Students' achievements in solving geometric problems using visual representations in a virtual learning environment

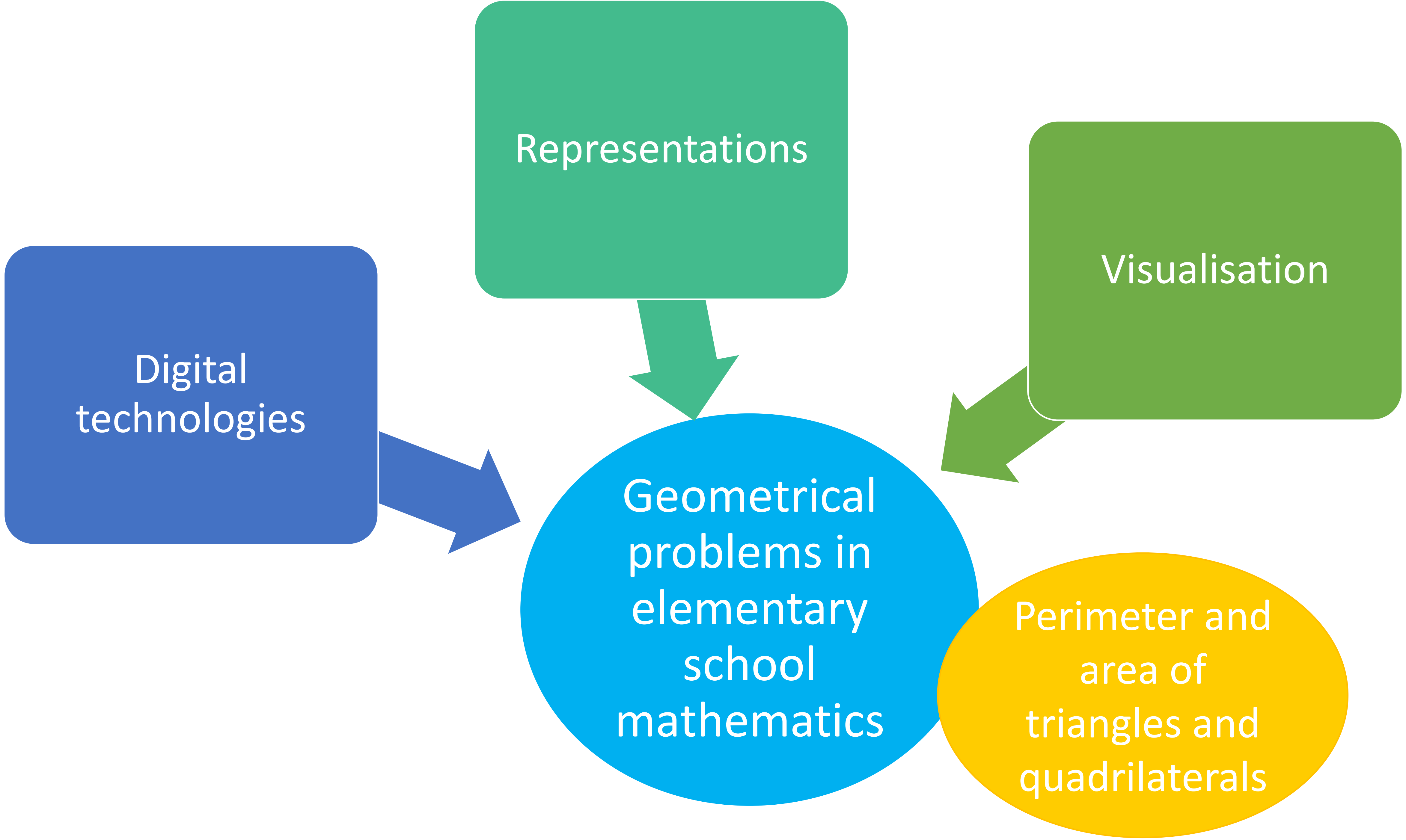
Amalija Žakelj
Faculty of Education, University of Primorska, Koper, Slovenia
amalija.zakelj@pef.upr.si

Andreja Klančar
Faculty of Education, University of Primorska, Koper, Slovenia
andreja.klancar@pef.upr.si

We present the results of an experimental study aimed at exploring

(1) whether geometry learning in a virtual learning environment (rich in various teaching material and activities, including dynamics geometry activities, and instructions) is reflected in higher student achievements in solving geometric problems and

(2) whether solving geometric problems with the aid of visual representations contributes to higher student achievements.

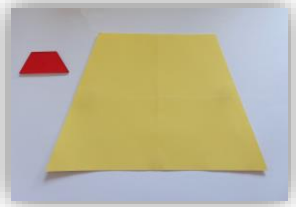


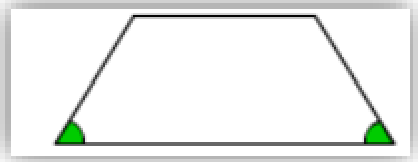
Representations
Visualisation
Digital technologies
Geometrical problems in elementary school mathematics
Perimeter and area of triangles and quadrilaterals

Visualization and development of geometric representations

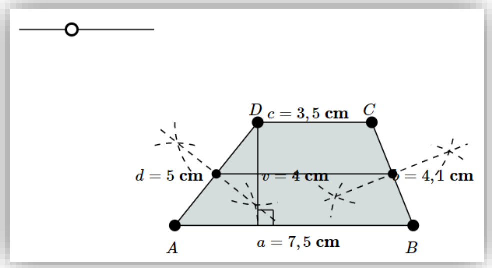
Geometric concepts can be visualized in different ways:

- with concrete physical models,



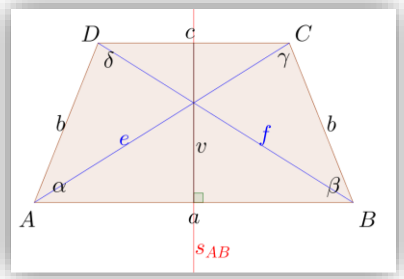


- with static graphic representations (images, schemes, displays ...),



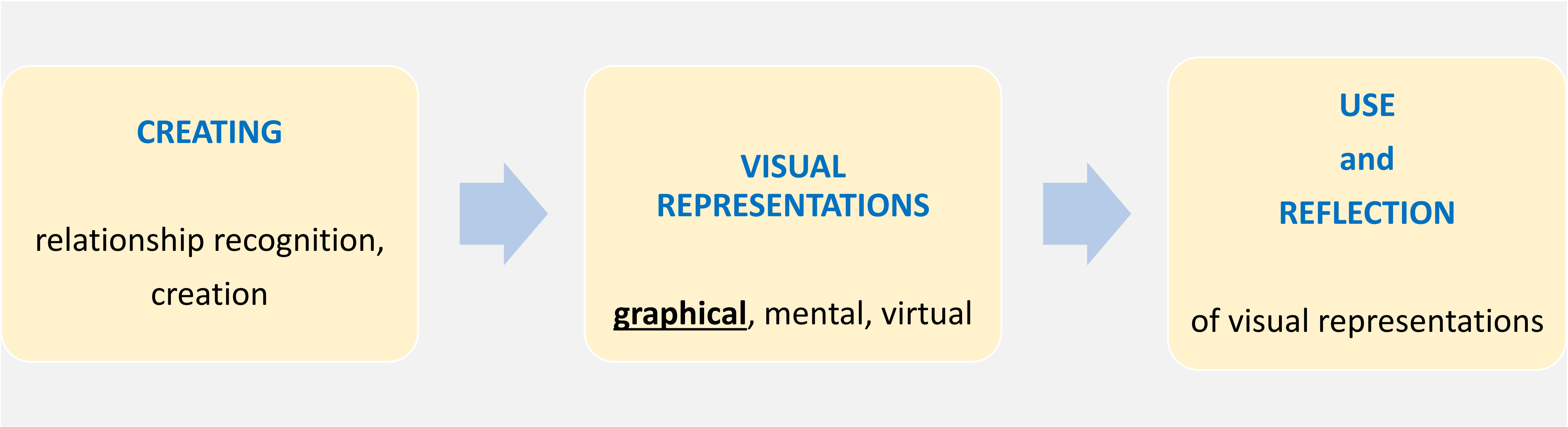
- with dynamic graphic representations (video, applet...),
- with constructed or drawn representations or with representations made with computer program (Kmetič, Miholič and Zobec, 2014).

Good visualization is also provided by computer programs.



Visualization of geometrical concepts

Visualization is the process of creating and using images in mathematical research and understanding of mathematical concepts and problems (Atanasov-Pachemska et. all, 2016).



Picture 2: Visualization process

Purpose of the research

The purpose of the study was to explore

(1) whether geometry learning in a virtual learning environment rich in various teaching materials, activities (including dynamics geometry activities) and instructions is reflected in higher student achievements in solving geometric problems compared to the group of students who studied in traditional way and

(2) whether there are differences in students' achievements in geometric problems with and without visual representations.

EG a model of geometry learning in virtual learning environment	**CG** a model of geometry learning in traditional way (using paper and pencil)
Use of different learning resources, dynamic geometry programs and applets, which fosters visualisation and the exploration of geometric concepts through the manipulation of interactive virtual representations.	In the geometry lesson of the control group, the students used geometric tools (ruler, geotriangle, compass), classic textbooks and notebooks.

Example: Real-life geometric problem: paving yards

- Students use the applet and determine its area using a non-standard unit.

- The applet enables students to manipulate objects to learn by trying to find an appropriate solution.

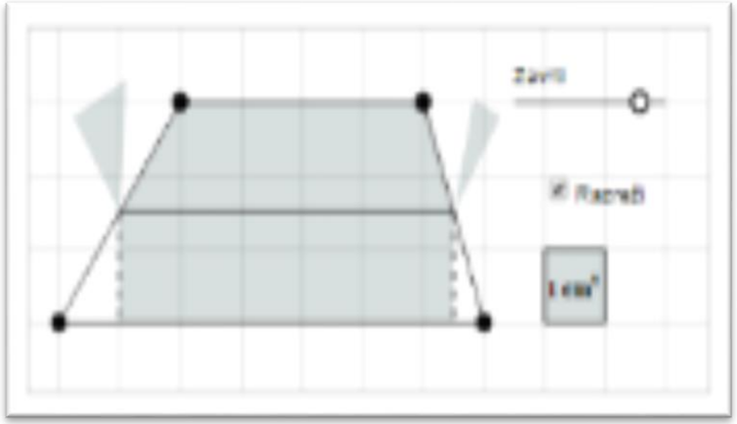
- Paving using an applet enables the development of the conceptual knowledge of trapezoid area.

Example: Using an applet to explore the trapezoid area

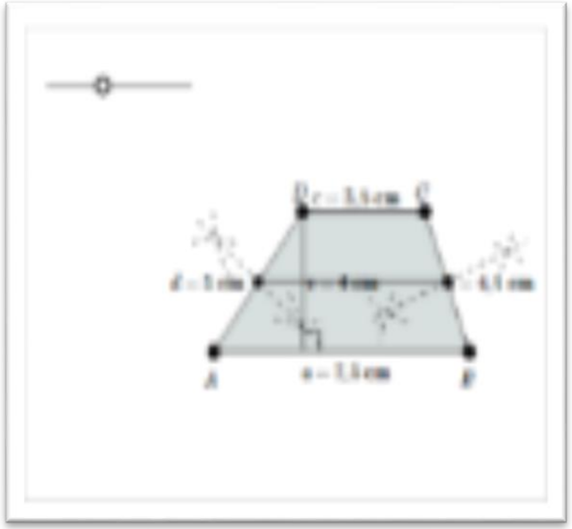
- Students choose between two possibilities of reshaping the trapezoid.

- Students learn the process of reshaping the trapezoid into a rectangle/parallelogram observing an simulation, through which they generalise the findings and write them in symbolic form as a formula.

- The simulation helps students develop conceptual knowledge and form strategies to solve the problem of the algebra text for the area of a trapezoid.



Reshaping the trapezoid into a rectangle



Reshaping the trapezoid into a parallelogram

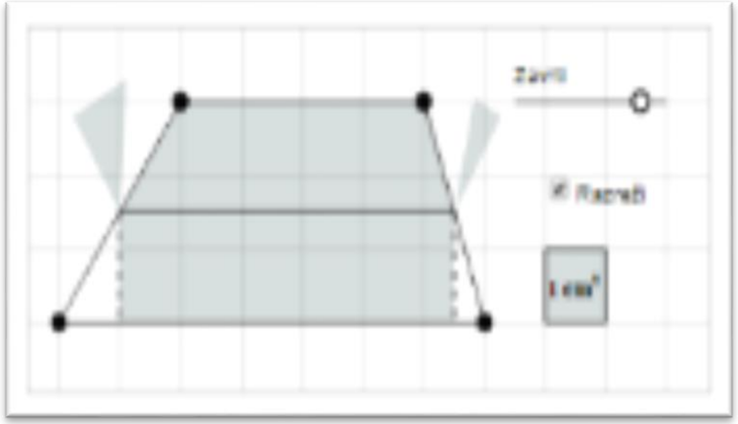
Research questions

- Is geometry learning in a virtual learning environment (rich in various teaching material and activities, including dynamics geometry activities, and instructions) reflected in higher student achievements in solving geometric problems?
- To explore and compare the achievements of students of the experimental group (EG) and control group (CG) in geometric tasks with graphical representations and in geometric tasks without graphical representation.
- To explore the role of visual representations in solving geometric tasks from the point of view of students achievements.

Metodology

A descriptive and causal experimental method of pedagogical research was used.

The learning process was focused on transformation of triangles and quadrilaterals into shapes for which we know how to calculate the area and to find and use a suitable strategy for calculating the area of the figure.



Eksperiment sample and data collection

Sample

n = 125 of 7th grade students (63 EG students and 62 CG students) and 6 math teachers.

Data collection - measurement characteristics

For each measuring instrument, we analyzed (a) objectivity, (b) reliability, (c) validity, and (d) difficulty as well as discriminatory of tasks.

Task with and without graphical representation (GR)

Tasks without graphical representation: data are given in text only; through the process of visualization, the student independently created a graphic representation (eg a sketch) as a solution, or the creation of a graphic representation was a task in itself.

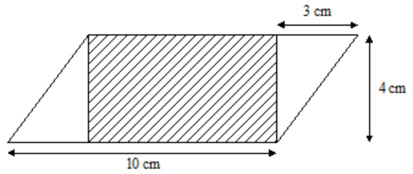
Example 1: An isosceles triangle

Calculate the perimeter and area of an isosceles triangle with a leg of 5 cm, a base of 6 cm, and a height on a base of 4 cm. Draw a sketch.

Tasks with graphical representation: to deduce data, relations between geometric objects, etc. from a GR, and to solve a problem.

Example 2: Rectangle

The sketch shows a shaded rectangle in a parallelogram. Take into account the data written on the sketch and calculate the area of the rectangle.



Slika : Pravokotnik v paralelogramu

Results and interpretation

1. Is geometry learning in a virtual learning environment (rich in various teaching material and activities, including dynamics geometry activities, and instructions) reflected in higher student achievements in solving geometric problems?

1.1 Analysis of differences in geometry knowledge between students of the experimental and control group in the achievement in overall score

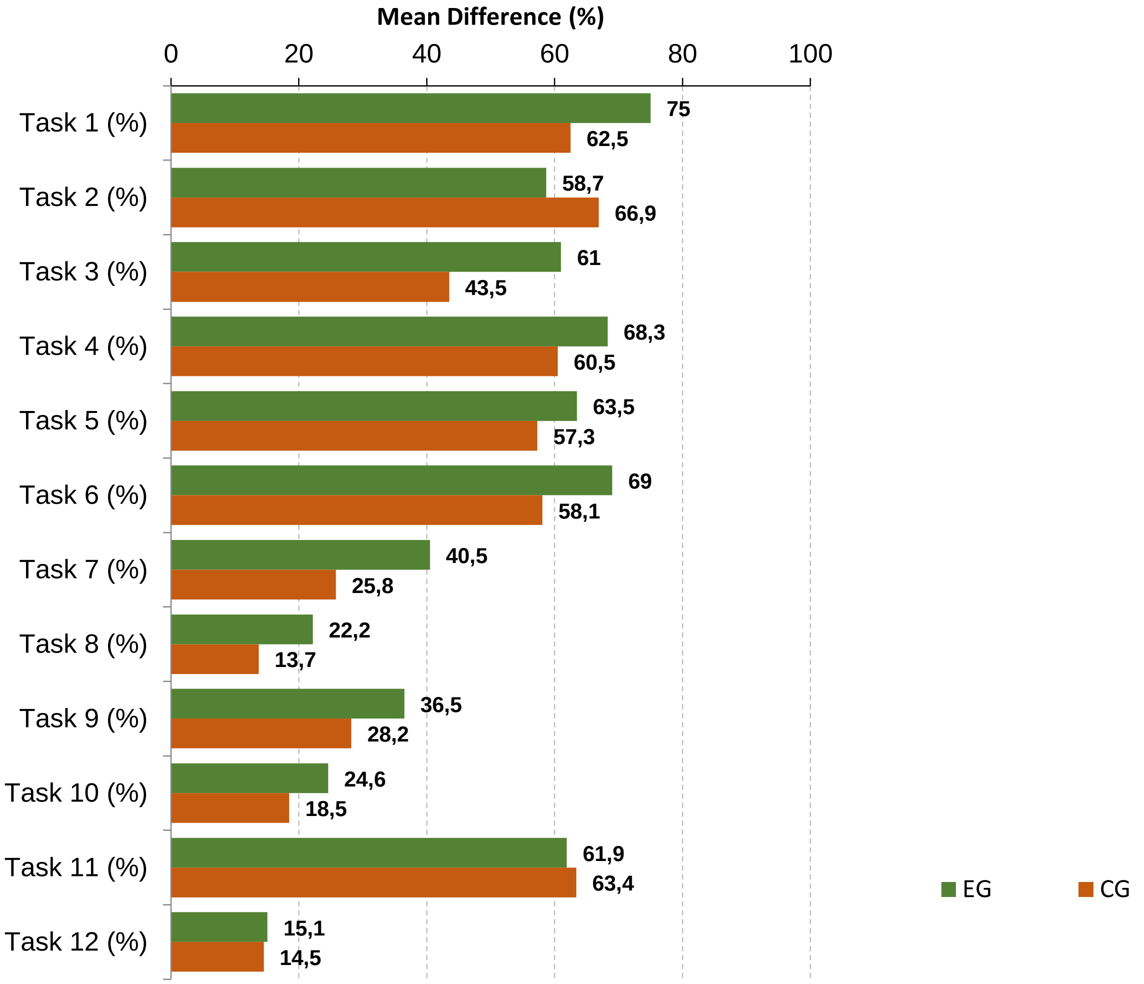
EG
was more successful in the final test
in the total number of points

- EG was more successful in the final test in the total number of points (EC 52.9 %, KS: 44.8 %). The differences are statistically significant (t (123) = 2.015, p = 0.046).
- The EG have benefited from a geometry teaching model using digital technologies in a virtual learning environment rich in a variety of learning materials (multimedia building blocks, e-materials, activities (including dynamic geometry activities).

EG
was more successful in applying the **appropriate strategy for area and perimeter calculation**

- EG was more successful in using an appropriate strategy to calculate the area of a triangle or quadrilateral than CG students. The differences are statistically significant (t (123) = 2.353, p = 0.020).
- EG is more successful in using an appropriate triangle or quadrilateral perimeter calculation strategy than CG students. The differences are statistically significant (t (123) = 2.150, p = 0.034).

1.2 Analysis of differences in geometry knowledge between students of the experimental and control groups in the final state according to the individual task of the final test



Compared to the CG, the EG achieved higher results in 10 of the 12 tasks.

In the three tasks of the final test, the groups differ statistically significantly depending on the success of solving the tasks:

Task 1: $t = 2.029$, $p = 0.045$;

Task 3: $t = 2.725$, $p = 0.007$;

Task 7: $t = 2.131$, $p = 0.035$.

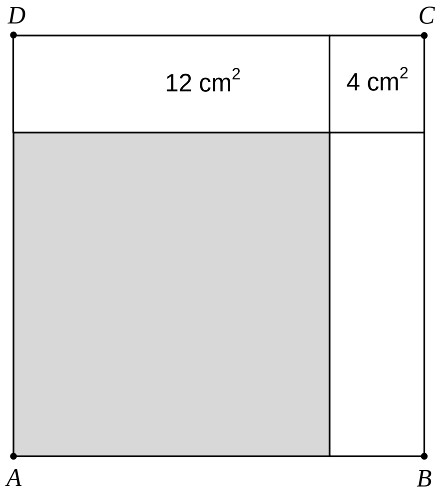
2. To explore and compare the achievements of students of the experimental group (EG) and control group (CG) in geometric tasks with graphical representation and in geometric tasks without graphical representation.

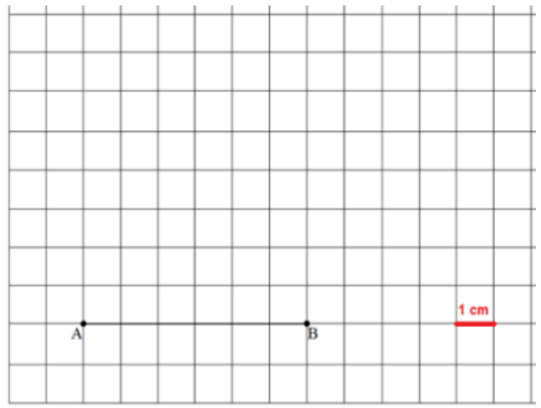
2.1 Analysis of differences in knowledge of geometry between students of the experimental and control groups in the final state according to the tasks with and without graphical representations.

Tasks **with** graphical representation	Tasks **without** graphical representation
Compared to the CG, the EG achieved higher results in most tasks (10/12), both in tasks with graphical representation (EG: 44.6%, CG: 40.6%) and in tasks without graphical representation (ES: 58.5 %; CG: 47.7%).	EG was more successful than CG in tasks without graphical representation, the differences were statistically significant (t-test for independent samples) ($t(123) = 2.409$, $p = 0.017$).

2.2 Achievements of EG and CG students in the analysis and creation of graphical representations

Achievements of EG and CG students in the **analysis** of graphical representations	Achievements of ES and KS students in **creating** graphical representations
EG students were more successful than CG students, but the differences are not statistically significant.	EG students were more successful than CG students, the differences are not statistically significant.





3. To explore the role of visual representations in solving geometric tasks from the point of view of students achievements.

a) Role of graphical representation (GR):

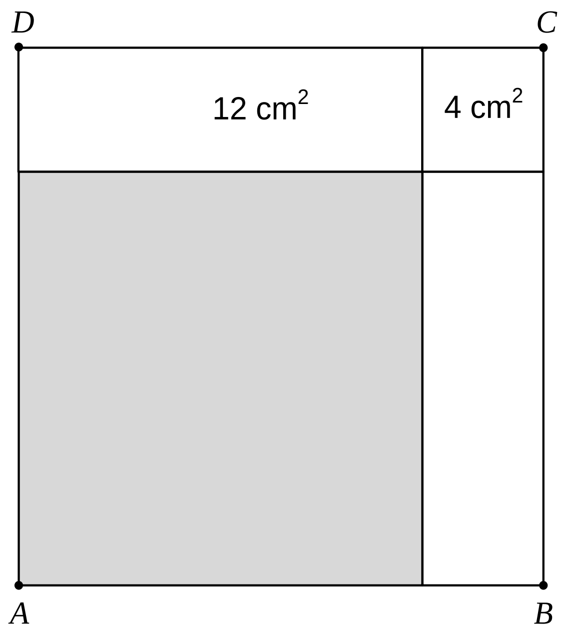
- GR is already created (student is an user)
- GR includes a data (direct or indirect)

The square ABCD is divided into two squares and two congruent rectangles. The area of the smaller square and the area of the rectangle are written in the figure. Calculate the perimeter of the square ABCD. Calculate the area of a shaded square.

The area of the smaller *square and the area of the rectangle are written in the figure.*

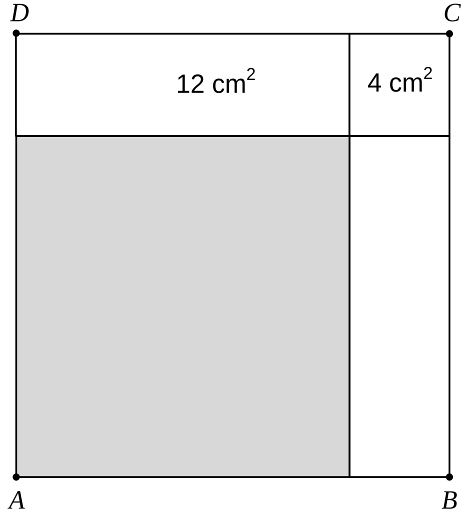
a) The perimeter of the square ABCD is____________ cm.

b) The area of the shaded square is____________ cm^2.



a) Role of graphical representation (GR):

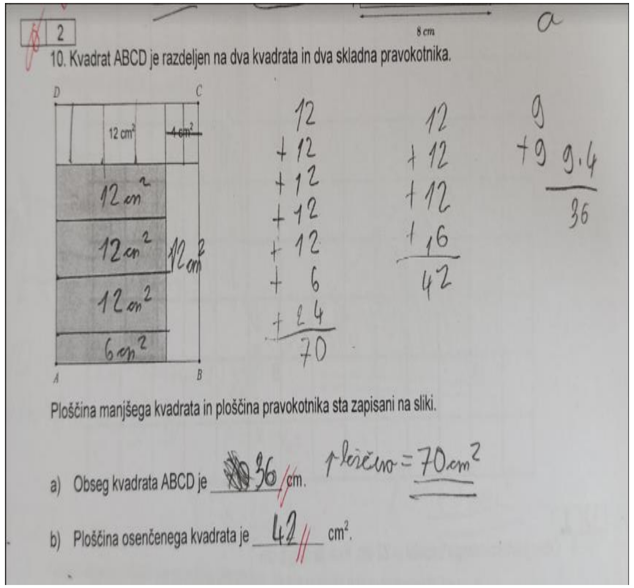
- GR is already created (student is an user)
- GR includes a data(direct or indirect)

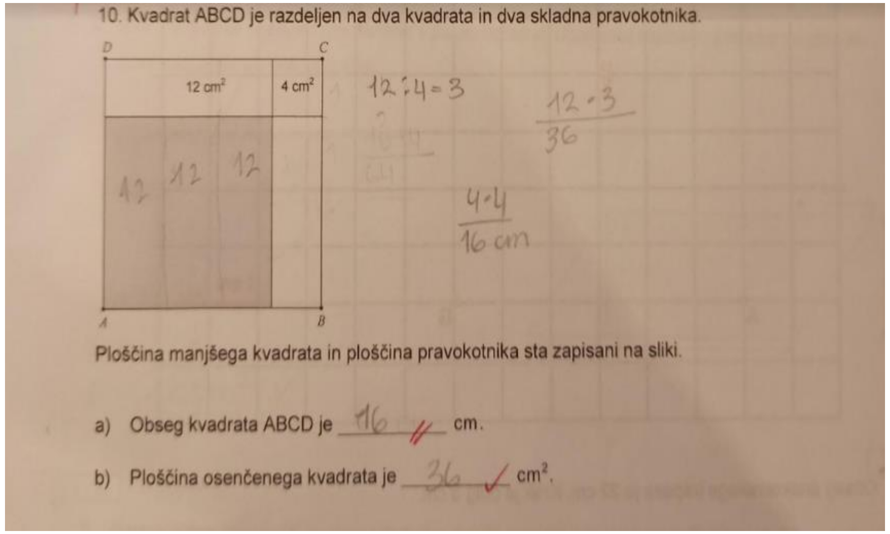


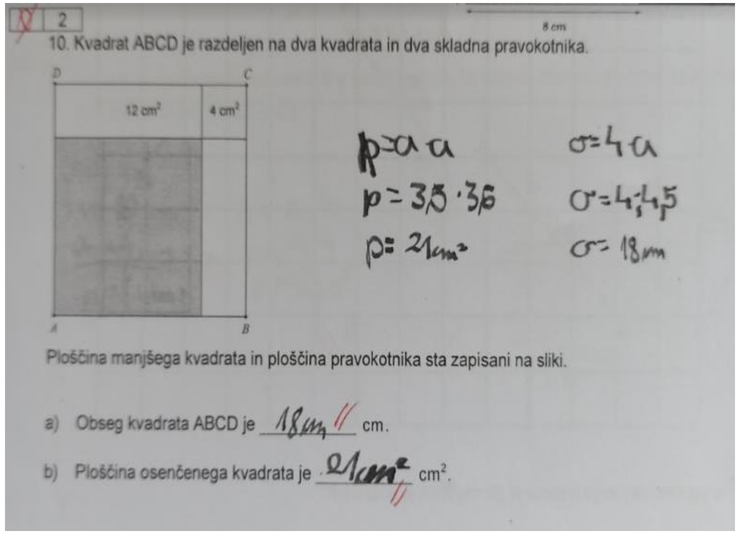
Results

- CG students averaged 18.5% points and EG students 24.6% points, according to the t-test the difference is not statistically significant ($t(123) = 0.937$, $p = 0.351$).
- EG students were more successful in calculating the **perimeter**, they scored 23.8% points, and CG students 17.7% points, according to the t-test the difference is not statistically significant ($t(123) = 0.831$, $p = 0.407$).
- EG students were more successful in calculating the **area**, they scored 25.4% points, and CG students 19.4% points, according to the t-test the difference is not statistically significant ($t(123) = 0.806$, $p = 0.422$).
- Overall students were more successful in calculating **area.**

Examples of student solvings: Critical points in reading data.







The use of formulas is based on the data that they had to obtain from the GR in the tasks and if this data was not obtained, they subsequently unsuccessfully solved the task.

The analysis of the results of the solution shows that aruond one third of the EG students correctly used the strategy for calculating a perimeter in case of incorrectly understood data in the GR (image analysis and recalculation were required). Some of them are e.g. data were obtained by measurement.

b) Role of graphical representation (GR):

- GR (sketch) created by student (student creator)
- the data are given directly in the text

Calculate the perimeter and area of an isosceles triangle with a leg of 5 cm, a base of 6 cm, and a height on a base of 4 cm. Draw a sketch.

Results:

- EG is statistically significantly more successful than CG in solving the **whole task** (t (123) = 2.725, p = 0.007).

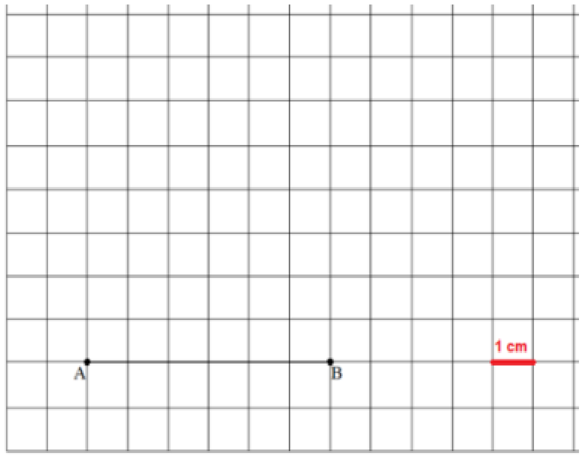
Results between EG in CG by items:

- There are **no differences** in performance when **drawing a sketch** between EG and CG students.
  - EG students are significantly more successful in using an appropriate triangle **perimeter** calculation strategy than KS students (p = 0.029, t (123) = 2.209).
  - EG students are also significantly more successful in using an appropriate triangle **area** calculation strategy than CG students (p = 0.039, t (123) = 2.089).
  - Pearson's correlation coefficient between acievements of EG and CG students for connections between 3.1 (sketch) and 3.2 (**perimeter** calculation strategy) and between 3.1 (sketch) and 3.4 (**area** calculation strategy) indicates that the is low to medium significant connection between sketch and the calculation of perimeter and area.

c) Role of graphical representation (GR):

- GR created by student (student creator)
- reading data from the image and drawing the image (a base for calculation/procedural part)

The side AB of triangle ABC is drawn. Determine the vertex C so that the area of triangle ABC is equal to 12 cm^2. Draw a triangle DEF which has the same area as triangle ABC but is not congruent with triangle ABC.



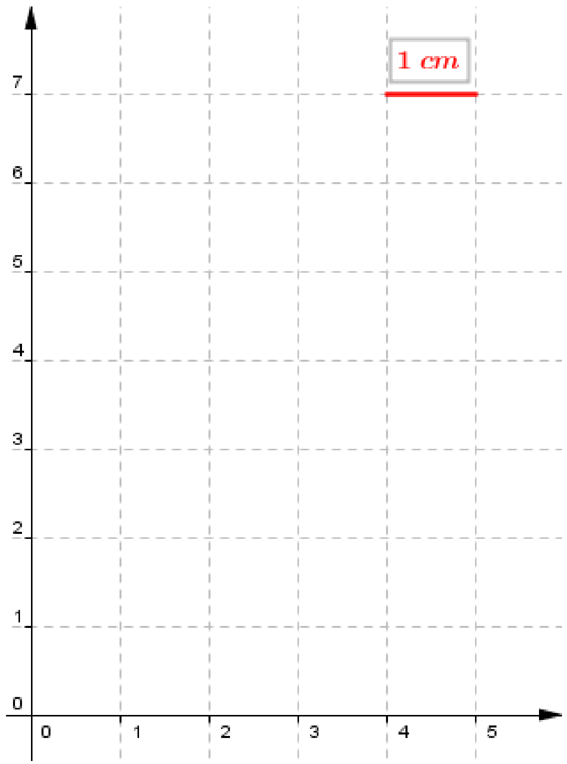
Results:

- CG students scored an average of 25.8% of the points, while **EG students** scored 40.5% of the points. According to the t-test, the differences between EG and CG are statistically significant ($t(114{,}006) = 2{,}131$, $p = 0{,}045$) in favor of the EG.

d) Role of graphical representation (GR):

- GR created by student (student creator)
- the GR contains data for the procedural part

Draw a figure with data A (0, 3), B (1, 0), C (2, 3) and D (1, 4) in the coordinate grid with a given unit. Name the figure. Read the necessary data from the GR and calculate the area of the figure.



Task objectives: drawing graphical representation (11.1) – GR contains data for the procedural part (11.3). The success of creating GR is a condition for calculating the area.

d) Role of graphical representation (GR):

- GR created by student (student creator)
- the GR contains data for the procedural part

Results:

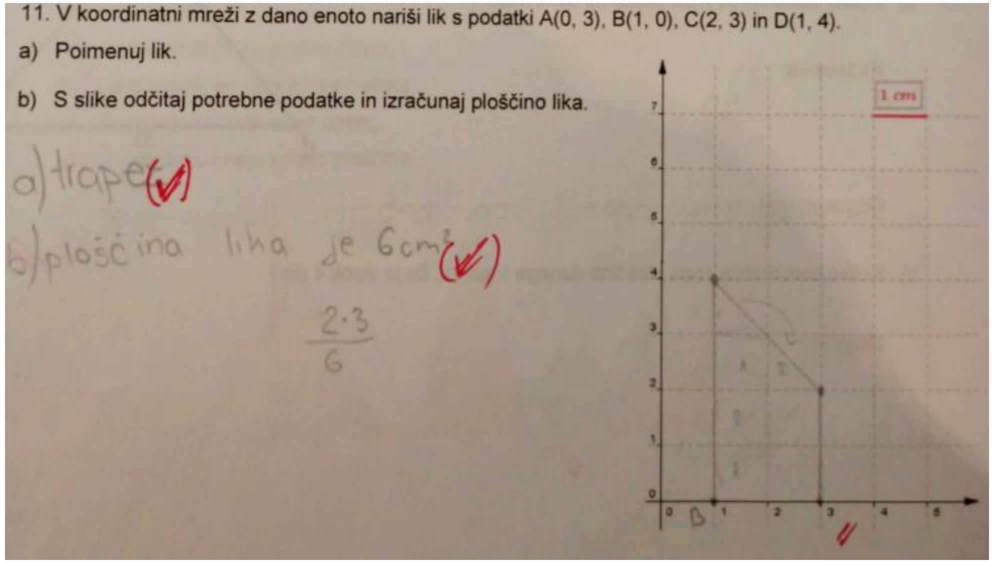
- **CG** students scored an average of **63.4 %** of the points, while EG students scored 61.9 % of the points.

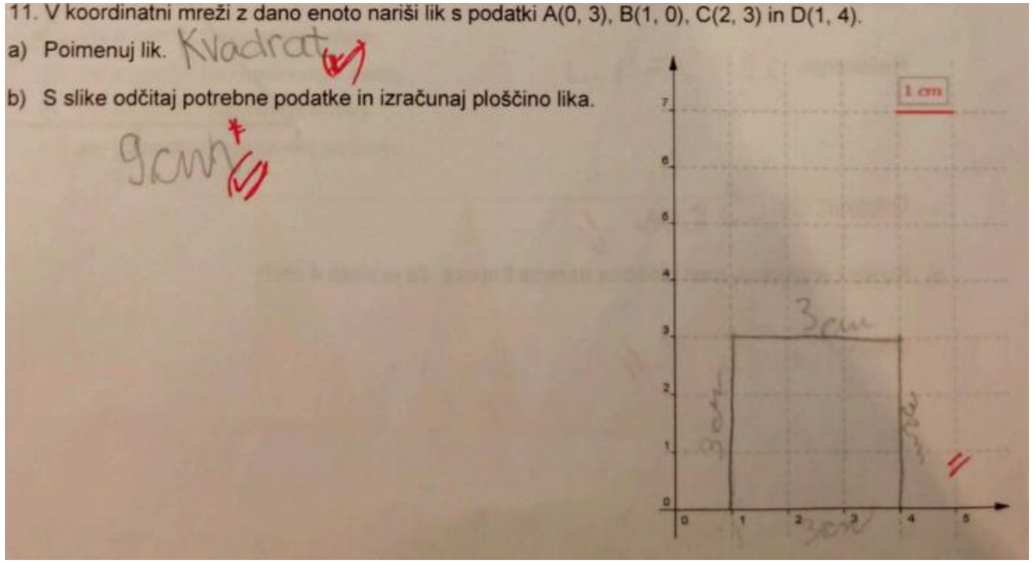
- image drawing:
  - **CG** students scored an average of **70.9 %** of the points, while EG students scored 53.9 % of the points.

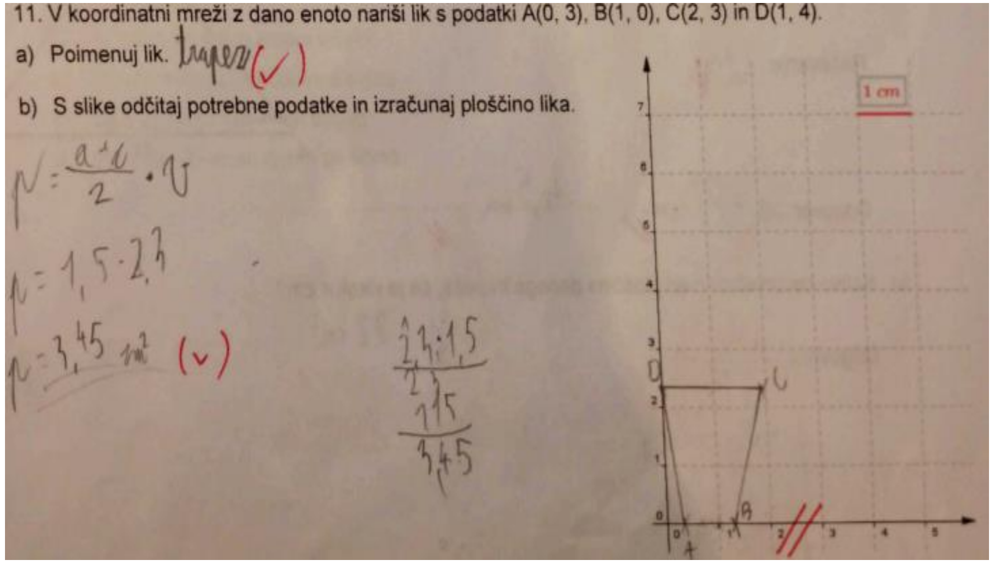
- **application of area calculation strategy:**
  - CG students scored an average of 37.1 % of the points, while **EG** students scored **60.3 %** of the points ($t = 2.649$, $\mathbf{p = 0.009}$)

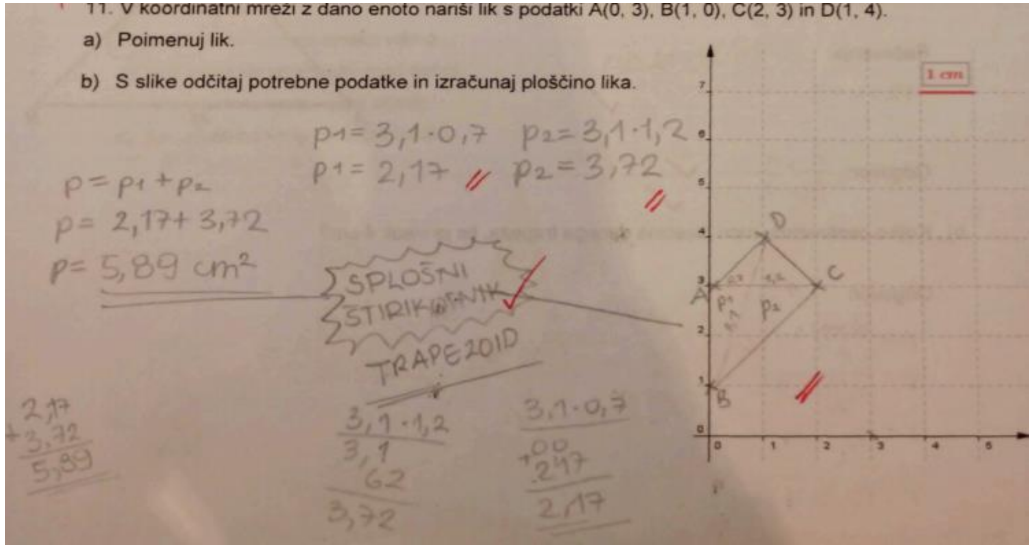
- For both EG and CG, we determine the **correlation** between graphical representations and the calculation of area.
  - EG: ($r = 0{,}619$, $2p = 0{,}000$) medium correlation
  - CG: ($r = 0{,}344$, $2p = 0{,}006$) weak correlation

Examples of students' solvings: Critical points









Role of graphical representations

Different types:

- The graphical representation (GR) is already created (student is an user)
    - GR includes the data (direct or indirect)

- The graphical representation is created by student (student in a creator, GR is a solution):
    - drawing a sketch + given direct data in the text
    - drawing the GR, which contains data for the procedural part

- Combination: reading data from a GR, then calculating and drawing GR (basis calculation or procedural part), GR is a solution

Conclusions

- a **positive impact of the model of teaching** (a model of geometry learning in virtual learning environment) **on EG students achievements.**
- **EG students achieved higher results** in most tasks, both in tasks with graphic representation and in tasks without graphic representation. For tasks (task as a whole) **without graphical representation, the differences are statistically significant** in favor of the EG.
- EG students are significantly more successful than CG students in **calculating area and perimeter**.
- EG students are <u>more successful in analyzing and reading data from graphical representations</u> than CG students, but the differences are not statistically significant.
- EG students are <u>more successful in creating graphic representations</u> (subtasks) than CG students, but the differences are not statistically significant.
- For both EG and CG, <u>we determine the correlation</u> (from low/weak to medium correlation) <u>between graphical representations and the calculation of area and perimeter.</u>

- The results point out **that visualizing a concept or problem is more challenging for students than the procedural part of the task itself.** Similar findings were made by the authors Hegarty and Kozhevnikov (1999), who state that the visualization of a concept or problem is often more difficult for students than the procedural part itself.

- Qualitative analysis of the results draws attention to **neuralgic points in students' knowledge,** especially in those segments of **visualization** in which:
  - the recognition of relationships between geometric objects in GR is expected and consequently the recognition of data GR and
  - independent image creation is expected.

- In case of difficulties in analyzing the GR, students resorted to other possibilities to access the data (the data were obtained by measurement).


Thank You
For Your Atten