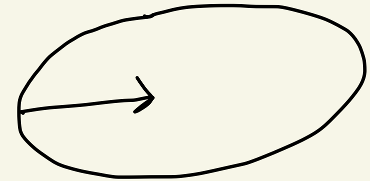
STABILITY OF NONLOCAL GEOMETRIC EVOLUTIONS

MATTEO NOVAGA – UNIV. OF PISA
(JOINT WORK WITH CESARONI, DE LUCA, PONSIGLIONE)

8ECM, 22/06/2021

$$V = \mathcal{H}$$

NONLOCAL MEAN CURVATURE

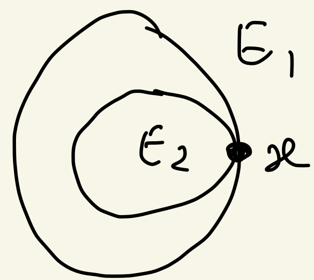


NORMAL VELOCITY

[CHAMBOLLE - MORINI - PONSIGLIONE]

ASSUMPTIONS:

- MONOTONICITY



$$\mathcal{H}_{E_1}(x) \leq \mathcal{H}_{E_2}(x)$$

- TRANSLATION INVARIANCE
- C^2-CONTINUITY: $\underset{\substack{\Downarrow \\ x_n}}{\partial E_n} \to \underset{\substack{\Downarrow \\ x}}{\partial E}$ IN C^2 $\Longrightarrow$ $\mathcal{H}_{E_n}(x_n) \xrightarrow[n]{} \mathcal{H}_E(x)$

EX: $V = H \leftarrow$ G.F. $P(E) = \int_{\partial E} 1$

$\uparrow$ MEAN CURVATURE

$K : \mathbb{R}^n \to [0, +\infty) \qquad K(x) = K(-x)$

$$P_K(E) = \int_E \int_{E^c} K(x-y)\, dx\, dy$$

$\downarrow$ NONLOCAL PER

$$\xrightarrow{\text{G.F.}} \quad V = H_K = \int_{\mathbb{R}^n} \left[\chi_{E^c}(y) - \chi_E(y) \right] K(x-y)\, dy$$

$\downarrow$ K-MEAN CURV.

$$K(x) = \frac{1}{|x|^{n+s}} \qquad s \in (0,1)$$

$H_{K_s} = H_s$ FRACTIONAL M.C.

$P_{K_s} = P_s$ " PERIMETER

$\boxed{V = H_s}$ FRACTIONAL M.C.F.

WHAT HAPPENS WHEN $s \to 0$ OR $s \to 1$?

LEVEL-SET FORM.

$$E(t) = \{u(x,t) \leq 0\} \quad (*)$$

$$u_t = |Du| \cdot \mathcal{H}\left(x, \{y : u(y,t) \leq u(x,t)\}\right)$$

TH: IF $u(x,0) \in UC(\mathbb{R}^n) \Rightarrow \exists!$ SOL.

$u(x,t) \in UC(\mathbb{R}^n \times [0,T]) \quad \forall T$

IN THE VISCOSITY SENSE

START FROM $E(0) \xrightarrow{(*)} u(x,0) \xRightarrow{\text{TH.}} u(x,t) \xrightarrow{(*)} E^{\pm}(t) = \begin{cases} \{u \leq 0\} \\ \{u < 0\} \end{cases}$

TH (STABILITY)

$\mathcal{H}_n \to \mathcal{H}$, I.E., $\partial E_n \to \partial E$ IN C^2 $\Rightarrow$ $\mathcal{H}_n(x_n) \to \mathcal{H}(x)$

$x_n \to x$

IF $u_n(x,t)$ IS A L.S. SOL.

AND $u_n(x,0) \to u(x,0)$ UNIFORMLY

$\Rightarrow$ $u_n(x,t) \to u(x,t)$ UNIF. &

u IS A SOL. OF $u_t = |Du| \cdot \mathcal{H}$

APPLICATION

$$H_s(x) = \int_{\mathbb{R}^n} \frac{\chi_{E^c} - \chi_E}{|x-y|^{n+s}}$$

$$\boxed{x \in \partial E \in C^2}$$

KNOWN:

$$s H_s(x) \xrightarrow[s \to 0]{} n\omega_n$$

MEAN CURVATURE

$$(1-s) H_s(x) \to n(n-1)\omega_n H$$

$$\Longrightarrow \quad v = s H_s \longrightarrow \begin{cases} v = n\omega_n & s \to 0 \\ v = n(n-1)\omega_n H & s \to 1 \end{cases}$$

unif. conv. of level-set solutions