

# Base sizes for the symmetric and alternating groups

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Symmetries of Graphs, Maps, and Polytopes, 8ECM

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# Base/base size and determining set/number

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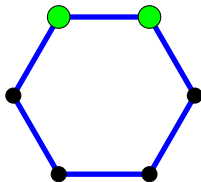
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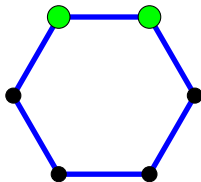
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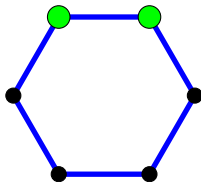
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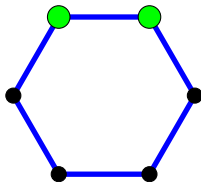
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**Note:** A determining set is not the same as a distinguishing set! (See above.)

# The actions

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*Then almost any pair of points is a base.*

So what about the action on subsets, or the action on partitions?

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If  $n = 12$ ,  $b = 4$ , and  $a = 3$ , and we have partitions

$\{1, 2, 3\}, \{4, 5, 6\}, \{7, 8, 9\}, \{10, 11, 12\}$

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then

$1 \rightarrow (0, 0, 0)$	$2 \rightarrow (0, 0, 1)$	$3 \rightarrow (0, 1, 0)$	$4 \rightarrow (1, 0, 0)$
$5 \rightarrow (1, 1, 1)$	$6 \rightarrow (1, 1, 2)$	$7 \rightarrow (2, 2, 1)$	$8 \rightarrow (2, 2, 2)$
$9 \rightarrow (2, 3, 3)$	$10 \rightarrow (3, 2, 3)$	$11 \rightarrow (3, 3, 2)$	$12 \rightarrow (3, 3, 3)$

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$b = 2$ ,  $a = 4$ ,  $\ell = 3$ . Partitions are:

$\{1, 3, 4, 7\}\{2, 5, 6, 8\}$ ;     $\{1, 2, 4, 6\}\{3, 5, 7, 8\}$ ;     $\{1, 2, 3, 5\}\{4, 6, 7, 8\}$ .

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It turns out that this bound or something very close to it can almost always be met.

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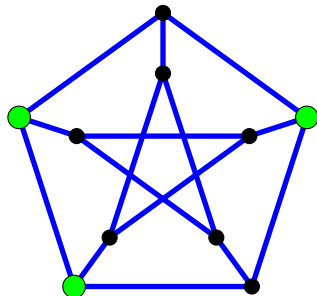
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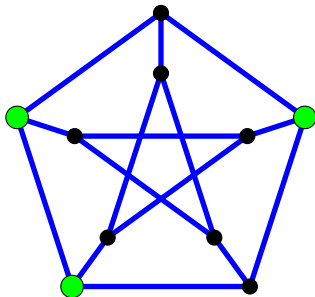
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This corresponds to the base  $\{1, 2\}, \{2, 3\}, \{3, 4\}$ .

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# Our results

## Theorem (M., Spiga, 2021+)

*We have exact values for the base size of the action of  $\text{Sym}(n)$  on  $k$ -sets whenever the base size is at least (approximately)  $3\sqrt{k}$ .*

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## Theorem (M., Spiga, 2021+)

*The base size of the alternating group of degree  $n = ab$  in its action on partitions into  $b$  parts of cardinality  $a$  is either the same as for the symmetric group except in some specified cases, when it is smaller by one.*

# Open Problems

What is the base size for the action on  $k$ -subsets when  $n$  is small relative to  $k$ ?

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Find more determining numbers.

Thank you!

