Spectral results on Identifying codes

Camino Balbuena[1], Cristina Dalfó[2], Berenice Martínez-Barona[1]

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[1]Universitat Politècnica de Catalunya, Barcelona, Spain
[2]Universitat de Lleida, Barcelona, Catalonia

Definition

Given a vertex subset $U \subset V$, let $N^-[U] = \bigcup_{u \in U} N^-[u]$. For a given integer $\ell \geq 1$, a dominating set $C \subset V$ is a $(1, \leq \ell)$-identifying code in a digraph D when, for all distinct subsets $X, Y \subset V$, with $1 \leq |X|, |Y| \leq \ell$, we get

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Remark
A digraph $D = (V, A)$ admits some $(1, \leq \ell)$-IdC if and only if for all distinct subsets $X, Y \subset V$ with $|X|, |Y| \leq \ell$, we have

$$N^-[X] \neq N^-[Y].$$

Introduction

Definition
Two distinct vertices u and v of D are called twins if $N^-[u] = N^-[v]$.

Remark
A digraph D admits a $(1, \leq 1)$-IdC (*identifying code*) if and only if D is twin-free.

Non-spectral results for digraphs

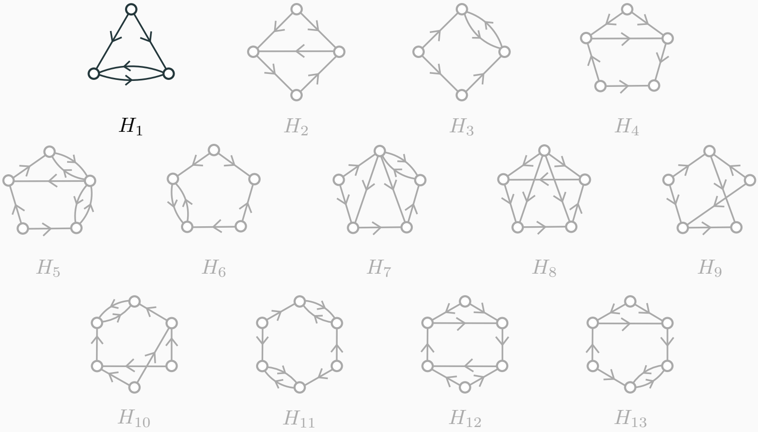
Non-spectral results for digraphs

Theorem (Balbuena, Dalfó, M-B., 2017)
Every 1-in-regular digraph D admits a $(1, \leq 2)$-IdC if and only if the girth of D is at least 5.

Non-spectral results for digraphs

Theorem (Balbuena, Dalfó, M-B., 2017)
Let D be a 2-in-regular digraph. Then,

(i) D admits an identifying code if and only if it is H_1-free.



Non-spectral results for digraphs

Theorem (Balbuena, Dalfó, M-B., 2017)
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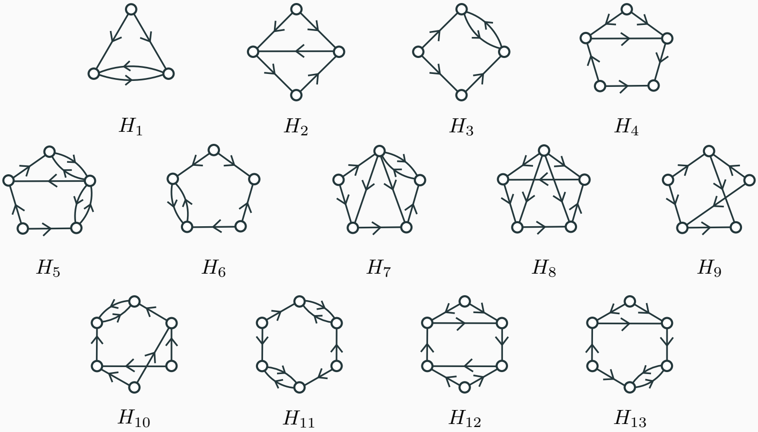
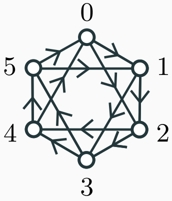
(ii) D admits a $(1, \leq 2)$-IdC if and only if it is $\mathcal{H}$-free.



Non-spectral results for digraphs

Example
Circulant digraph $C(6;1,2)$.



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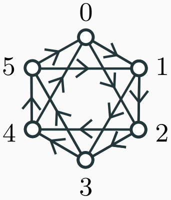

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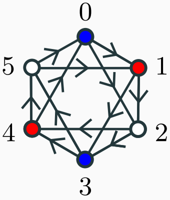

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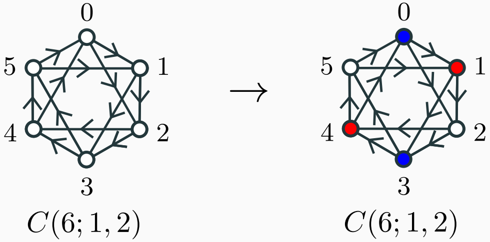


$X = \{0, 3\}$ and $Y = \{1, 4\}$

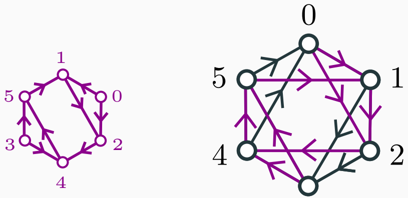
$N^-[X] = \{0, 1, 2, 3, 4, 5\} = N^-[Y]$

$\Downarrow$

$C(6;1,2)$ does not admit a $(1, \leq 2)$-IdC.

Example
Circulant digraph $C(6;1,2)$ does not admit a $(1,\leq 2)$-IdC.

- $H_{12} \subset C(6;1,2)$.



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Spectral results for digraphs

Recall that a digraph with adjacency matrix $M = (a_{uv})$ has eigenvalue λ and eigenvector $\boldsymbol{x} = (x_u)$ if and only if

$$M\boldsymbol{x} = \lambda\boldsymbol{x} \quad \Leftrightarrow \quad \sum_{v \in V} a_{uv} x_v = \sum_{v \in N^+(u)} x_v = \lambda x_u \qquad \text{for all } u \in V.$$

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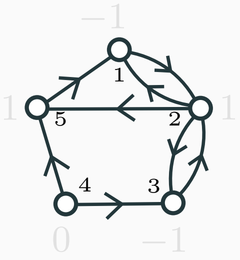

H_5

$$\lambda = -1$$

$$x_\lambda = (-1, 1, -1, 0, 1)^t$$

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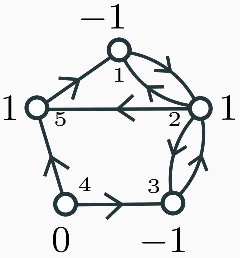

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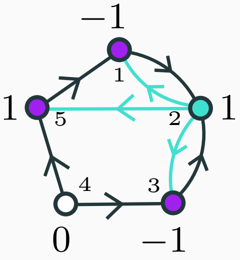

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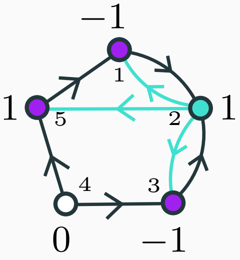

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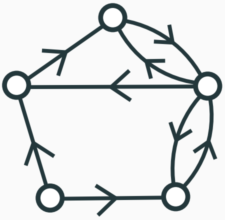

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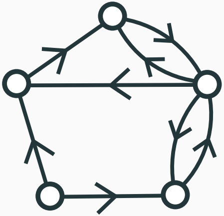

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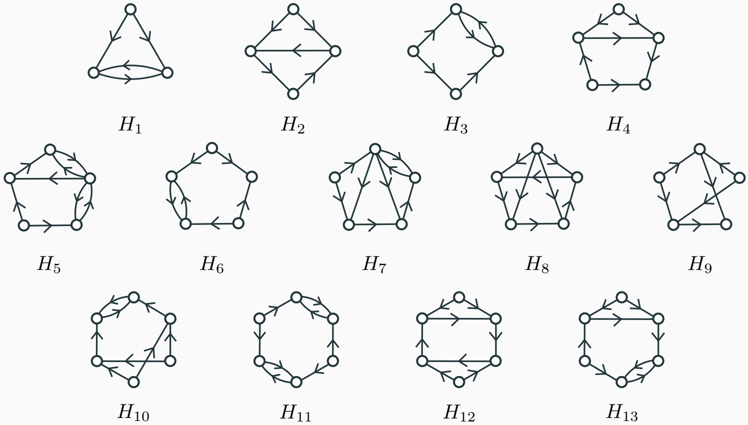

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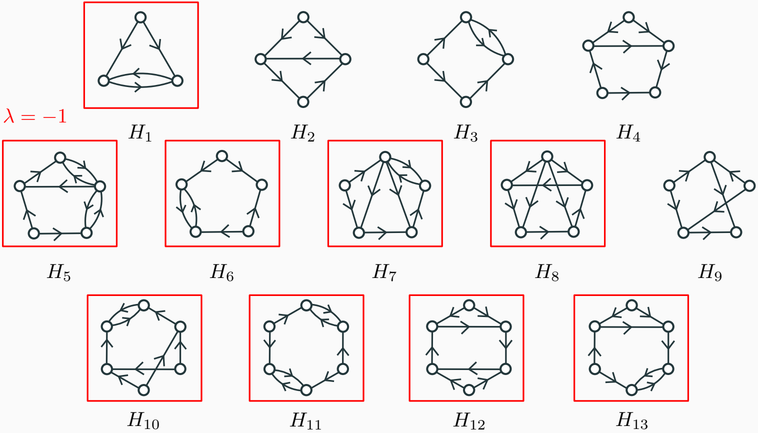

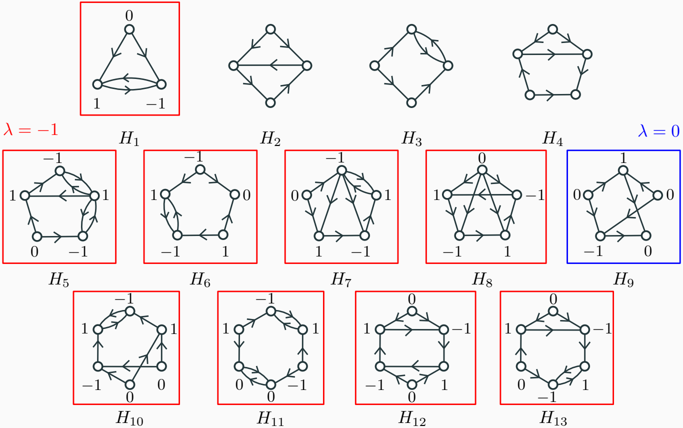
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H_1
H_2
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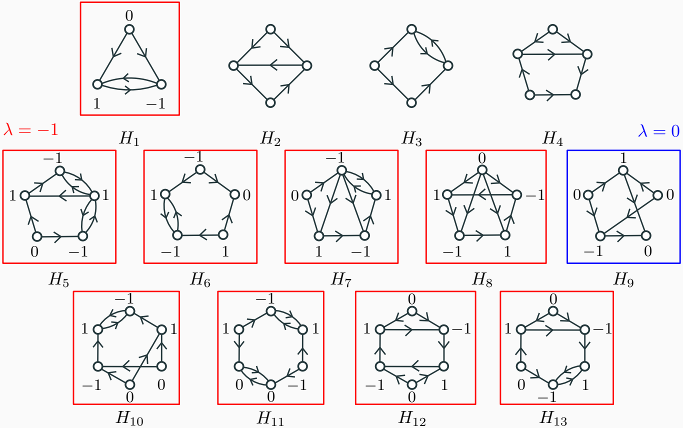
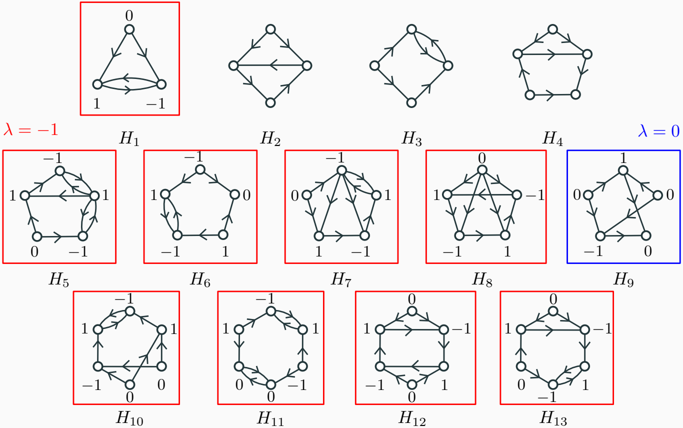
H_{13}



Lemma (Balbuena, Dalfó, M-B., 2019)
Let D' be a digraph with maximum in-degree Δ^- having an eigenvalue λ with eigenvector $\boldsymbol{x}' = (x'_u)$, such that $x'_v = 0$ for any vertex $v \in V(D')$ with $d^-(v) < \Delta^-$. Then, any digraph D with maximum in-degree Δ^- containing D' as a subdigraph has also the eigenvalue λ.



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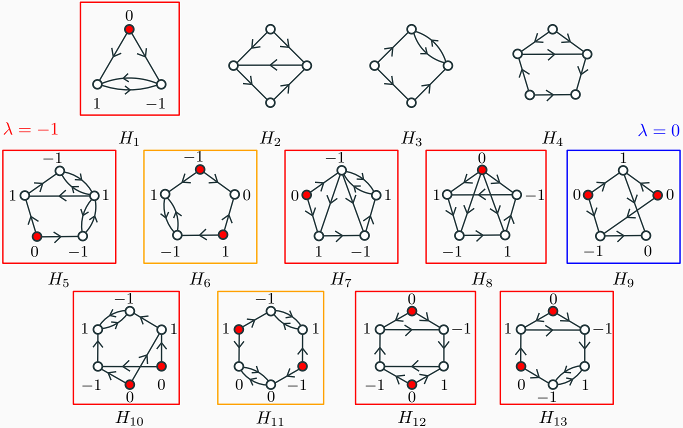

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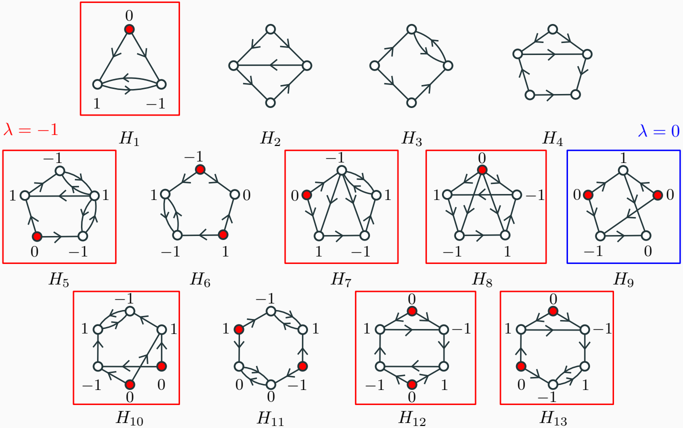

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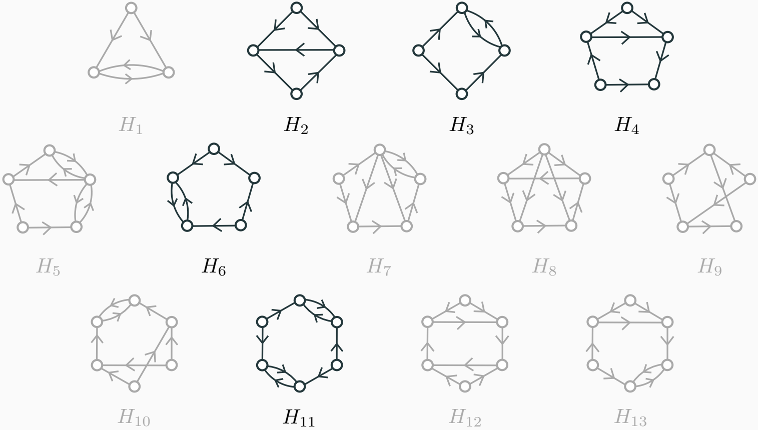
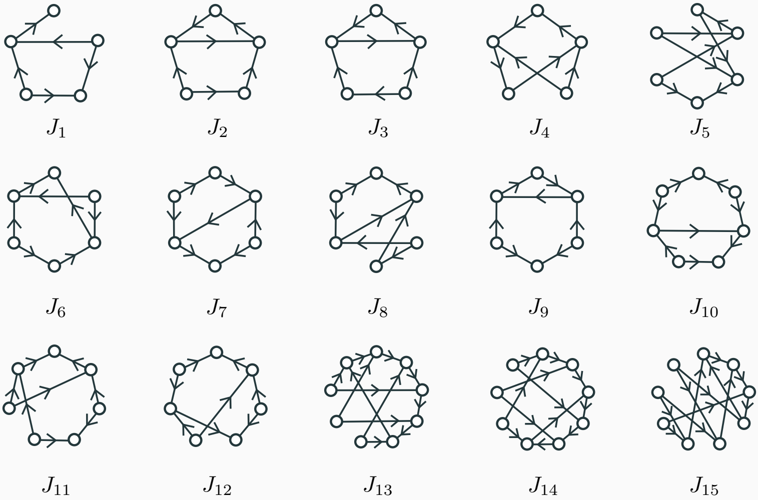
Theorem

Let D be a 2-in-regular digraph with adjacency matrix M.

(*i*) *If $-1, 0 \notin \mathrm{ev}(M)$ and D is $\{H_2, H_3, H_4, H_6, H_{11}\}$-free, then D admits a $(1, \leq 2)$-IdC.*



Theorem (Balbuena, Dalfó, M-B., 2019)
Let D be a 2-in-regular digraph. Then, D admits a $(1, \leq 3)$-IdC if and only if it is $\{TT_3\} \cup \mathcal{J}$-free oriented graph.



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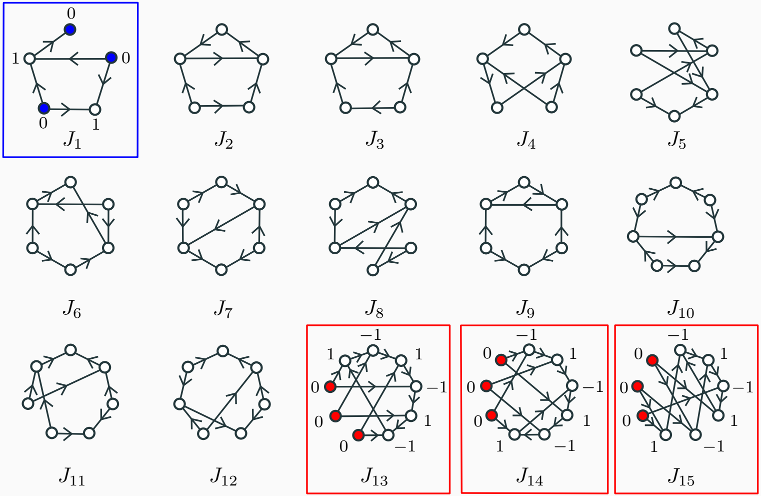
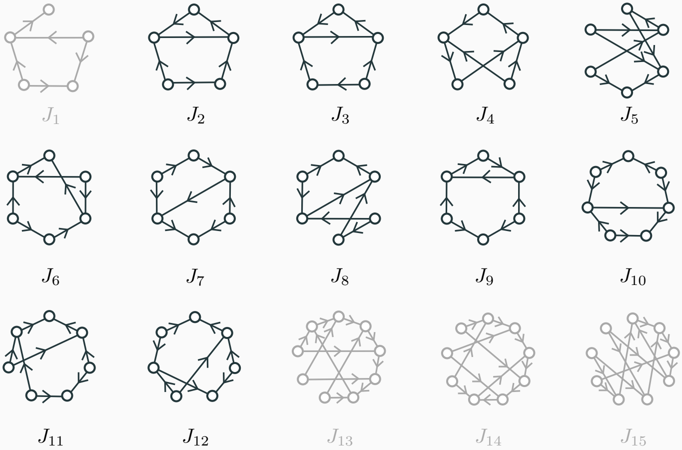


Theorem (Balbuena, Dalfó, M-B., 2019)
Let D be an oriented 2-in-regular TT_3-free graph with adjacency matrix M.

(ii) If $-1, 0 \notin \mathrm{ev}(M)$ and D is $\{J_i \in \mathcal{J} \mid 2 \leq i \leq 12\}$-free, then D admits a $(1, \leq 3)$-IdC.



Algebraic upper bound for ℓ

$$(1, \leq \ell) - \text{identifying code.}$$

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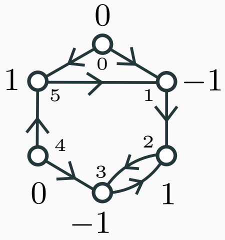
Let $\mathcal{P}(\boldsymbol{x}) = \{i : x_i > 0\}$, $\mathcal{N}(\boldsymbol{x}) = \{i : x_i < 0\}$, and $\mathcal{O}(\boldsymbol{x}) = \{i : x_i = 0\}$.



H_{13}

$\mathrm{sp}(M) = \{0^4, 1^1, -1^1\}$.
$\lambda = -1$; $x_\lambda = (0, -1, 1, -1, 0, 1)^t$.

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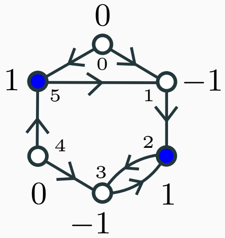

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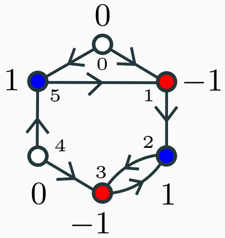

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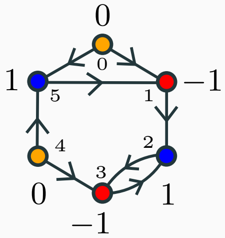

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Proposition (Balbuena, Dalfó, M-B., 2019)
Let D be a digraph with adjacency matrix M having $\lambda \in \mathrm{ev}(M) \cap \mathbb{R}$, with an associated eigenvector $\boldsymbol{x} = (x_u)_{u \in V}$ such that $X = \mathcal{P}(\boldsymbol{x}) \neq \emptyset$ and $Y = \mathcal{N}(\boldsymbol{x}) \neq \emptyset$. Then, depending on the sign of λ, the following holds:

(a) *If $\lambda < 0$, then $N^-[X] = N^-[Y]$.*

(b) *If $\lambda > 0$, then $X \cup N^-(Y) = Y \cup N^-(X)$.*

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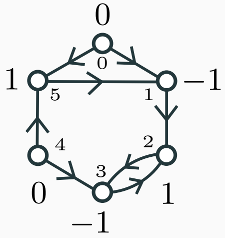
Corollary
Let D be a digraph or a graph admitting a $(1, \leq \ell)$-IdC, and let M be its adjacency matrix having at least one negative eigenvalue λ. Then,

$$\ell < \min_{\boldsymbol{x} \in E_\lambda(M)} \max\{|\mathcal{P}(\boldsymbol{x})|, |\mathcal{N}(\boldsymbol{x})|\}$$

Example



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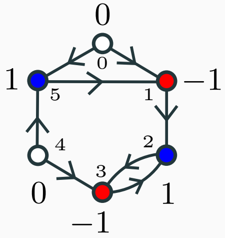

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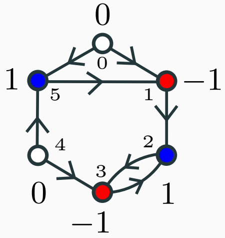

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$X = \{2, 5\}$, and $Y = \{1, 3\}$.

$N^-[X] = \{0, 1, 2, 3, 4, 5\} = N^-[Y]$.

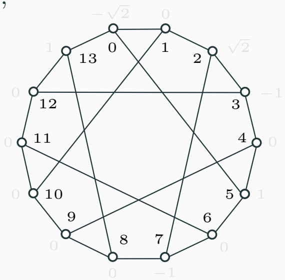
$\Downarrow$

H_{13} does not admit a $(1, \leq 2)$-IdC.

Example (Graphs case)

Heawood graph: $\mathrm{sp}(M) = \{3^1, \sqrt{2}^6, -\sqrt{2}^6, -3^1\}$. Eigenvectors corresponding to -3 and $-\sqrt{2}$, respectively:

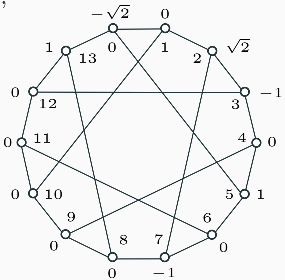
$$(-1,1,-1,1,-1,1,-1,1,-1,1,-1,1,-1,1),$$
$$(-1,0,1,0,-1,\sqrt{2},0,-\sqrt{2},1,0,0,0,0,0)^t,$$
$$(-1,0,1,-\sqrt{2},0,\sqrt{2},-1,0,0,0,0,0,1,0)^t,$$
$$(0,-1,\sqrt{2},-1,0,1,-\sqrt{2},0,0,0,0,1,0,0)^t,$$
$$(0,-1,\sqrt{2},0,-\sqrt{2},1,0,-1,0,1,0,0,0,0)^t,$$
$$(0,-\sqrt{2},1,0,-1,\sqrt{2},-1,0,0,0,1,0,0,0)^t,$$
$$(-\sqrt{2},0,\sqrt{2},-1,0,1,0,-1,0,0,0,0,0,1)^t.$$



Example (Graphs case)

Heawood graph: $\mathrm{sp}(M) = \{3^1, \sqrt{2}^6, -\sqrt{2}^6, -3^1\}$. Eigenvectors corresponding to -3 and $-\sqrt{2}$, respectively:

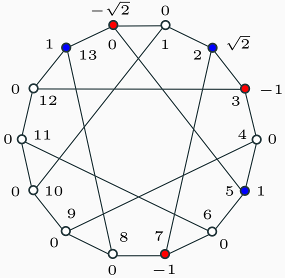
$$(-1, 1, -1, 1, -1, 1, -1, 1, -1, 1, -1, 1, -1, 1),$$
$$(-1, 0, 1, 0, -1, \sqrt{2}, 0, -\sqrt{2}, 1, 0, 0, 0, 0, 0)^t,$$
$$(-1, 0, 1, -\sqrt{2}, 0, \sqrt{2}, -1, 0, 0, 0, 0, 0, 1, 0)^t,$$
$$(0, -1, \sqrt{2}, -1, 0, 1, -\sqrt{2}, 0, 0, 0, 0, 1, 0, 0)^t,$$
$$(0, -1, \sqrt{2}, 0, -\sqrt{2}, 1, 0, -1, 0, 1, 0, 0, 0, 0)^t,$$
$$(0, -\sqrt{2}, 1, 0, -1, \sqrt{2}, -1, 0, 0, 0, 1, 0, 0, 0)^t,$$
$$(-\sqrt{2}, 0, \sqrt{2}, -1, 0, 1, 0, -1, 0, 0, 0, 0, 0, 1)^t.$$



Example (Graphs case)

$X = \{2, 5, 13\}$, $Y = \{0, 3, 7\}$.
$N[X] = V(H) = N[Y]$

- H does not admit a $(1, \leq 3)$-IdC
- Since $g(H) \geq 5$, it follows that H admits a $(1, \leq 2)$-IdC



Proposition (Balbuena, Dalfó, M-B., 2019)
Let D be a digraph with adjacency matrix M having $\lambda \in \mathrm{ev}(M) \cap \mathbb{R}$, with an associated eigenvector $\boldsymbol{x} = (x_u)_{u \in V}$ such that $X = \mathcal{P}(\boldsymbol{x}) \neq \emptyset$ and $Y = \mathcal{N}(\boldsymbol{x}) \neq \emptyset$. Then, depending on the sign of λ, the following holds:

(a) *If $\lambda < 0$, then $N^-[X] = N^-[Y]$.*

(b) *If $\lambda > 0$, then $X \cup N^-(Y) = Y \cup N^-(X)$.*

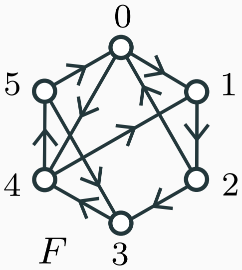
(c) *If $\lambda = 0$, then $N^-(X) = N^-(Y)$.*

Corollary
Let $D = (V, E)$ be a digraph with $\delta(D) \geq 1$ admitting a $(1, \leq \ell)$-IdC. Let M be its adjacency matrix. If $0 \in \mathrm{ev}(M)$, then

$$\ell < \min_{\boldsymbol{x} \in E_0(M)} \{\max\{|\mathcal{P}(\boldsymbol{x})|, |\mathcal{N}(\boldsymbol{x})|\} + |\mathcal{O}(\boldsymbol{x})|\}.$$

Example

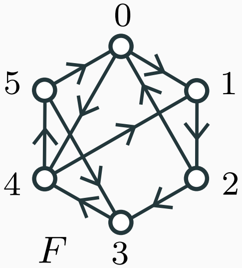
$$\mathrm{sp}(F) = \{\tfrac{1}{2}(\sqrt{5}+1), 0, \tfrac{1}{2}(-\sqrt{5}+1), \tfrac{1}{2}(-1+i\sqrt{7}), \tfrac{1}{2}(-1-i\sqrt{7})\}.$$



Example

$$\mathrm{sp}(F) = \{\tfrac{1}{2}(\sqrt{5}+1), \boxed{0, \frac{1}{2}(-\sqrt{5}+1),} \tfrac{1}{2}(-1+i\sqrt{7}), \tfrac{1}{2}(-1-i\sqrt{7})\}.$$

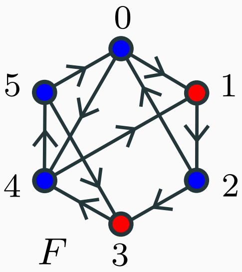
- $\lambda' = \frac{1}{2}(-\sqrt{5}+1) \to \boldsymbol{x'} = (1, \frac{1}{2}(-\sqrt{5}-1), 1, \frac{1}{2}(-\sqrt{5}-1), 1, 1)^t$
- $\lambda = 0 \to \boldsymbol{x} = (-1, 0, 0, 1, 0, 0)^t$



Example

$$\mathrm{sp}(F) = \{\tfrac{1}{2}(\sqrt{5}+1), \boxed{0, \frac{1}{2}(-\sqrt{5}+1),} \tfrac{1}{2}(-1+i\sqrt{7}), \tfrac{1}{2}(-1-i\sqrt{7})\}.$$

- $\lambda' = \frac{1}{2}(-\sqrt{5}+1) \to \boldsymbol{x'} = (1, \frac{1}{2}(-\sqrt{5}-1), 1, \frac{1}{2}(-\sqrt{5}-1), 1, 1)^t$
- $\lambda = 0 \to \boldsymbol{x} = (-1, 0, 0, 1, 0, 0)^t$

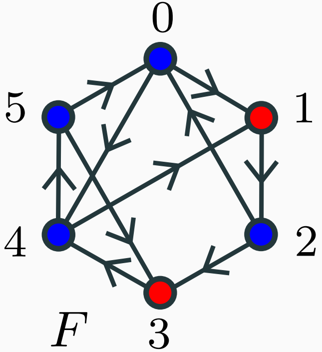


Example

$$\mathrm{sp}(F) = \{\tfrac{1}{2}(\sqrt{5}+1), \boxed{0, \frac{1}{2}(-\sqrt{5}+1),} \tfrac{1}{2}(-1+i\sqrt{7}), \tfrac{1}{2}(-1-i\sqrt{7})\}.$$

- $\lambda' = \frac{1}{2}(-\sqrt{5}+1) \to \boldsymbol{x'} = (1, \frac{1}{2}(-\sqrt{5}-1), 1, \frac{1}{2}(-\sqrt{5}-1), 1, 1)^t$
- $\lambda = 0 \to \boldsymbol{x} = (-1, 0, 0, 1, 0, 0)^t$

$|\mathcal{P}(\boldsymbol{x'})| = 1,$ and $|\mathcal{N}(\boldsymbol{x'})| = 4,$



Example

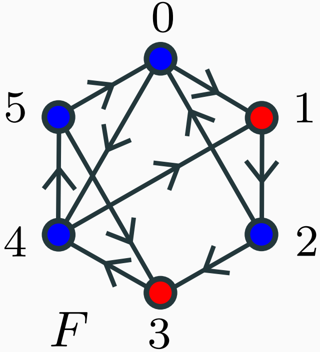
$$\mathrm{sp}(F) = \{\tfrac{1}{2}(\sqrt{5}+1), \boxed{0, \frac{1}{2}(-\sqrt{5}+1),} \tfrac{1}{2}(-1+i\sqrt{7}), \tfrac{1}{2}(-1-i\sqrt{7})\}.$$

- $\lambda' = \frac{1}{2}(-\sqrt{5}+1) \to \boldsymbol{x'} = (1, \frac{1}{2}(-\sqrt{5}-1), 1, \frac{1}{2}(-\sqrt{5}-1), 1, 1)^t$
- $\lambda = 0 \to \boldsymbol{x} = (-1, 0, 0, 1, 0, 0)^t$

$|\mathcal{P}(\boldsymbol{x'})| = 1, \text{ and } |\mathcal{N}(\boldsymbol{x'})| = 4,$

$$\Downarrow$$

F does not admit a $(1, \leq 4)$-IC



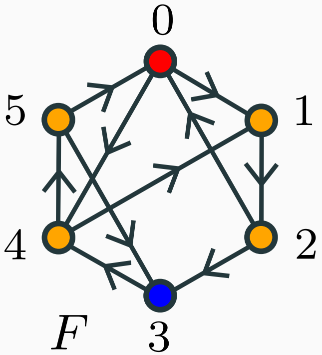
Example

$$\mathrm{sp}(F) = \{\tfrac{1}{2}(\sqrt{5}+1), \boxed{0, \frac{1}{2}(-\sqrt{5}+1),} \tfrac{1}{2}(-1+i\sqrt{7}), \tfrac{1}{2}(-1-i\sqrt{7})\}.$$

- $\lambda' = \frac{1}{2}(-\sqrt{5}+1) \to \boldsymbol{x'} = (1, \frac{1}{2}(-\sqrt{5}-1), 1, \frac{1}{2}(-\sqrt{5}-1), 1, 1)^t$
- $\lambda = 0 \to \boldsymbol{x} = (-1, 0, 0, 1, 0, 0)^t$

$|\mathcal{P}(\boldsymbol{x'})| = 1,\ |\mathcal{N}(\boldsymbol{x'})| = 1,$

and $|\mathcal{O}(\boldsymbol{x'})| = 4.$



Example

$\mathrm{sp}(F) = \{\frac{1}{2}(\sqrt{5}+1), \boxed{0, \frac{1}{2}(-\sqrt{5}+1),} \frac{1}{2}(-1+i\sqrt{7}), \frac{1}{2}(-1-i\sqrt{7})\}.$

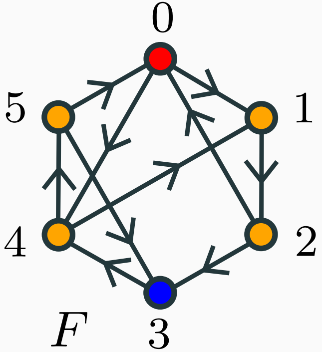
- $\lambda' = \frac{1}{2}(-\sqrt{5}+1) \to \boldsymbol{x'} = (1, \frac{1}{2}(-\sqrt{5}-1), 1, \frac{1}{2}(-\sqrt{5}-1), 1, 1)^t$
- $\lambda = 0 \to \boldsymbol{x} = (-1, 0, 0, 1, 0, 0)^t$

$|\mathcal{P}(\boldsymbol{x'})| = 1,\ |\mathcal{N}(\boldsymbol{x'})| = 1,$

and $|\mathcal{O}(\boldsymbol{x'})| = 4.$

$\Downarrow$

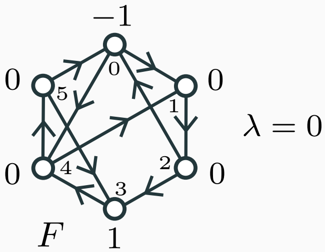
F does not admit a $(1, \leq 5)$-IC



Example

$\text{sp}(F) = \{\frac{1}{2}(\sqrt{5}+1), \boxed{0, \frac{1}{2}(-\sqrt{5}+1),} \frac{1}{2}(-1+i\sqrt{7}), \frac{1}{2}(-1-i\sqrt{7})\}.$

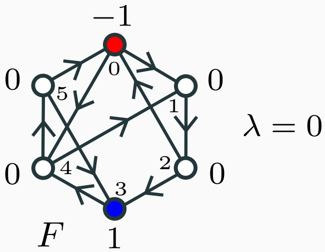
- $\lambda' < 0 \to F$ admits at most a $(1, \leq 3)$-IdC
- $\lambda = 0 \to F$ admits at most a $(1, \leq 4)$-IdC
- $\hat{\delta}^- = 1 \to F$ admits at most a $(1, \leq 2)$-IdC



Example

$$\mathrm{sp}(F) = \{\tfrac{1}{2}(\sqrt{5}+1), \boxed{0, \frac{1}{2}(-\sqrt{5}+1),} \tfrac{1}{2}(-1+i\sqrt{7}), \tfrac{1}{2}(-1-i\sqrt{7})\}.$$

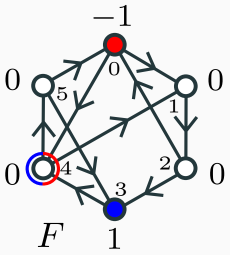
- $\lambda' < 0 \to F$ admits at most a $(1, \leq 3)$-IdC
- $\lambda = 0 \to F$ admits at most a $(1, \leq 4)$-IdC
- $\hat{\hat{\delta}}^{-} = 1 \to F$ admits at most a $(1, \leq 2)$-IdC
- $N^{-}(3) = N^{-}(0)$



Example

$$\mathrm{sp}(F) = \{\tfrac{1}{2}(\sqrt{5}+1), \boxed{0, \frac{1}{2}(-\sqrt{5}+1),} \tfrac{1}{2}(-1+i\sqrt{7}), \tfrac{1}{2}(-1-i\sqrt{7})\}.$$

- $\lambda' < 0 \rightarrow F$ admits at most a $(1, \leq 3)$-IdC
- $\lambda = 0 \rightarrow F$ admits at most a $(1, \leq 4)$-IdC
- $\hat{\delta}^- = 1 \rightarrow F$ admits at most a $(1, \leq 2)$-IdC
- $N^-[\{3, 4\}] = N^-[\{0, 4\}]$

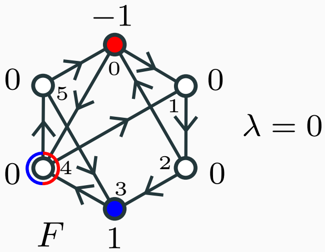


$\lambda = 0$

Example

$$\mathrm{sp}(F) = \{\tfrac{1}{2}(\sqrt{5}+1), \boxed{0, \frac{1}{2}(-\sqrt{5}+1),} \tfrac{1}{2}(-1+i\sqrt{7}), \tfrac{1}{2}(-1-i\sqrt{7})\}.$$

- $\lambda = 0 \to F$ admits an identifying code and does not admit a $(1, \leq \ell)$-IdC for any $\ell \geq 2$
- $N^-[\{3, 4\}] = N^-[\{0, 4\}]$



Thank you

Hvala