad. is bipartite.
- 4-colorability of T is a sufficient condition.

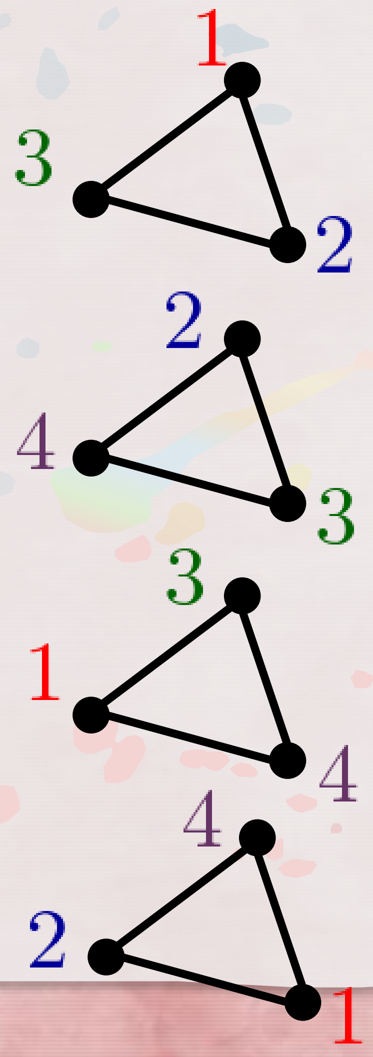
[3]

Bipartiteness

Proposition B

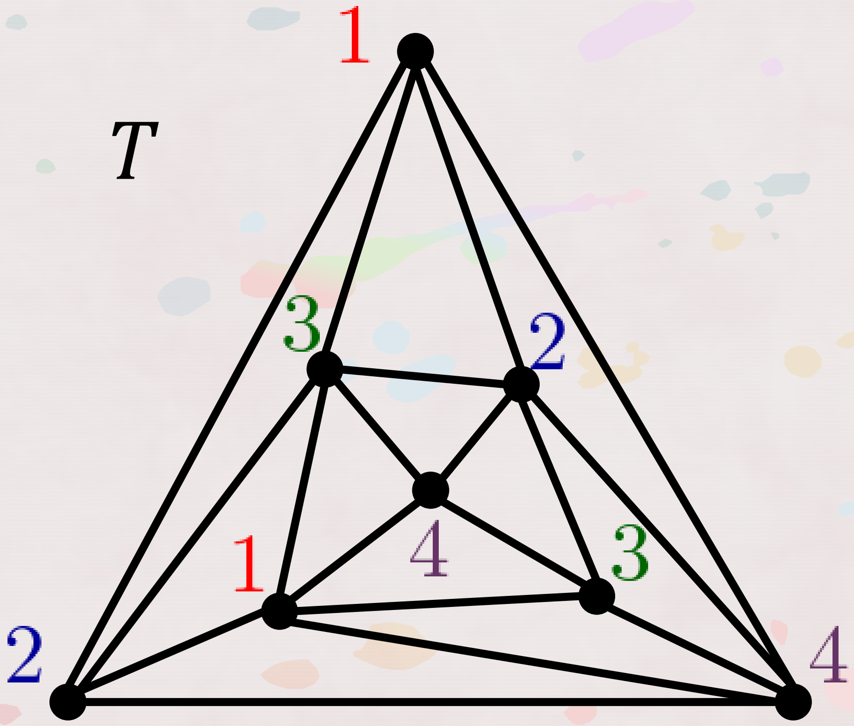
If T is a 4-colorable tri. on a surface, then T has a spanning bipartite quad.

[3] Proof



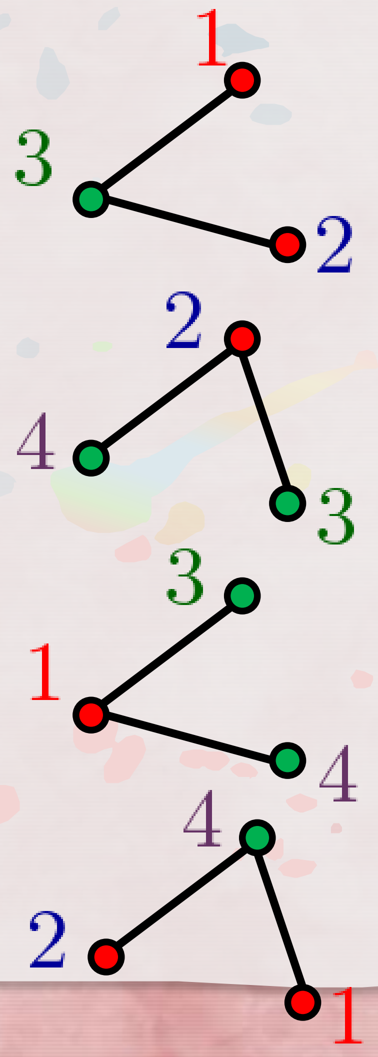
Proposition B

If T is a 4-colorable tri. on a surface, then T has a spanning bipartite quad.



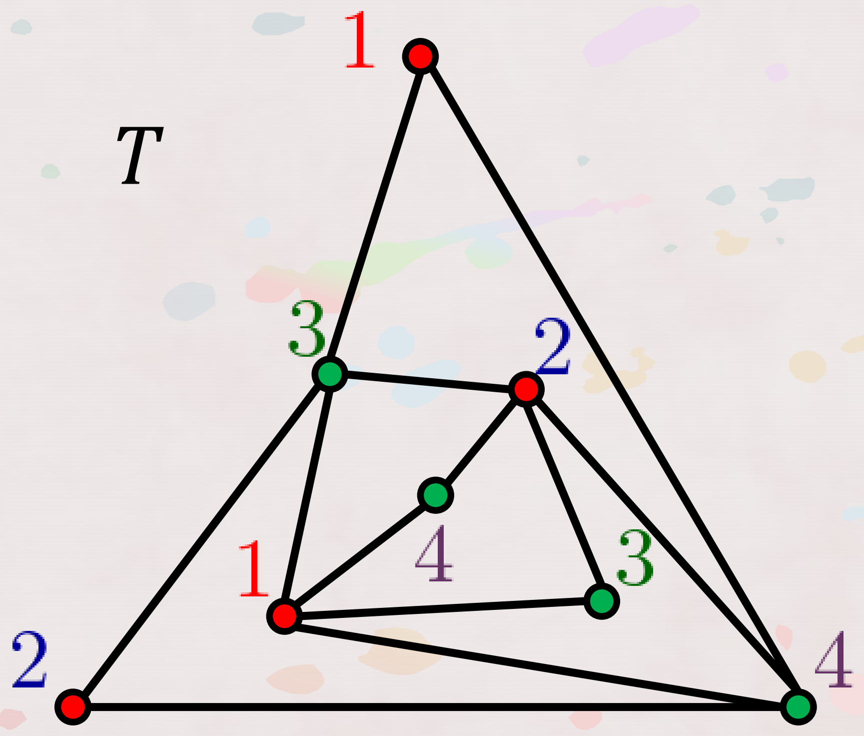
[3]

Proof



<u>Proposition B</u>

If T is a 4-colorable tri. on a surface, then T has a spanning bipartite quad.



[3]

On the projective plane

Theorem 1 (Kündgen, Thomassen, 2017; Nakamoto, N., Ozeki, 2019)

Let T be an Eulerian tri. of the projective plane. If T is 3-colorable, then every spanning quad. of T is bipartite. If T is not 3-colorable, then T has both bipartite and non-bipartite spanning quads.

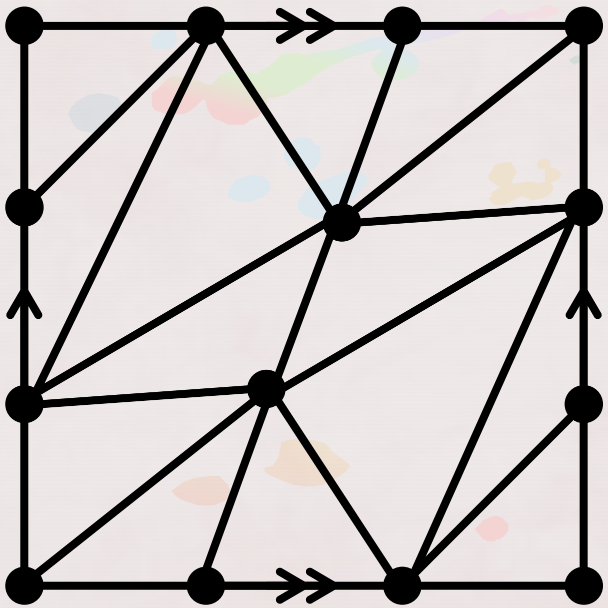
[3]

On the torus

Fact

Even tri. K_7 of the torus has no spanning bipartite quad.

$K_7 \subset \mathbb{S}_1$



[3]

A known theorem

Theorem C (Kündgen, Thomassen, 2017)
Let T be a loopless Eulerian tri. of the torus.
Then T has a spanning non-bipartite quad.
Furthermore, if T has sufficiently large edge width, then T has a spanning bipartite quad.

[3]

Main theorem

Theorem 2 (Nakamoto, N., Ozeki, 2019)
Let T be a loopless Eulerian tri. of the torus.
T has a spanning bipartite quad. if and only if
T does not have K_7 as a subgraph.

[3]

Outline of the Proof of Thm 2

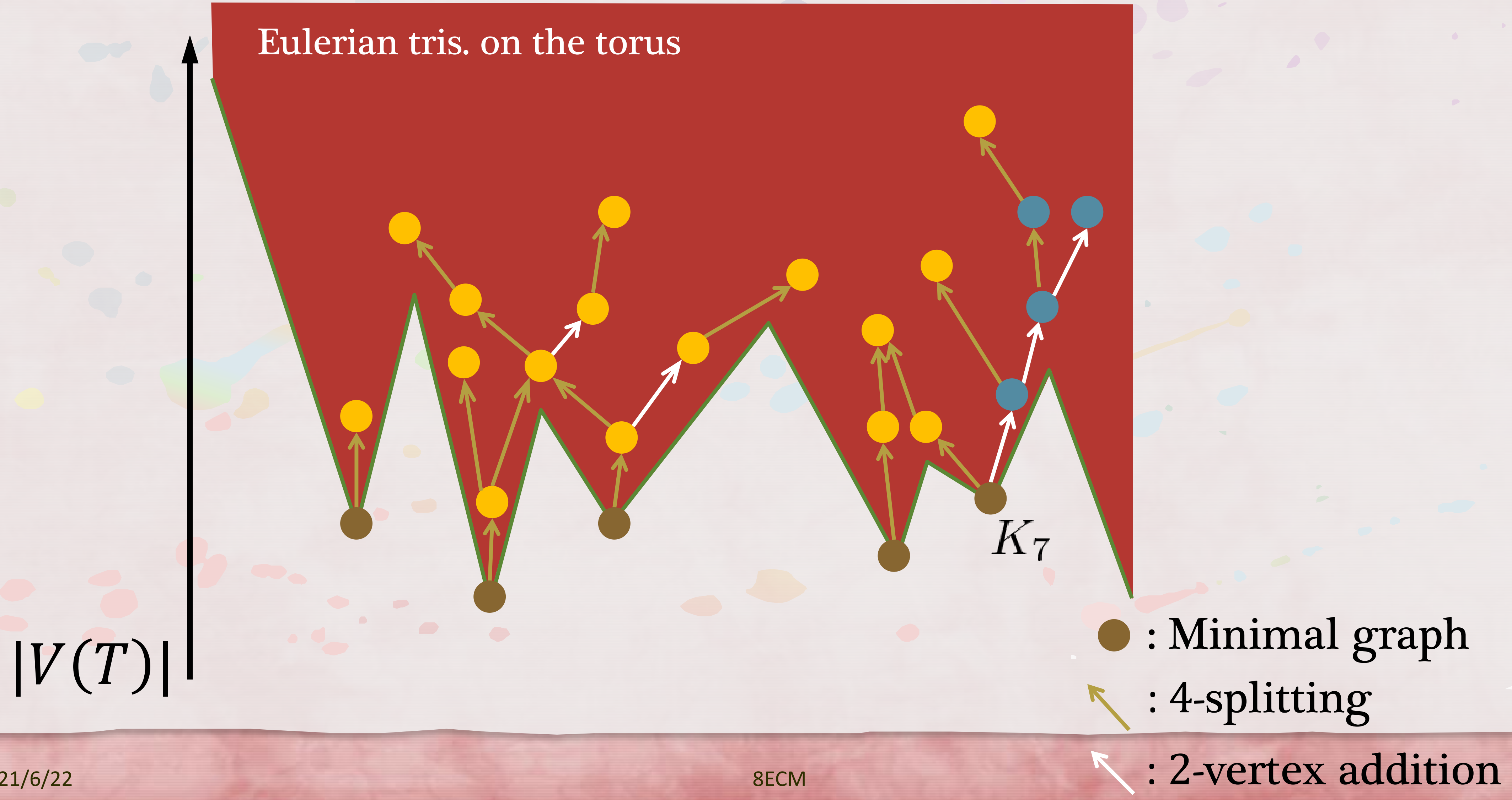
Theorem 2 (Nakamoto, N., Ozeki, 2019)
Let T be a loopless Eulerian tri. of the torus.
T has a spanning bipartite quad. if and only if T does not have K_7 as a subgraph.

We use a "generating" theorem.

Theorem D (Matsumoto, Nakamoto, Yamaguchi, 2018)
Every loopless Eulerian tri. of the torus is generated from one of 27 minimal graphs and 6-regular tris. by using 4-splittings and 2-vertex additions.

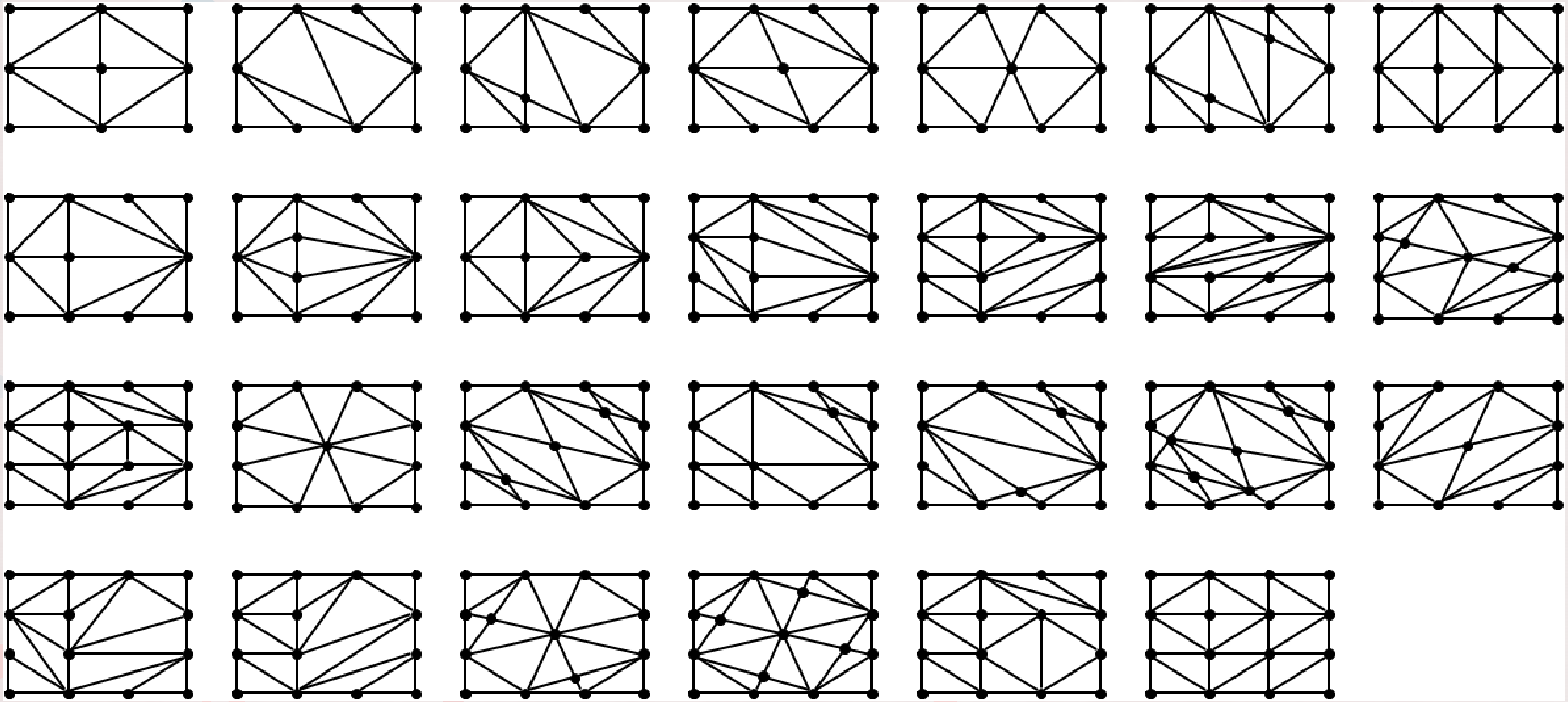
[3]

Generating Eulerian tris.



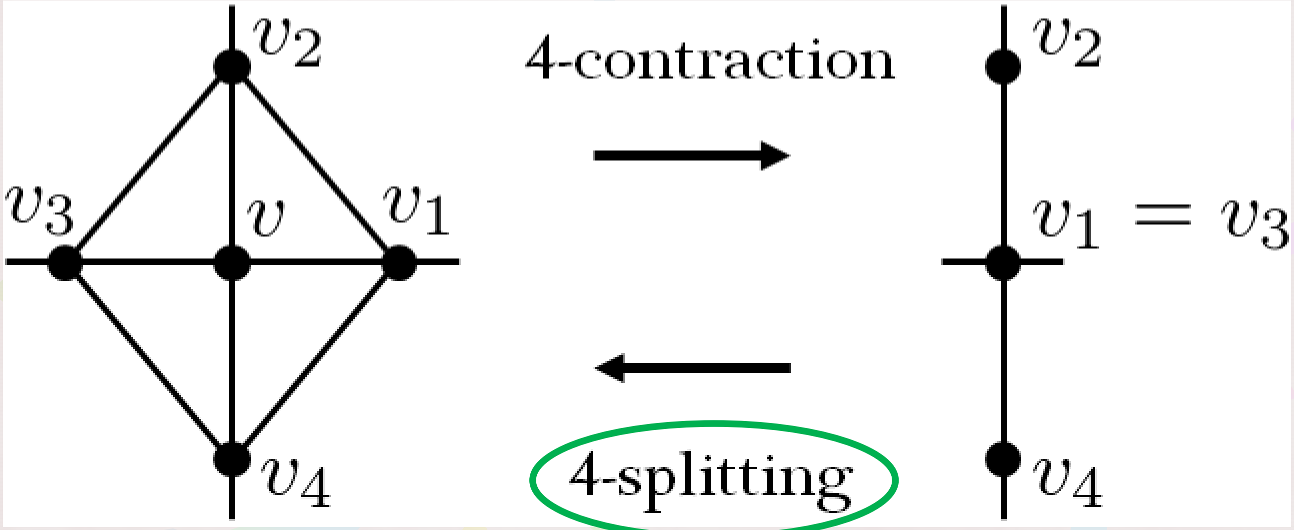
[3]

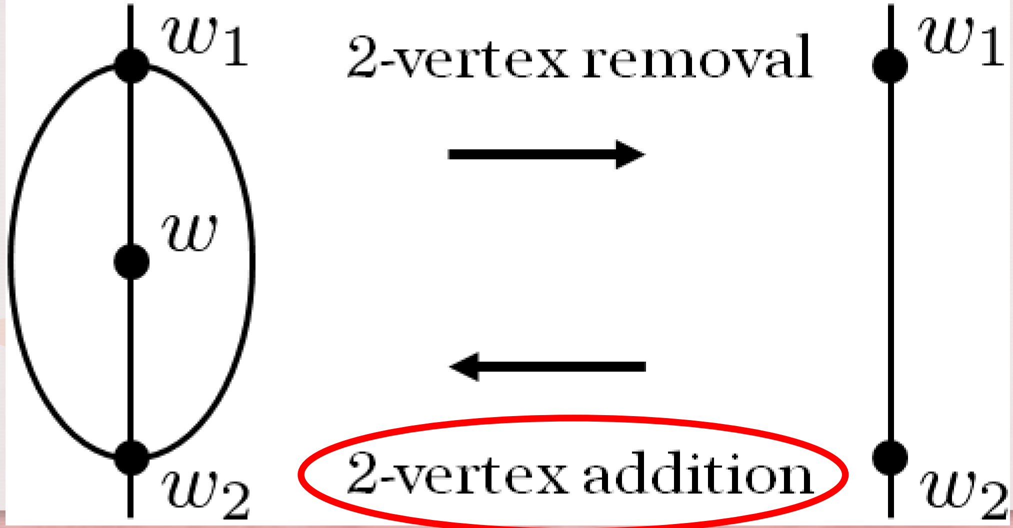
27 minimal graphs



[3]

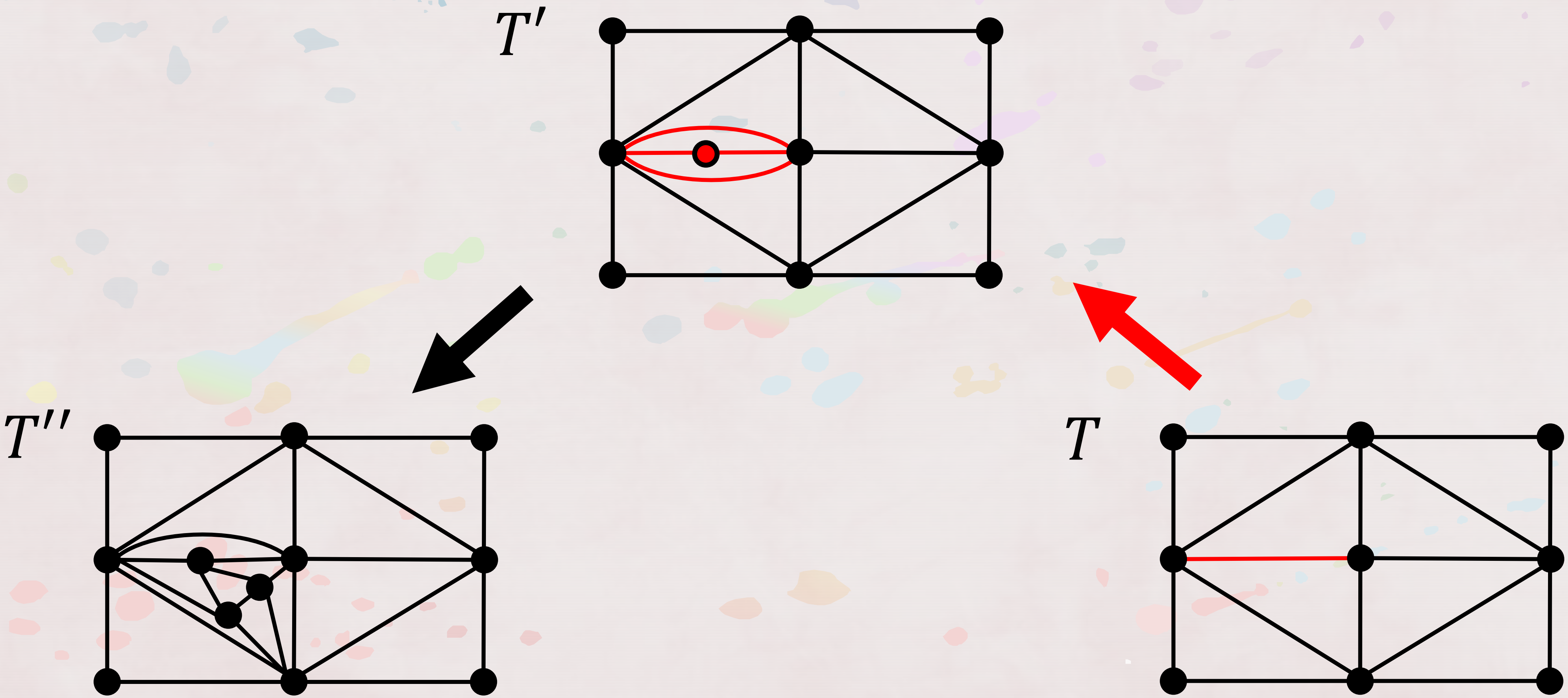
Two operations for the generating theorem





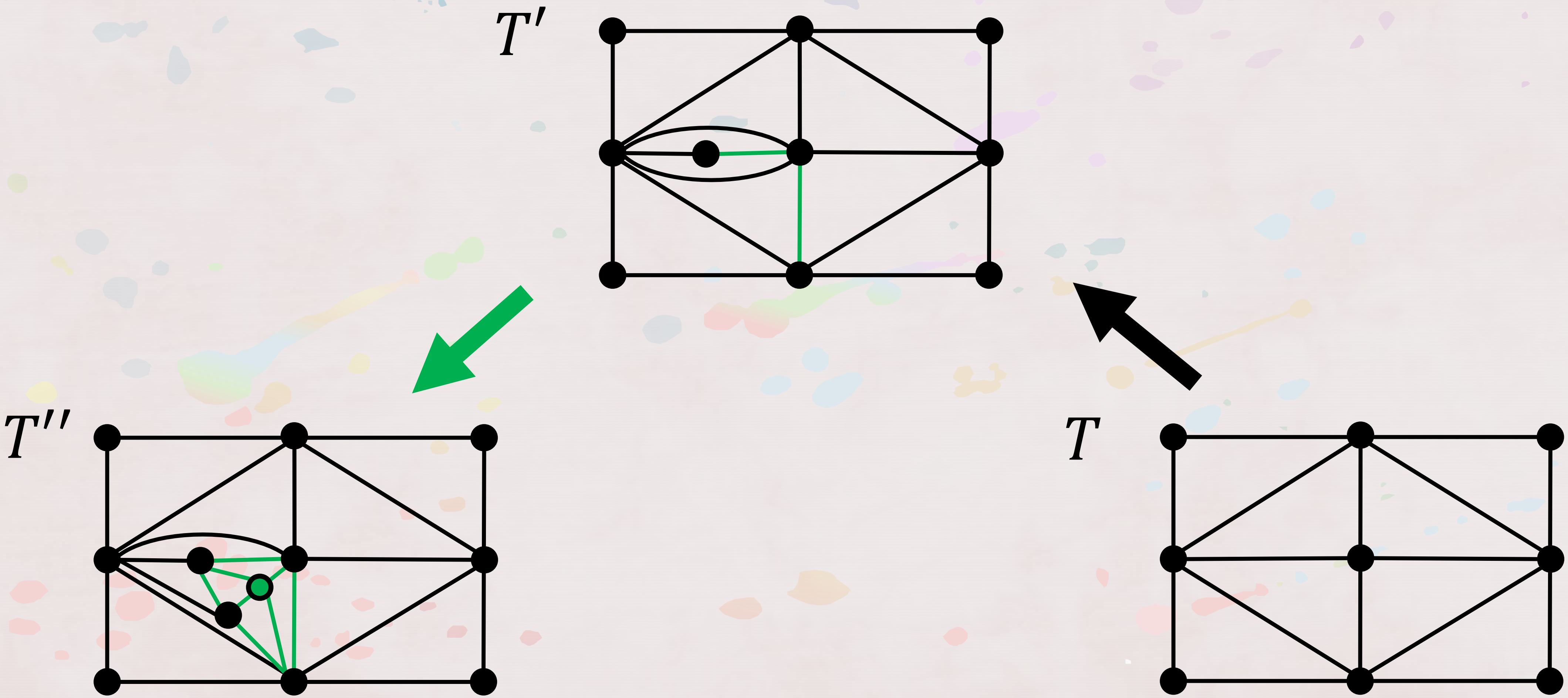
[3]

2-vertex addition



[3]

4-splitting



[3]

Outline of the Proof of Thm 2

Theorem 2 (Nakamoto, N., Ozeki, 2019)
Let T be a loopless Eulerian tri. of the torus.
T has a spanning bipartite quad. if and only if T does not have K_7 as a subgraph.

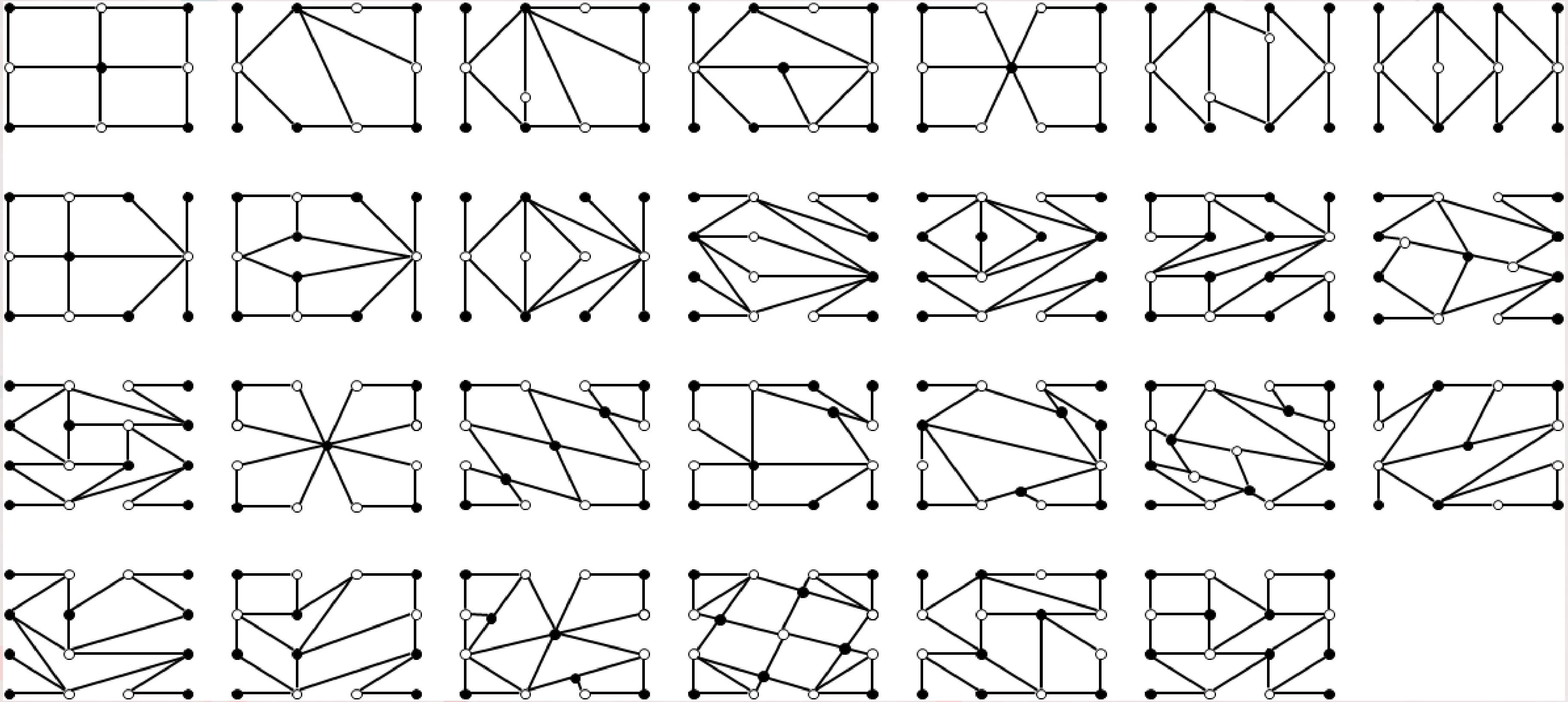
We use the generating theorem.

(i) Confirming that all minimal graphs other than K_7 have a spanning bipartite quad.

(ii) Showing that the bipartiteness of a spanning quad. is preserved under the two operations.

[3]

Spanning bipartite quads.



Conclusion

For a given quad., can we extend it to

[2]	• Eulerian tri.?	Yes
[4]	• 3-colorable tri.?	∃ iff condition
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Thank you!