Topological explorations in neuroscience

Public Lecture

8th European Congress of Mathematicians

21 June 2021

Blue Brain Project

EPFL

$\rightarrow$ Bimod

$M \tilde{\otimes} N$

$P \otimes Q$

$\rightarrow O_P$

$$F_P(V) \tilde{\otimes} F_Q(W) \cong F_{P \otimes Q}(V \square W)$$

$$(P \circ V \circ P) \square (Q \circ W \circ Q) \xrightarrow{\tau_{V,W}} (P \otimes Q) \circ (V \square W) \circ (P \otimes Q)$$

$$((p; v_1, \dots, v_k; p_1, \dots, p_m), (q; w_1, \dots, w_l; q_1, \dots, q_n)) \mapsto (p \otimes q; (v_i w_j)_{i,j}; (p_r \otimes q_s)_{r,s})$$

$$\forall\ \frac{a : F_P V \longrightarrow F_P V'}{a^\# : V \longrightarrow P \circ V' \circ P}, \quad \frac{b : F_Q W \longrightarrow F_Q W'}{b^\# : W \longrightarrow Q \circ W' \circ Q},$$

$$\exists\ V \square W \xrightarrow{a^\# \square b^\#} (P \circ V' \circ P) \square (Q \circ W' \circ Q) \xrightarrow{\tau_{V',W'}} (P \otimes Q) \circ \dots$$

$$\underbrace{\left(\tau_{V',W'} \circ (a^\# \square b^\#)\right)^\flat}_{=: a \tilde{\otimes} b} : F_{P \otimes Q}(V \square W) \longrightarrow F_{P \otimes Q}(V' \square W')$$

$$\dots) \xrightarrow{\tau_{V,W}} (P \otimes Q) \circ (V \square W) \circ (P \otimes Q)$$

$$\downarrow (P \otimes Q) \circ (a^\# \square b^\#) \circ (P \otimes Q)$$

$$\dots Q^{\circ 2}) \rightarrow (P \otimes Q) \circ ((P \circ V' \circ P) \square (Q \circ W' \circ Q)) \circ (P \otimes Q)$$

$$\downarrow (P \otimes Q) \circ \tau_{V',W'} \circ (P \otimes Q)$$

$$\circlearrowright \qquad (P \otimes Q)^{\circ 2} \circ (V' \square W') \circ (P \otimes Q)^{\circ 2}$$

$$\downarrow \mu_{P \otimes Q} \circ (V' \square W') \circ \mu_{P \otimes Q}$$

$$\dots V' \circ Q) \xrightarrow{\tau_{V',W'}} (P \otimes Q) \circ (V' \square W') \circ (P \otimes Q)$$

$$\mathcal{L}_P(P) \tilde{\otimes} Q \cup P \tilde{\otimes} \mathcal{L}_Q(Q)$$

$$\mathcal{L}_P(P) \tilde{\otimes} \mathcal{L}_Q(Q) \qquad P \otimes Q$$

$$\wedge\!\!\! h$$

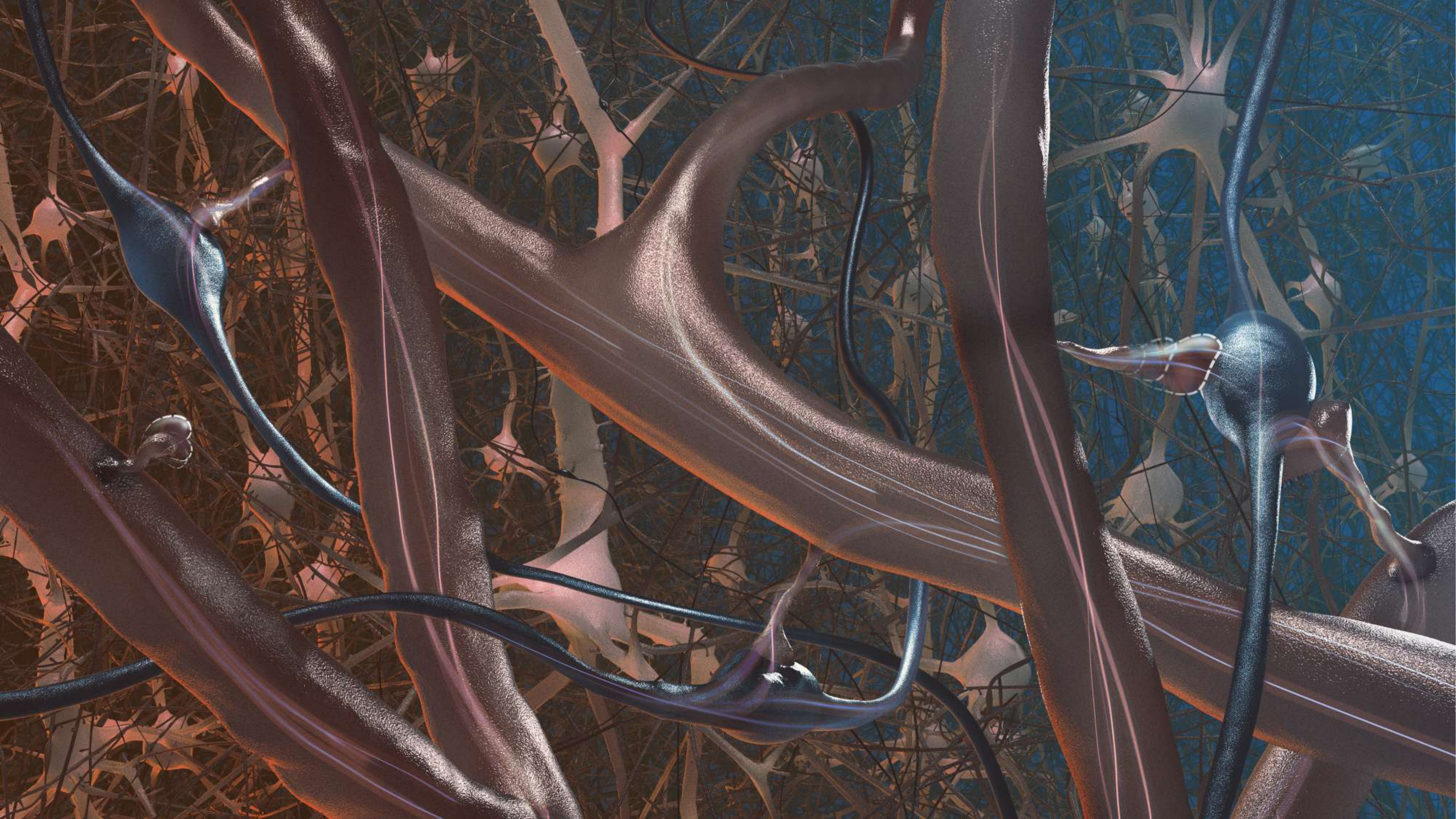
$$\mathcal{L}_{P \otimes Q}(P \otimes Q)$$

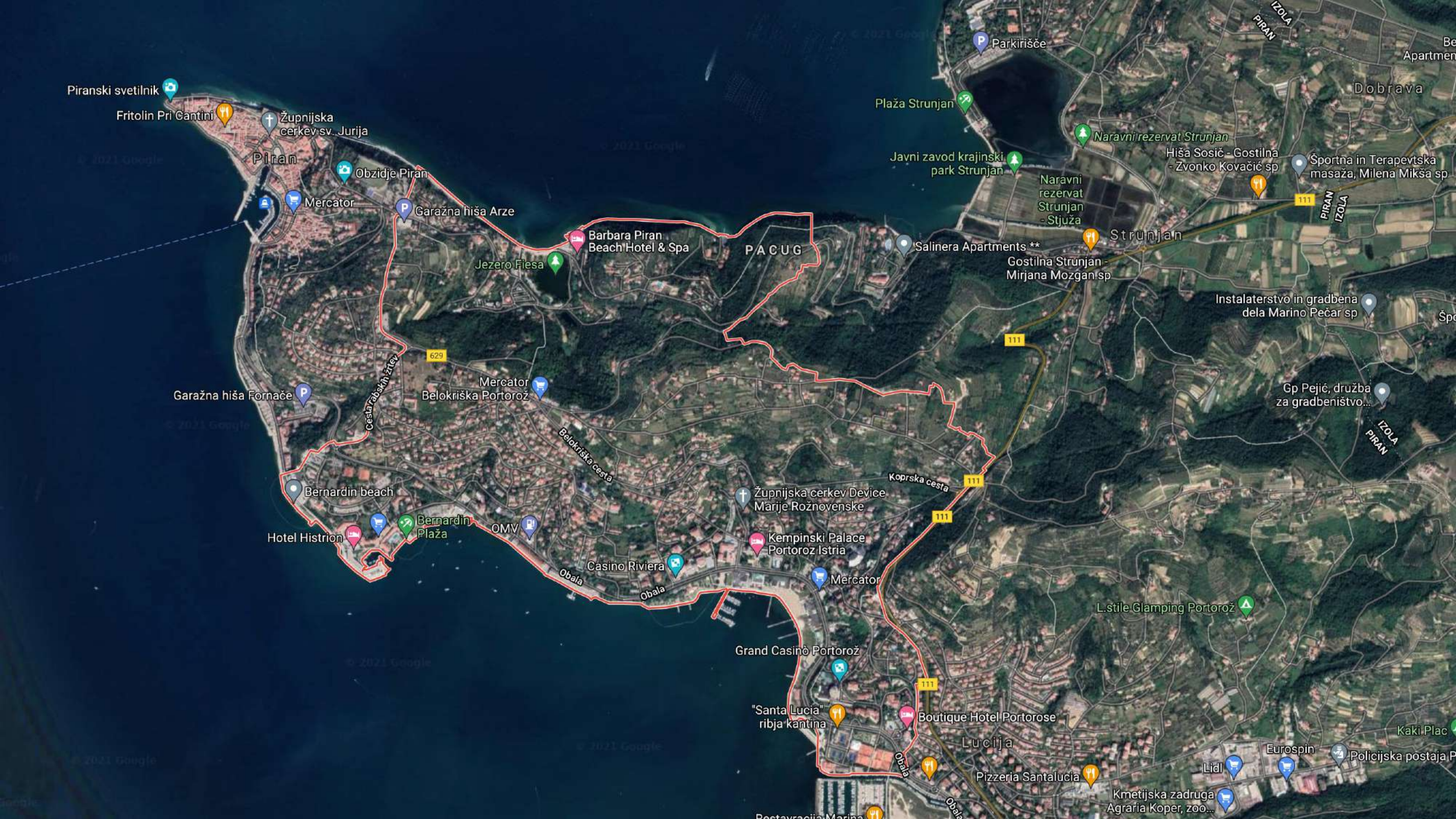
$$(a' \tilde{\otimes} b') \circ (a \tilde{\otimes} b) \stackrel{?}{=} (a' \circ a) \tilde{\otimes} (b' \circ b)$$

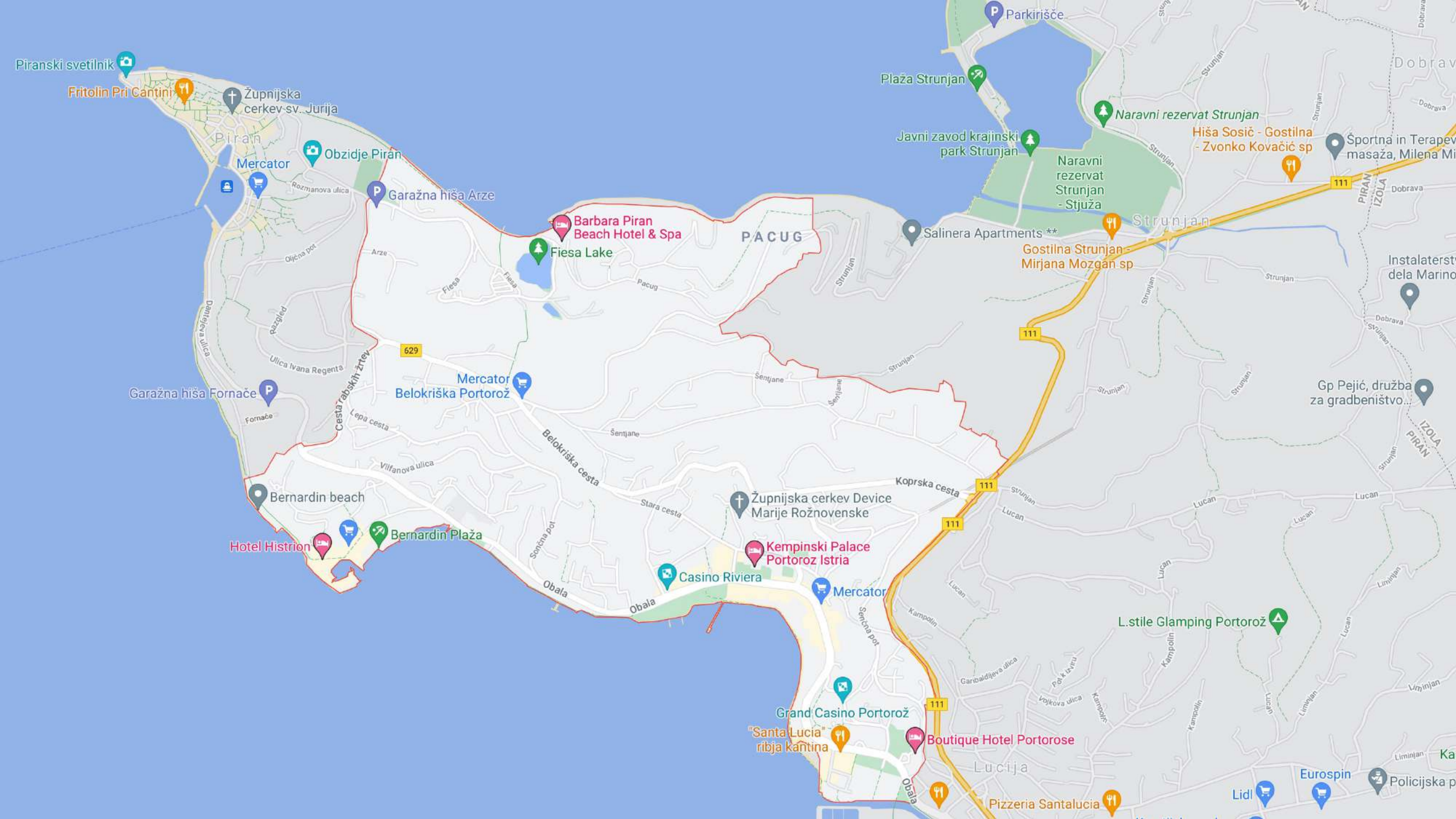
$$V \xrightarrow{a^\#} F_P V' \xrightarrow{a'} F_P V'' \qquad \circlearrowright \qquad (a'a)^\#$$

Me

?




Piranski svetilnik
Fritolin Pri Cantini
Župnijska cerkev sv. Jurija
Piran
Obzidje Piran
Mercator
Garažna hiša Arze
Barbara Piran Beach Hotel & Spa
Jezero Fiesa
PACUG
Parkirišče
Plaža Strunjan
Naravni rezervat Strunjan
Javni zavod krajinski park Strunjan
Naravni rezervat Strunjan - Stjuža
Hiša Sosič - Gostilna - Zvonko Kovačič sp
Športna in Terapevtska masaža, Milena Mikša sp
Dobrava
Salinera Apartments **
Strunjan
Gostilna Strunjan - Mirjana Mozgan sp
Instalaterstvo in gradbena dela Marino Pečar sp
629
111
Mercator Belokriška Portorož
Garažna hiša Fornače
Cesta Tabskih žrtev
Belokriška cesta
Gp Pejić, družba za gradbeništvo...
IZOLA
PIRAN
Koprska cesta
Bernardin beach
Župnijska cerkev Device Marije Rožnovenske
Bernardin Plaža
OMV
Hotel Histrion
Kempinski Palace Portoroz Istria
Casino Riviera
Mercator
Obala
L.stile Glamping Portorož
Grand Casino Portorož
"Santa Lucia" ribja kantina
Boutique Hotel Portorose
Lucija
Kaki Plac
Eurospin
Lidl
Policijska postaja
Pizzeria Santalucia
Kmetijska zadruga Agraria Koper, zoo...
Restavracija Marina


Piranski svetilnik
Fritolin Pri Cantini
Župnijska cerkev sv. Jurija
Piran
Obzidje Piran
Mercator
Rozmanova ulica
Garažna hiša Arze
Barbara Piran Beach Hotel & Spa
Fiesa Lake
PACUG
Pacug
Arze
Fiesa
Oljčna pot
Razgled
Dantejeva ulica
Ulica Ivana Regenta
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Fornače
Lepa cesta
Šentjane
Belokriška cesta
Vilfanova ulica
Bernardin beach
Hotel Histrion
Bernardin Plaža
Stara cesta
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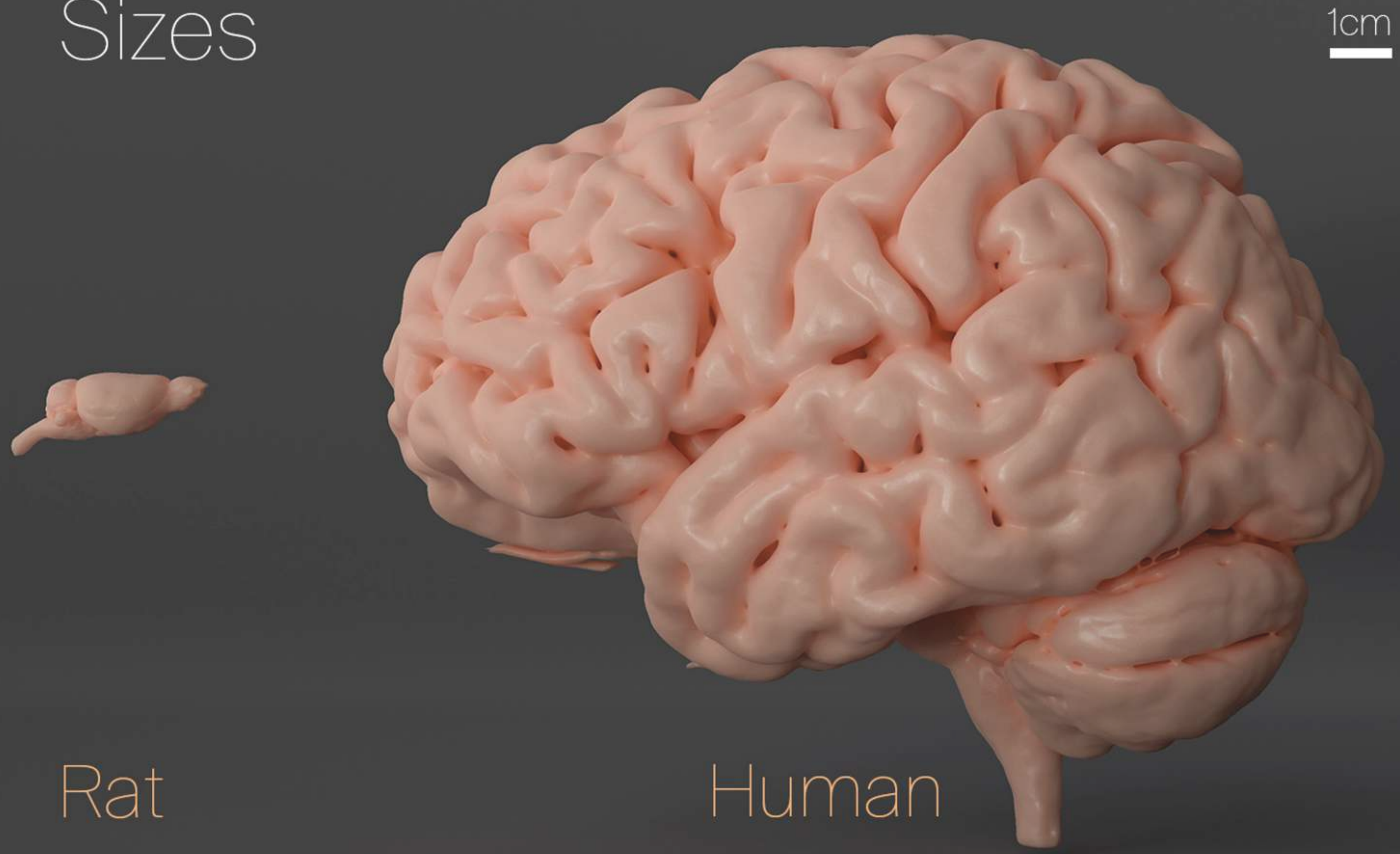
Gp Pejić, družba za gradbeništvo...
Eurospin
Lidl
Liminjan

What is the Blue Brain Project??

Sizes



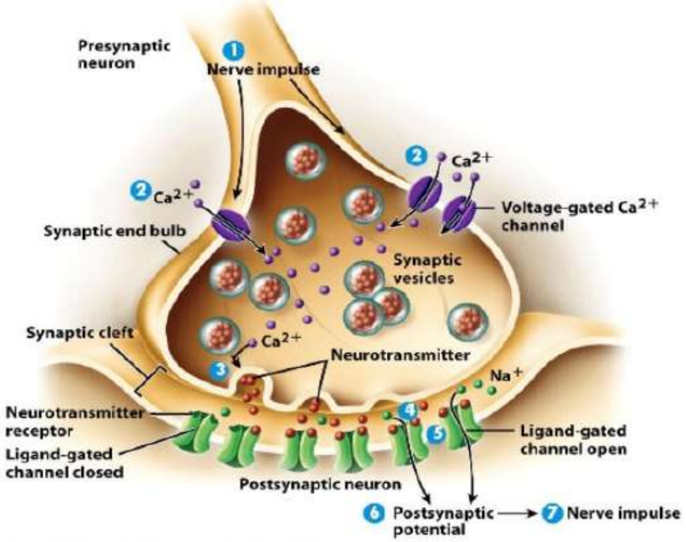
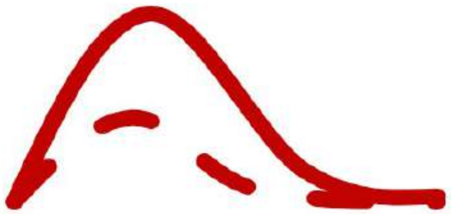
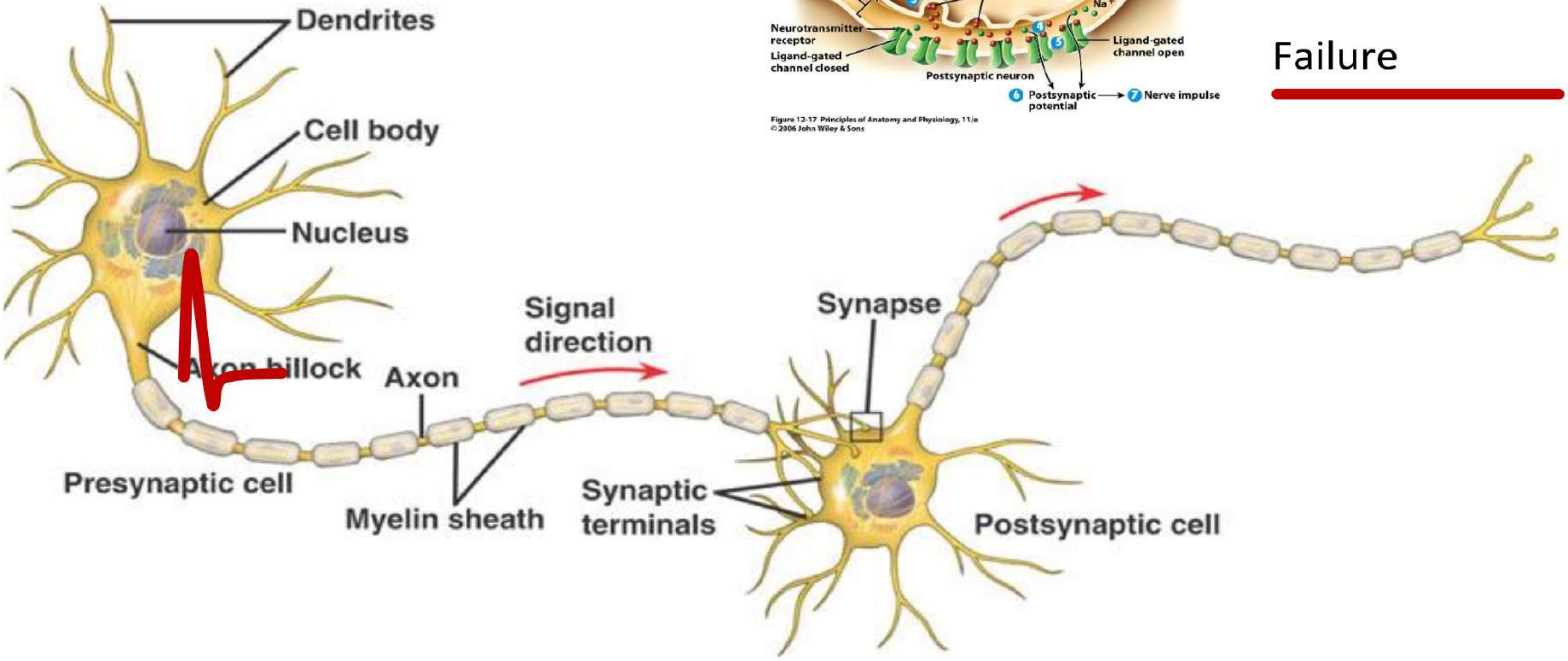


Figure 12-17 Principles of Anatomy and Physiology, 11/e

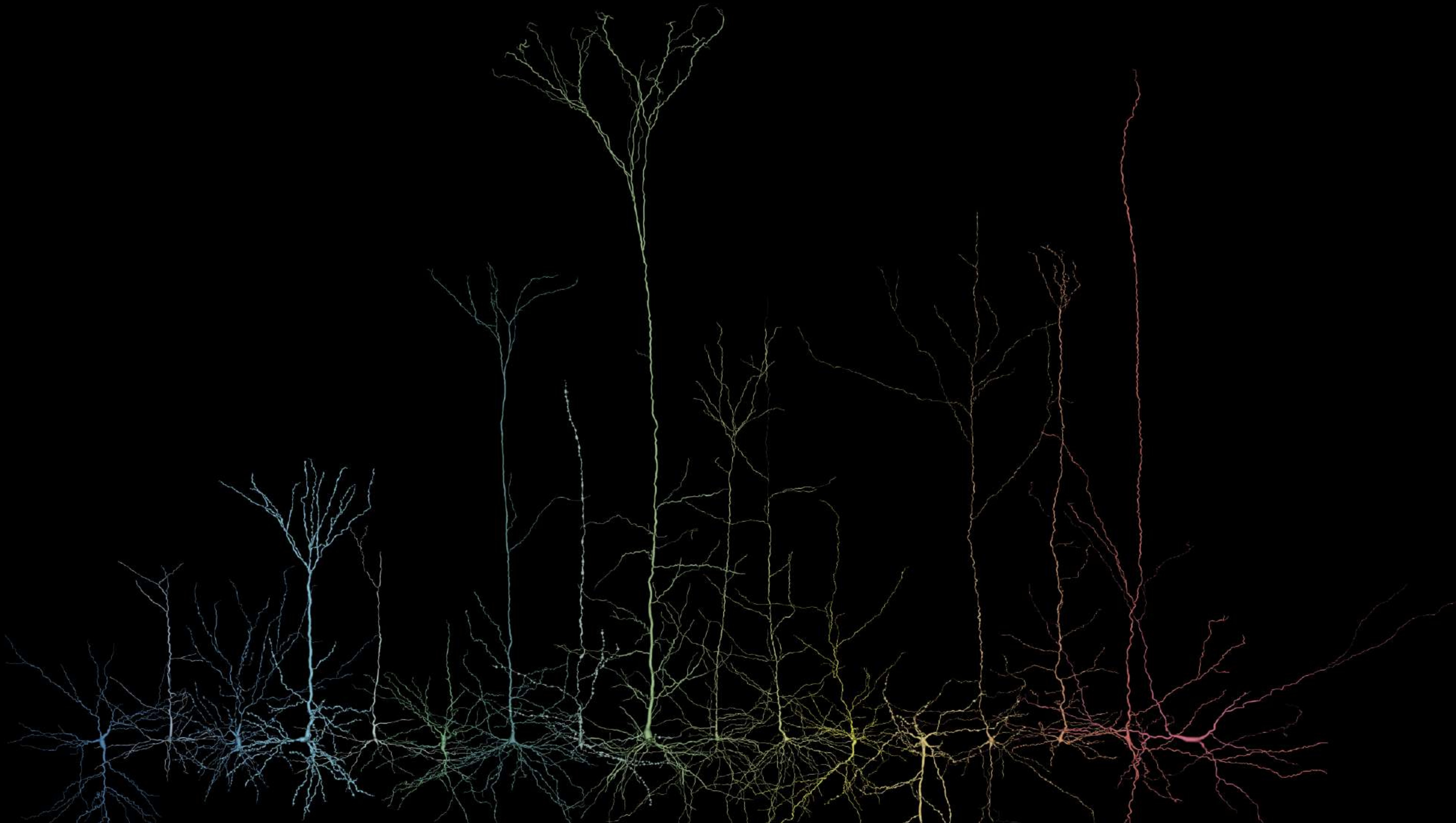
Response

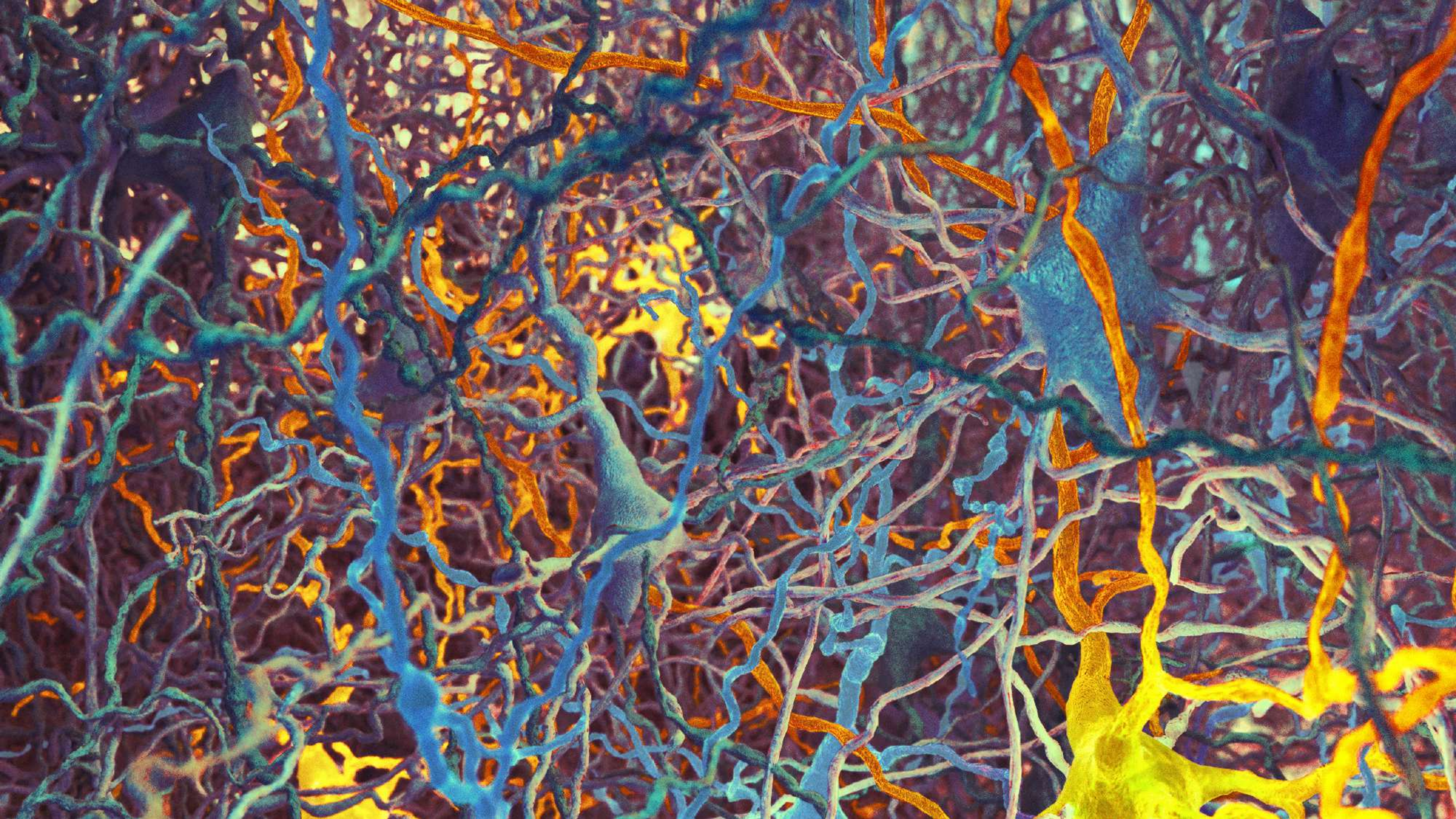


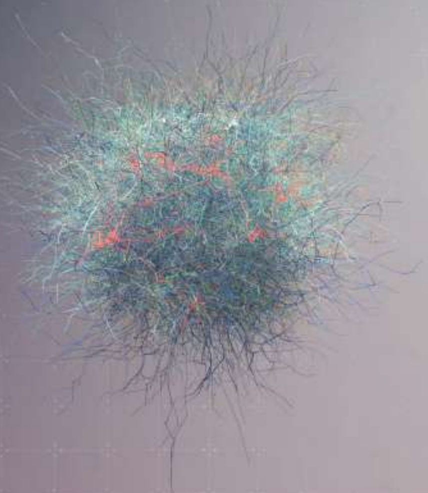
Failure



From pmgbiology.com



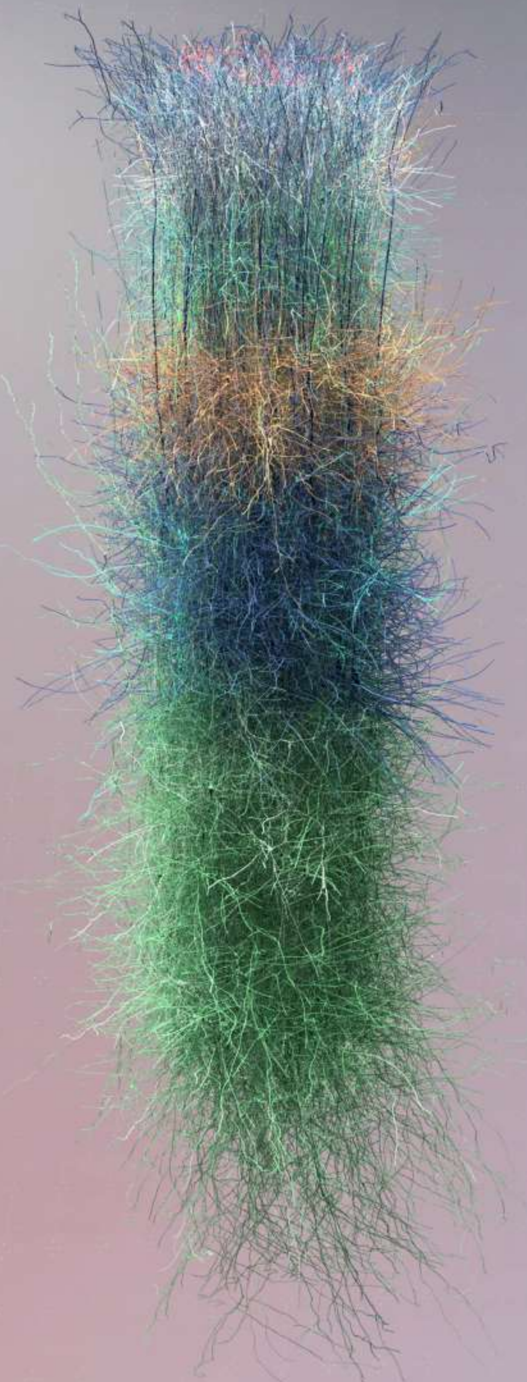


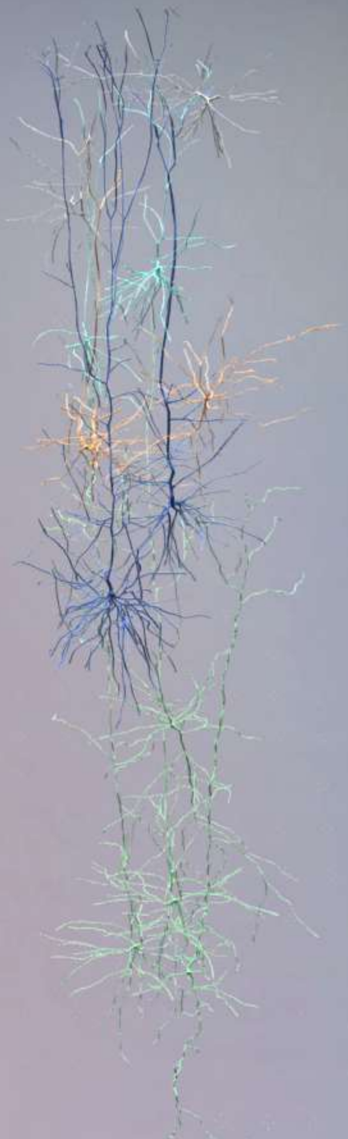


Neuronal distribution in the rat somatosensory neocortex

Circuit:	cxs1_v5.r0
Size:	31'346 neurons
Target:	mc2_Column
Visualisation:	1'000 neurons
Coloring:	per layer 1 to 6 and per synaptic class

Vis. eng. & design: Nicolas Antille
Scientific owners: Henry et al. Cell, 2015





Excitatory samples

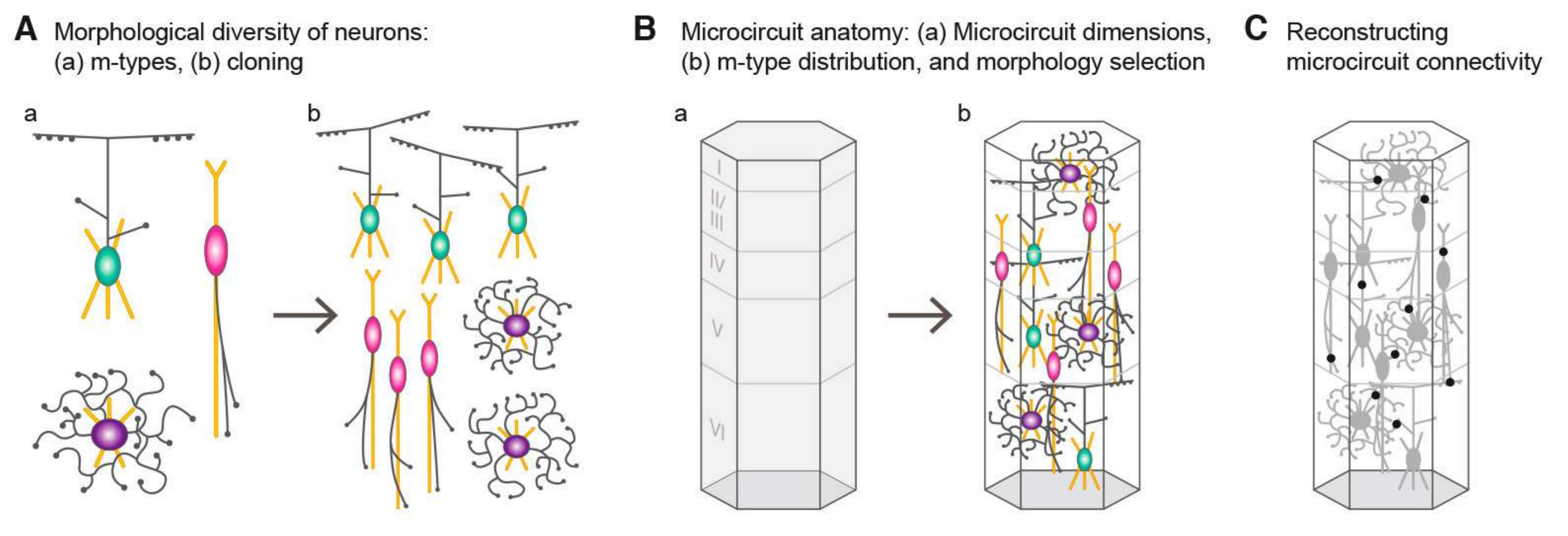
~80% of neural cells are excitatory

Inhibitory samples

~20% of neural cells are inhibitory

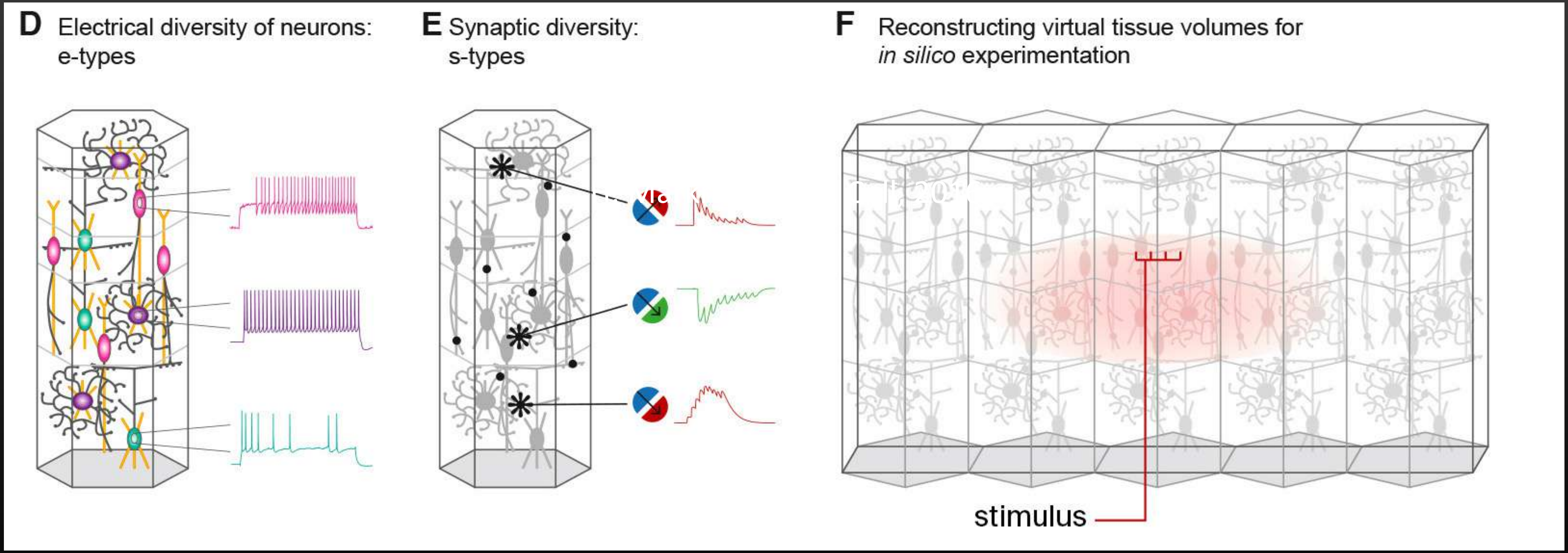
1000μm

Workflow: anatomy



H. Markram et al., Cell, 2015

Workflow: physiology



H. Markram et al., Cell, 2015

Why?

Why?

Study the emergent structural and functional properties of the microcircuit.

Why?

Study the emergent structural and functional properties of the microcircuit.

Study neurological disorders and neuroprostheses *in silico*.

Why?

Study the emergent structural and functional properties of the microcircuit.

Study neurological disorders and neuroprostheses *in silico*.

Reduce the need for animal testing in laboratory experiments.

Topological analysis of the microcircuit

Reimann et al. Frontiers in Computational Neuroscience, 2017.


shape
deformation
continuous
connectivity
path
cavity
invariants
geometry
connected
donut
classification
simplex
open
topology
proximity
mug
homology
equivalence
algebra
closed
continuity
complex

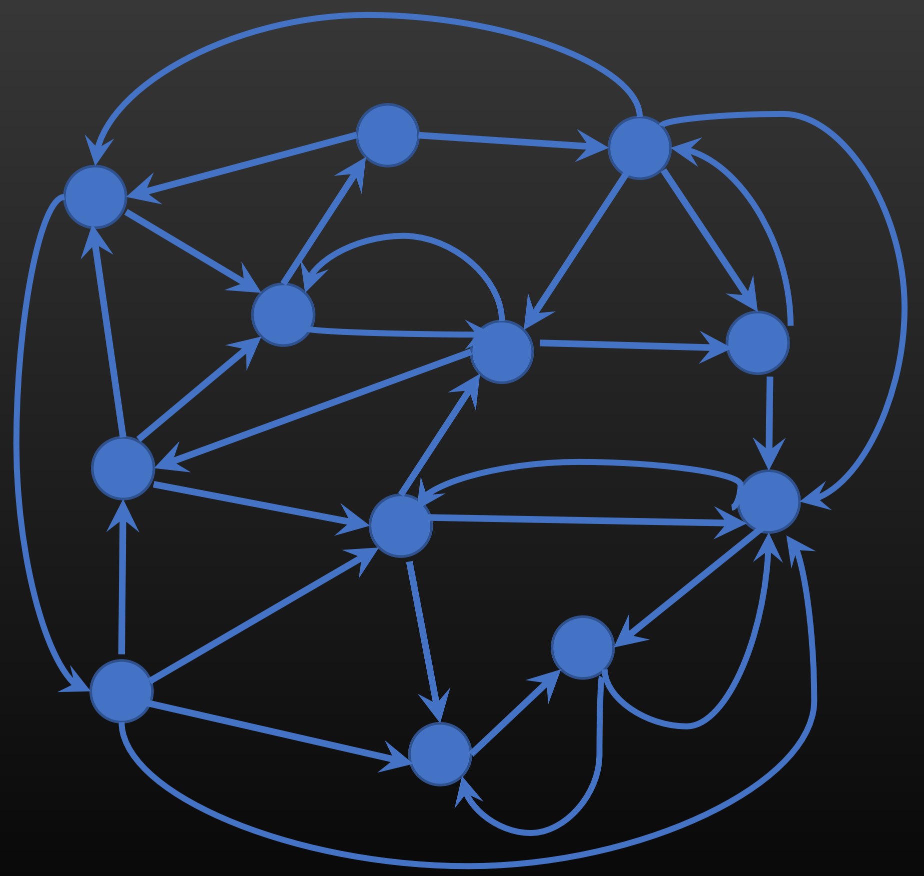
Topology is…

- the mathematics of shape;

Wikipedia, no license

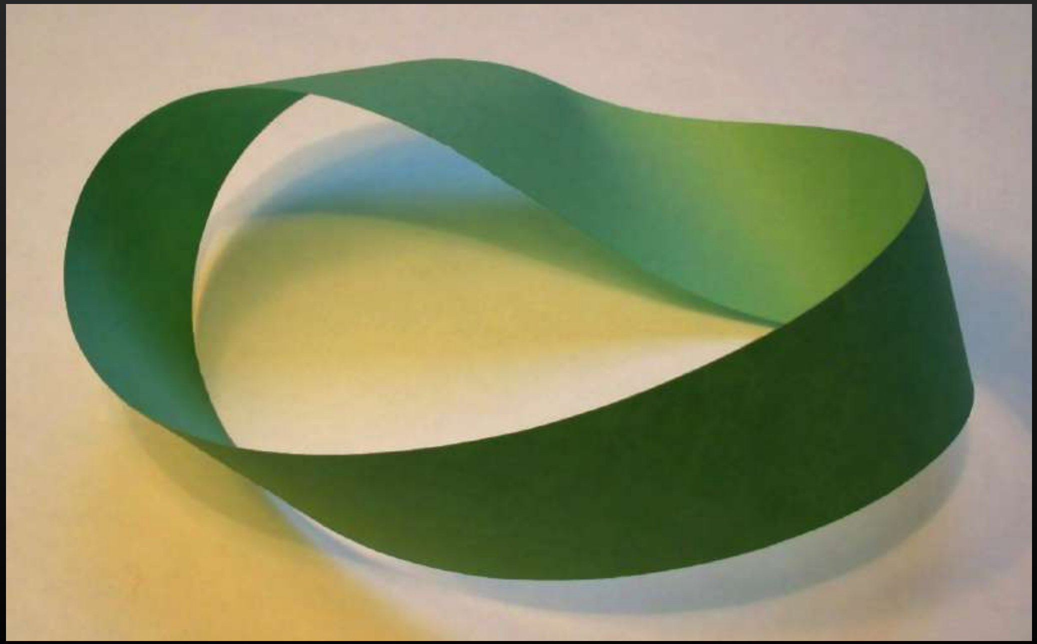
Topology is…

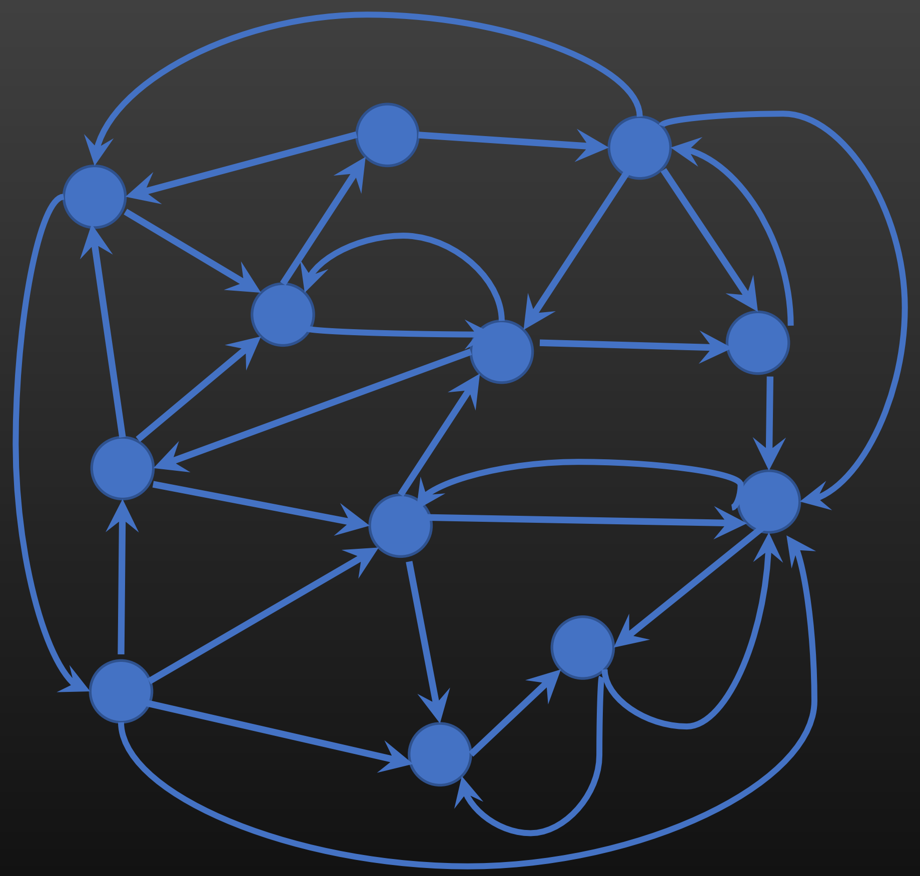
- the mathematics of shape;
- the mathematics of connectivity;



Topology is…

- the mathematics of shape;
- the mathematics of connectivity;
- the mathematics of emergence of global structure from local constraints.

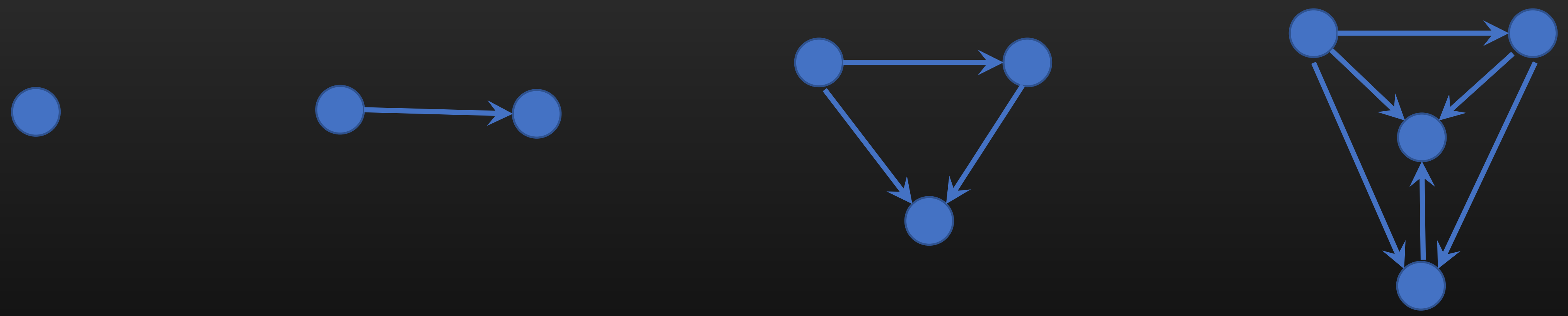




From networks to topology

From networks to topology

- Analyze the huge network of directed connections among neurons in terms of much smaller significant subnetworks.



Directed simplices of dimensions 0, 1, 2, and 3

From networks to topology

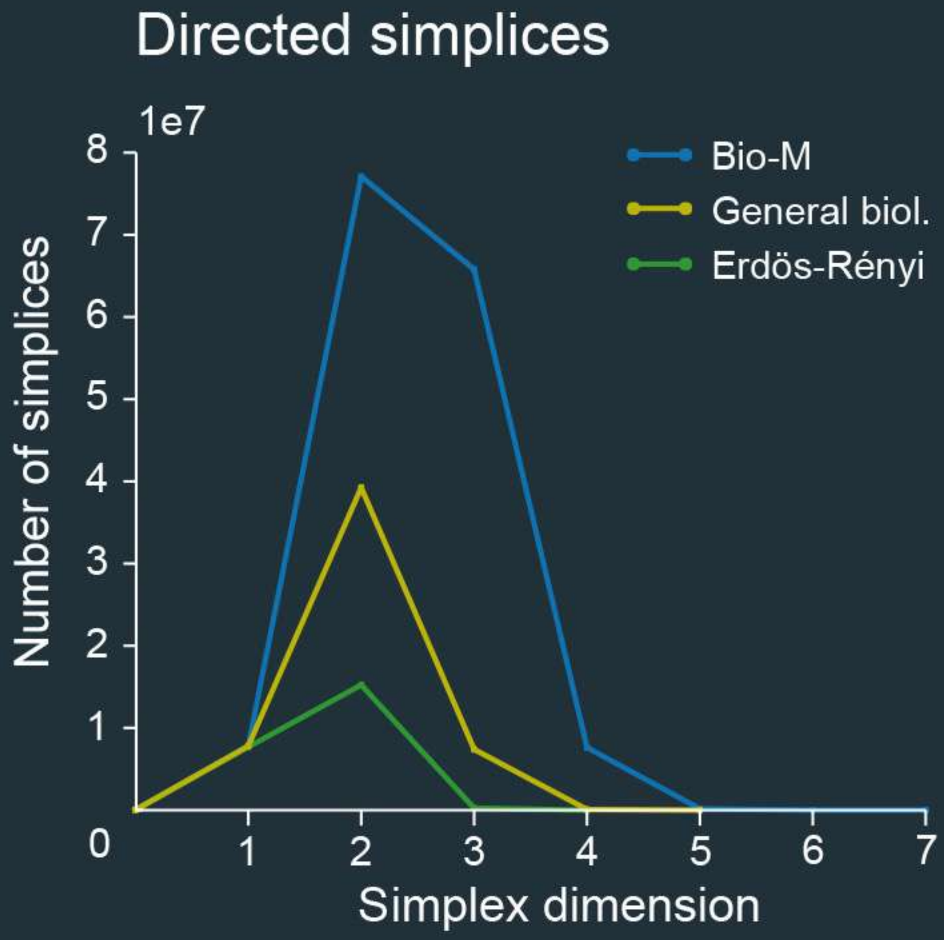
- Analyze the huge network of directed connections among neurons in terms of much smaller significant subnetworks.

- The numbers of different types of significant subnetworks provide important local information about the whole network.

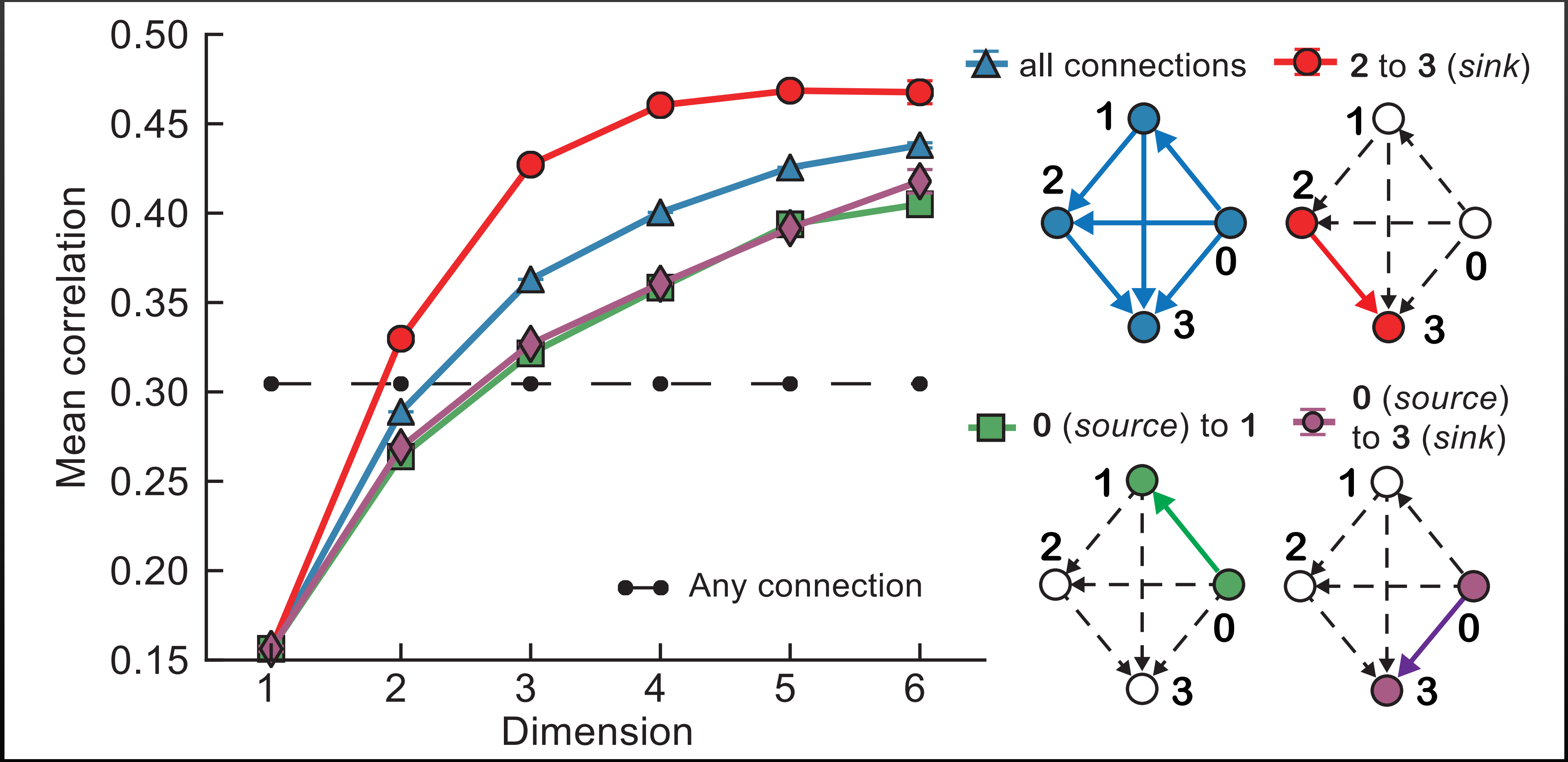
From networks to topology

- Analyze the huge network of directed connections among neurons in terms of much smaller significant subnetworks.
- The numbers of different types of significant subnetworks provide important local information about the whole network.
- Quantify how the significant subnetworks overlap in the larger network to obtain important global information.

Measuring structure

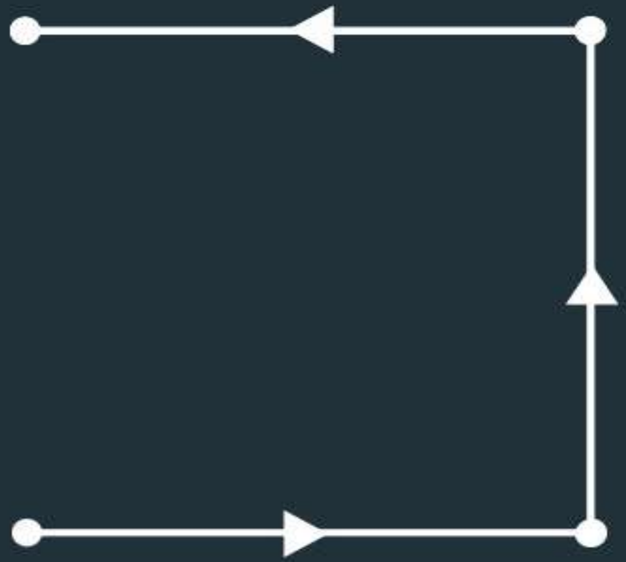


The functional importance of simplices

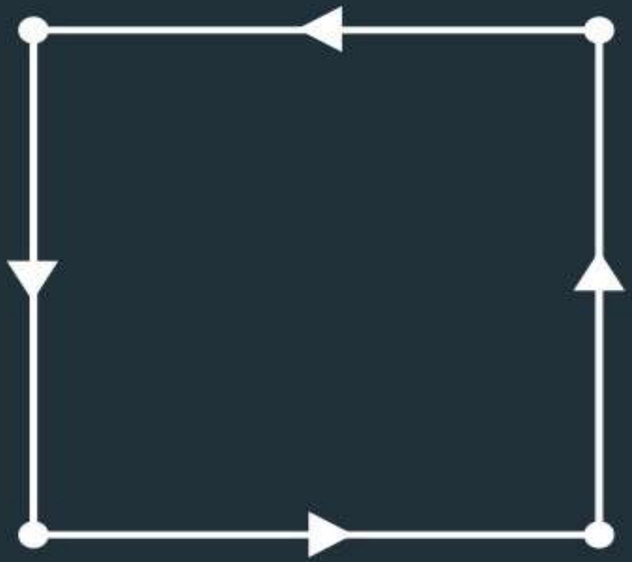


The idea of a cavity

1 simplex



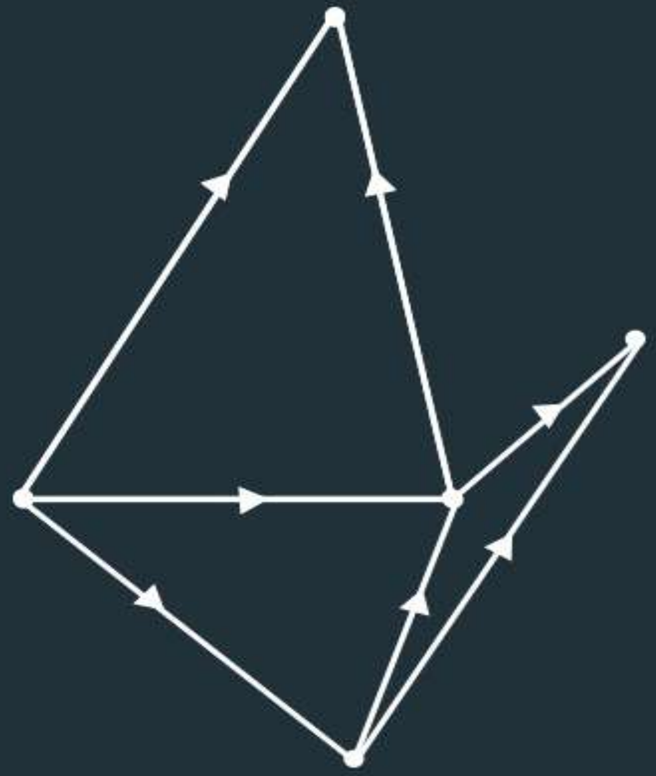
3 simplices...



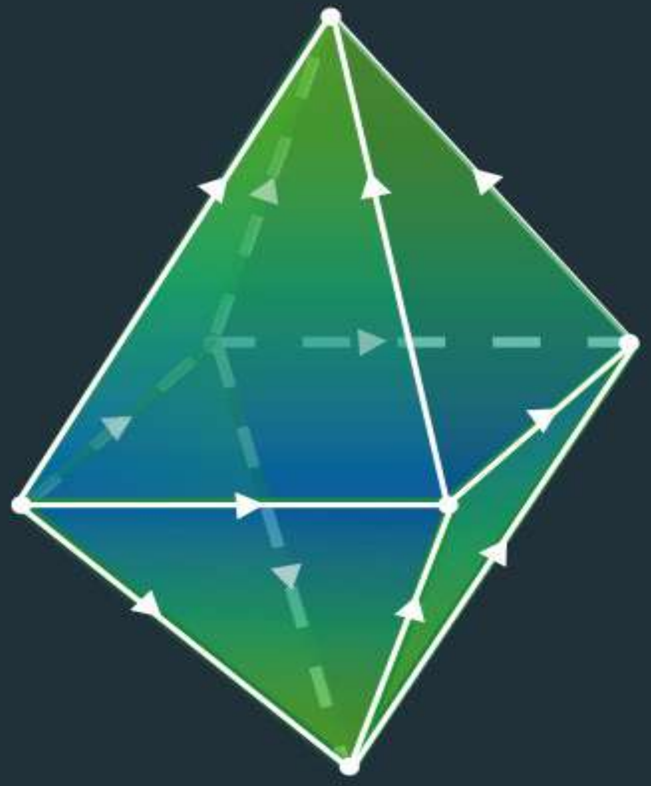
a cavity made of 4 simplices

Higher dimensions for a cavity

1 simplex

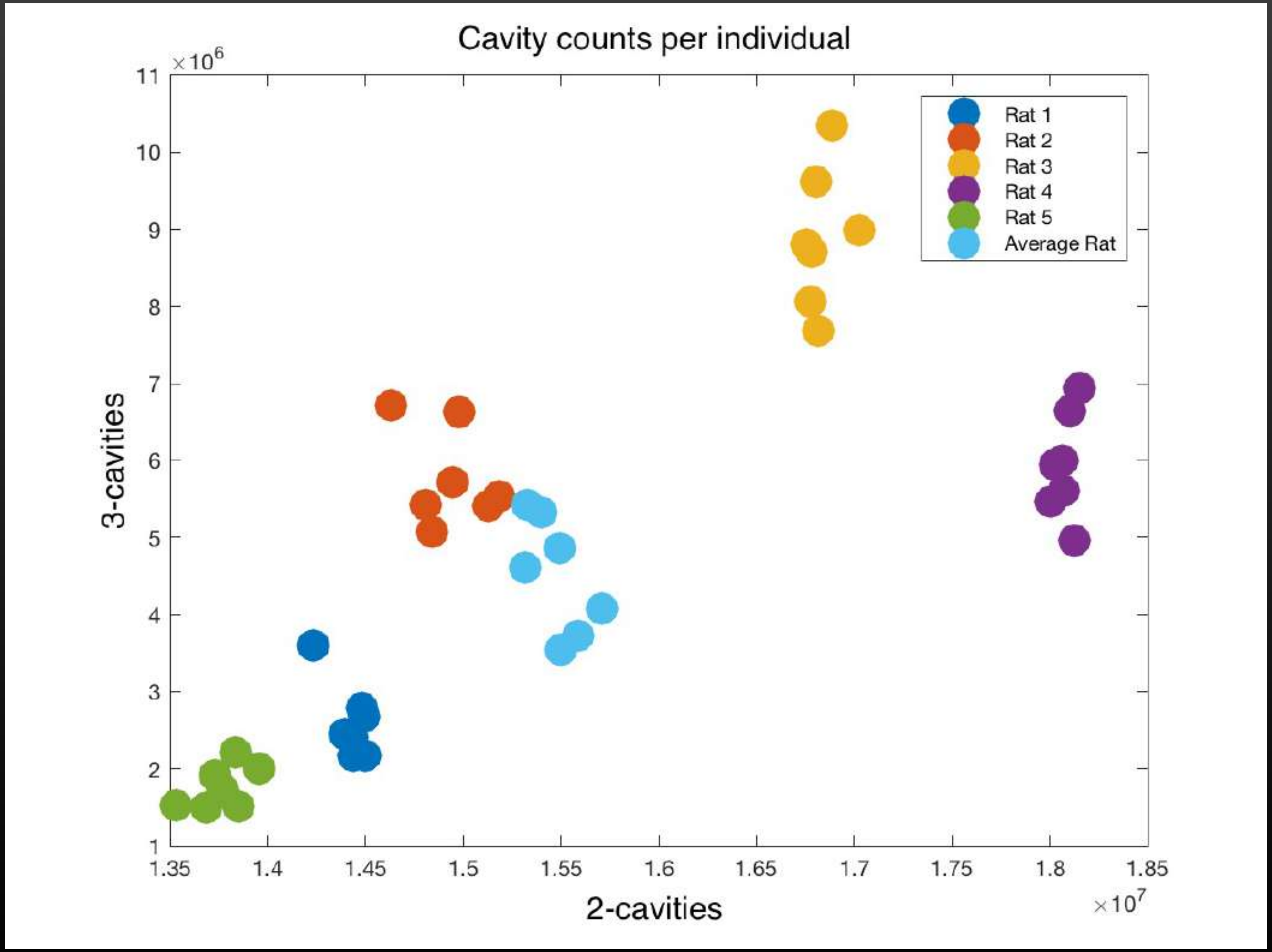


3 simplices...

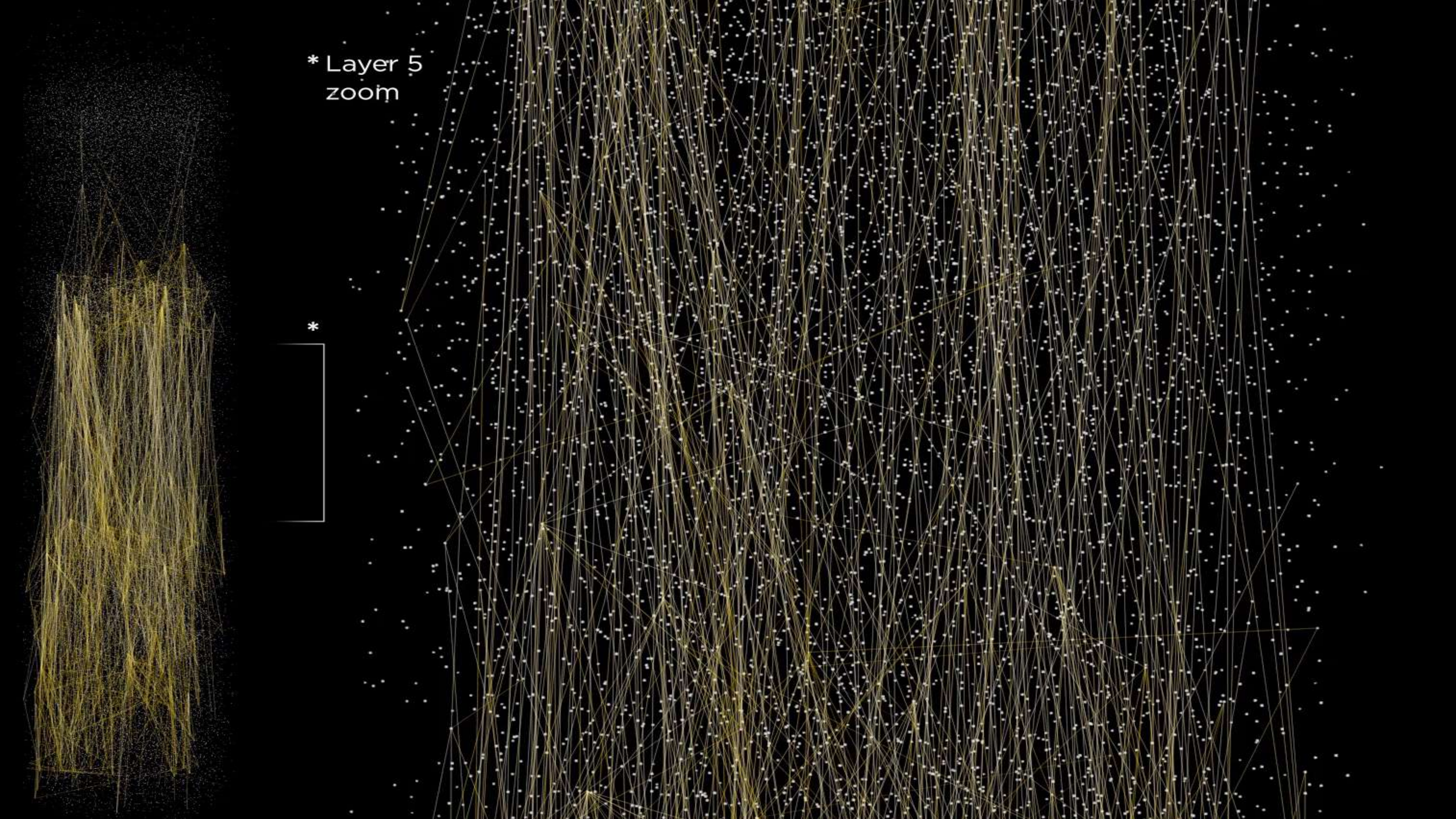


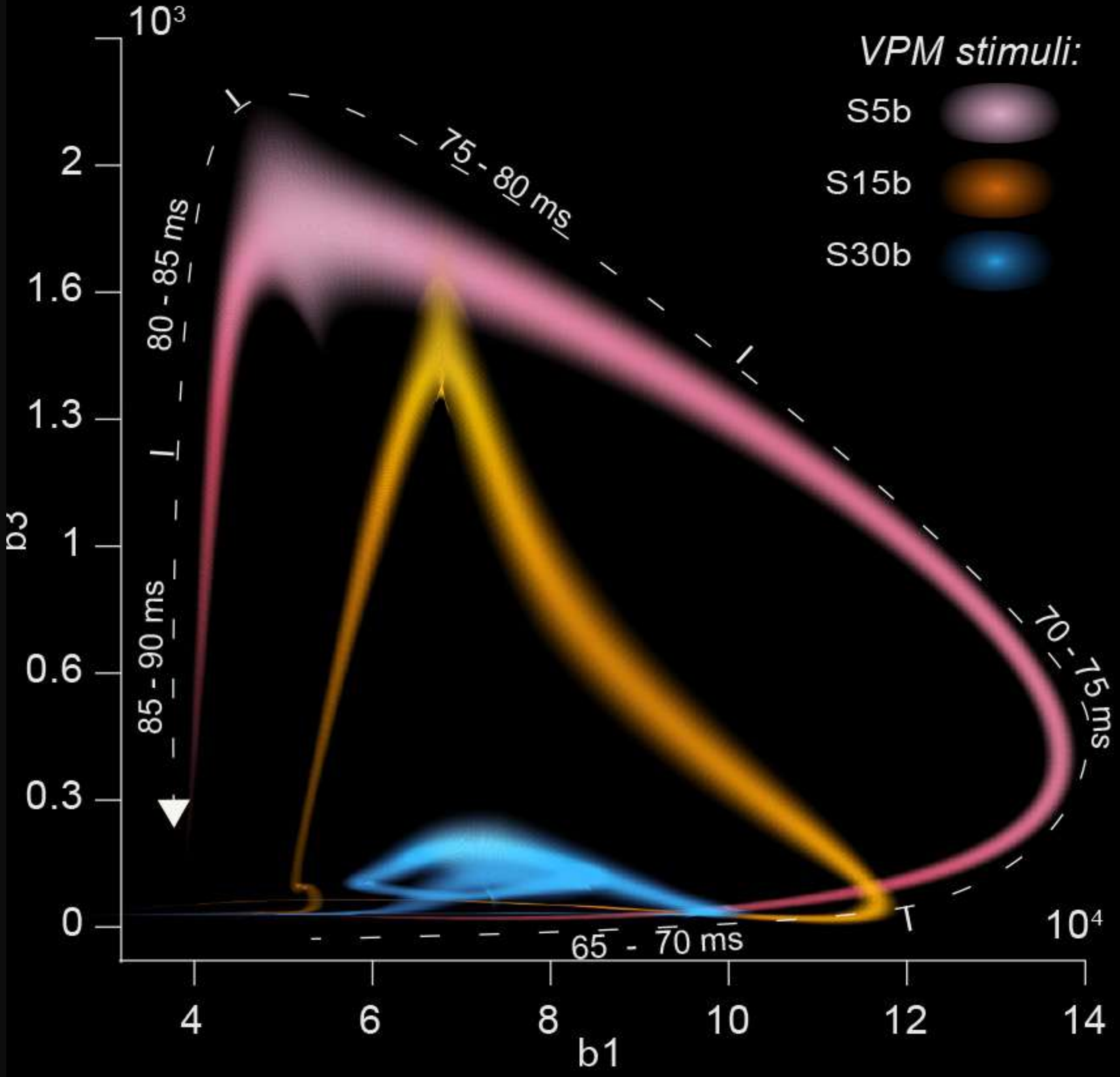
a cavity made of 8 simplices

Topology faithfully reflects biology




Neurons membrane potential
-10mV
-80mV
Time (ms): 562
blue brain


* Layer 5
zoom
*



VPM stimuli:
S5b
S15b
S30b
75 - 80 ms
80 - 85 ms
85 - 90 ms
70 - 75 ms
65 - 70 ms
10^3
10^4
b3
b1
2
1.6
1.3
1
0.6
0.3
0
4
6
8
10
12
14

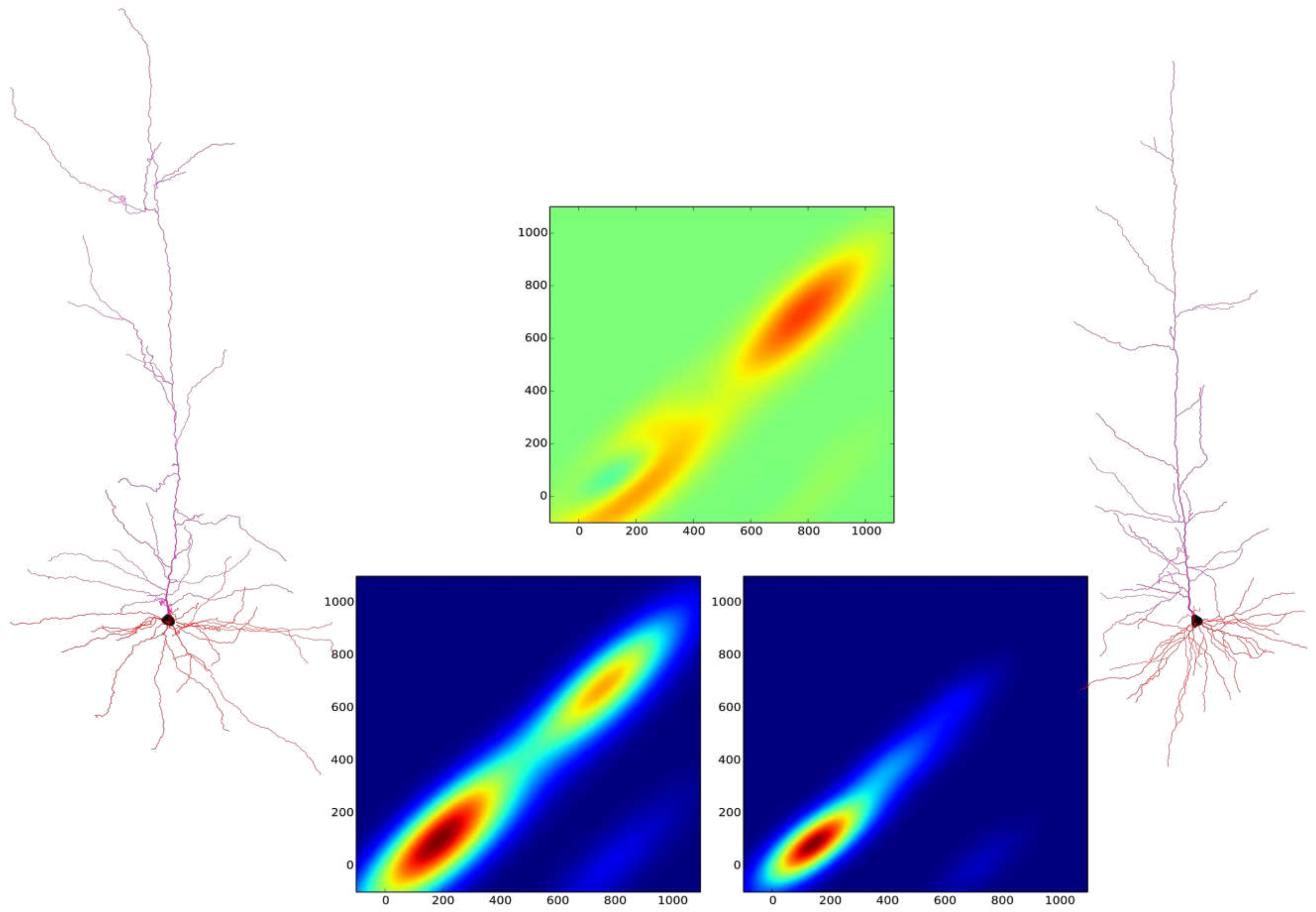
Classification of neuron morphologies

Y. Deitcher et al, Cerebral Cortex, 2017.

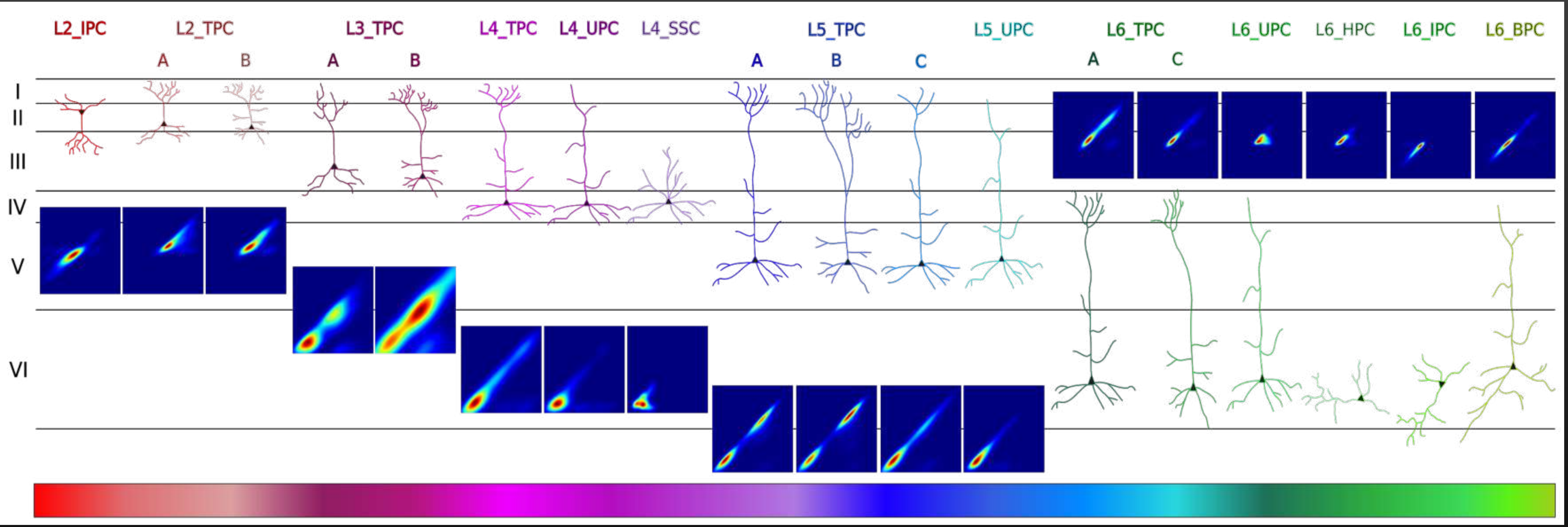
L. Kanari et al, Neuroinformatics, 2018.

L. Kanari et al, Cerebral Cortex, 2019.

VS



0
200
400
600
800
1000



L2_IPC
L2_TPC
A
B
L3_TPC
A
B
L4_TPC
L4_UPC
L4_SSC
L5_TPC
A
B
C
L5_UPC
L6_TPC
A
C
L6_UPC
L6_HPC
L6_IPC
L6_BPC
I
II
III
IV
V
VI

Collaborators

- Stefania Ebli, Daniela Egas Santander, Adélie Garin, Celia Hacker, Nicolas Ninin, Martina Scolamiero (now KTH), Gard Spreemann, Katharine Turner (now ANU), and Dimitri Zaganidis (Laboratory for Topology and Neuroscience, EPFL)
- Nicolas Antille, Giuseppe Chindemi, Cyrille Favreau, Lida Kanari, Henry Markram, Taylor Newton, Max Nolte, and Michael Reimann (Blue Brain Project, EPFL)
- Dejan Govc, Ran Levi, Daniel Lütgehetmann, and Jason Smith (Aberdeen)
- Pawel Dlotko (Swansea)
- Rodrigo Perin (Laboratory of Neural Microcircuitry, EPFL)


Thank you!