

A family of non-Cayley cores that is constructed  
from vertex-transitive or strongly regular  
self-complementary graphs

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$\Gamma$  finite simple/undirected graph with  $|V(\Gamma)| \geq 1$

$\Lambda$  finite simple/undirected graph with  $|V(\Lambda)| \geq 0$

A *homomorphism* between graphs  $\Gamma_1$  and  $\Gamma_2$  is a map  $\Phi : V(\Gamma_1) \rightarrow V(\Gamma_2)$  such that

$$\{u, v\} \in E(\Gamma_1) \implies \{\Phi(u), \Phi(v)\} \in E(\Gamma_2).$$

If  $\Gamma_1 = \Gamma_2$ , then homomorphism = *endomorphism*.

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A graph is a *core* if all its endomorphisms are automorphisms.

Basic examples:

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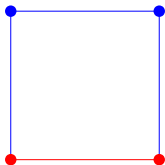
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Example:  $\text{core}(C_4) = K_2$

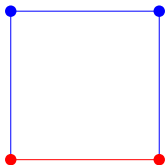


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**Proposition (cf. Godsil & Royle)**

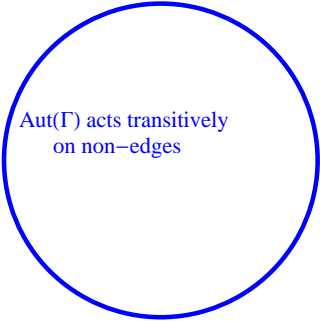
Every graph  $\Gamma$  has a core, which is an induced subgraph and is unique up to isomorphism.

Many 'nice' graphs are cores or their cores are complete



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Cameron, Kazanidis 2008  
J. Aust. Math. Soc.



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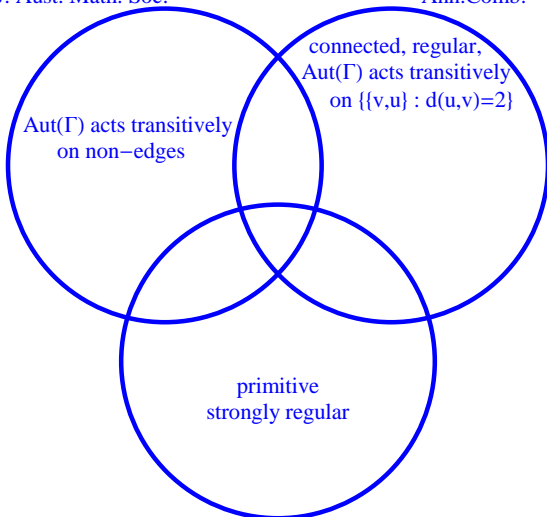
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connected, regular,  
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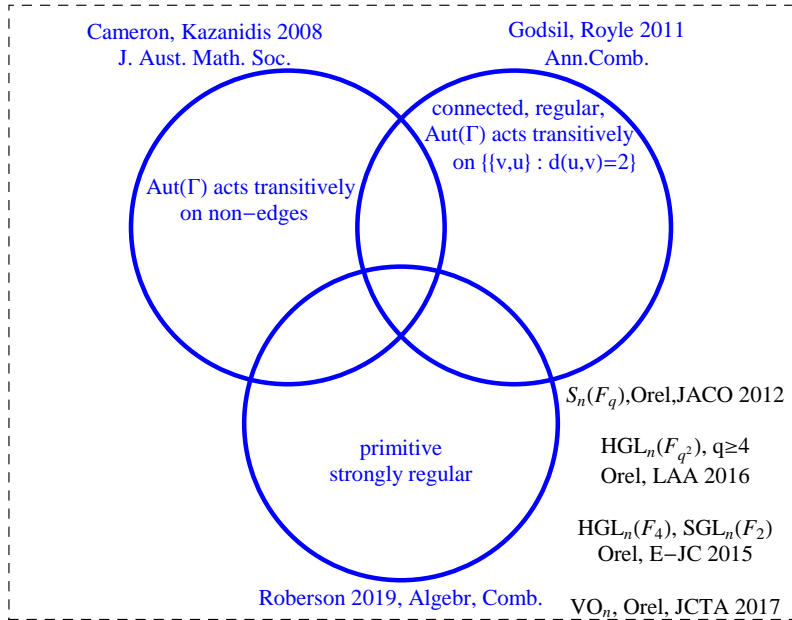
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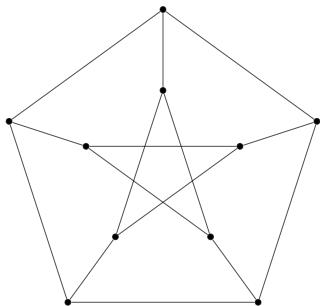


Roberson 2019, Algebr, Comb.

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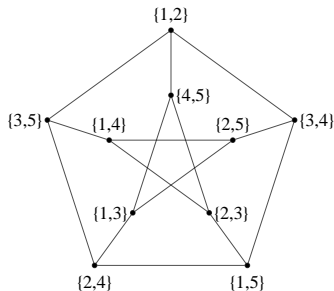
# A famous core: the Petersen graph



# A generalization: Kneser graphs $K(v, r)$

$$V(K(v, r)) = \{S \subseteq \{1, \dots, v\} : |S| = r\}$$

$$E(K(v, r)) = \{\{S_1, S_2\} : S_1 \cap S_2 = \emptyset\}$$



cf. Godsil, Royle 2001

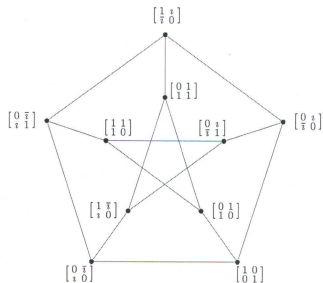
If  $v > 2r$ , then  $K(v, r)$  is a core.

# Another generalization: $HGL_n(\mathbb{F}_4)$

$$\mathbb{F}_4 = \{0, 1, \iota, 1 + \iota\} = \mathbb{F}_2 + \iota\mathbb{F}_2, \quad \iota^2 = 1 + \iota = \bar{\iota}$$

$$V(HGL_n(\mathbb{F}_4)) = \{\text{invertible Hermitian } n \times n \text{ matrices over } \mathbb{F}_4\}$$

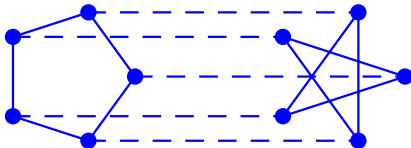
$$E(HGL_n(\mathbb{F}_4)) = \{\{A_1, A_2\} : \text{rank}(A_1 - A_2) = 1\}$$



Orel, E-JC 2015

If  $n \geq 2$ , then  $HGL_n(\mathbb{F}_4)$  is a core.

# Yet another generalization: $\Gamma \equiv \bar{\Gamma}$



If  $\Gamma$  is a graph with  $V(\Gamma) = \{v_1, \dots, v_n\}$  let  $V(\Gamma \equiv \bar{\Gamma}) = W_1 \cup W_2$ ,

$$W_1 = \{(v_1, 1), \dots, (v_n, 1)\} \quad \text{and} \quad W_2 = \{(v_1, 2), \dots, (v_n, 2)\},$$

and let  $E(\Gamma \equiv \bar{\Gamma})$  be:

$$\begin{aligned} & \left\{ \{(u, 1), (v, 1)\} : \{u, v\} \in E(\Gamma) \right\} \\ & \cup \left\{ \{(u, 2), (v, 2)\} : \{u, v\} \in E(\bar{\Gamma}) \right\} \\ & \cup \left\{ \{(u, 1), (u, 2)\} : u \in V(\Gamma) \right\}. \end{aligned}$$

# Yet another generalization: $\Gamma \equiv \bar{\Gamma}$

## Main question

When is  $\Gamma \equiv \bar{\Gamma}$  a core?

## Other problems

- Spectrum of  $\Gamma \equiv \bar{\Gamma}$
- $\text{Aut}(\Gamma \equiv \bar{\Gamma})$ ; related to
  - self-complementary vertex-transitive graphs
  - non-Cayley vertex-transitive graphs
- Hamiltonicity of  $\Gamma \equiv \bar{\Gamma}$

# Diameter and spectrum

## Proposition (Orel 2021+)

$\Gamma \equiv \bar{\Gamma}$  is a connected graph with  $\text{diam}(\Gamma \equiv \bar{\Gamma}) \leq 3$ . Moreover,

$$\text{diam}(\Gamma \equiv \bar{\Gamma}) = 1 \iff \Gamma \cong K_1,$$

$$\text{diam}(\Gamma \equiv \bar{\Gamma}) = 2 \iff \text{diam}(\Gamma) = 2 = \text{diam}(\bar{\Gamma}).$$

## Theorem (Orel 2021+)

If  $\Gamma$  is a connected  $k$ -regular graph on  $n$  vertices with eigenvalues  $k = \lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n$ , then the eigenvalues of  $\Gamma \equiv \bar{\Gamma}$  equal

$$\left\{ \frac{n-1 \pm \sqrt{(n-1)^2 - 4((n-k-1)k-1)}}{2} \right\} \cup \left\{ \frac{-1 \pm \sqrt{1 + 4(\lambda_i^2 + \lambda_i + 1)}}{2} : 2 \leq i \leq n \right\}.$$

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## Two special families of graphs: $C_5(\Lambda)$ , $A(\Lambda)$

$\text{Aut}(\Gamma) = \{\text{automorphisms of } \Gamma\}$

$\overline{\text{Aut}(\Gamma)} = \{\text{antimorphisms of } \Gamma\} = \{\text{isomorphisms } \Gamma \rightarrow \bar{\Gamma}\}$

### Definition

$\Gamma$  is self-complementary (s.c.) if  $\overline{\text{Aut}(\Gamma)} \neq \emptyset$

### Examples of s.c. graphs

$C_5$ ,  $A$ -graph



$C_5(\Lambda) =$  constructed from  $C_5$  by replacing one vertex with graph  $\Lambda$

$A(\Lambda) =$  constructed from  $A$  by replacing the 'top' vertex with  $\Lambda$

FACT: if  $\Lambda$  is s.c., then  $C_5(\Lambda)$  and  $A(\Lambda)$  are s.c.

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- $S_5$  if  $|V(\Lambda)| = 1$
- $(\text{Aut}(\Gamma) \cup \overline{\text{Aut}(\Gamma)}) \rtimes \mathbb{Z}_2$  if  $\Lambda$  is s.c. with  $|V(\Lambda)| \neq 1$
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## Corollary (Orel 2021+)

For each graph  $\Gamma$ ,  $\frac{|\text{Aut}(\Gamma \equiv \bar{\Gamma})|}{|\text{Aut}(\Gamma)|} \in \{1, 2, 4, 12\}$ .

## Corollary (Orel 2021+)

$\Gamma \equiv \bar{\Gamma}$  is vertex-transitive if and only if  $\Gamma$  is vertex-transitive and s.c.

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# Aut( $\Gamma \equiv \bar{\Gamma}$ ) & corollaries; Hamiltonicity

It follows from a result of Muzychuk (Bull. London Math. Soc. 1999) that the orders of graphs  $\Gamma \equiv \bar{\Gamma}$ , which are vertex-transitive (and non-Cayley) are precisely the values

$$2p_1^{\alpha_1} \cdots p_s^{\alpha_s}, \text{ where } p_i^{\alpha_i} \equiv 1 \pmod{4} \text{ for all } i$$

( $p_1, \dots, p_s$  are distinct primes,  $\alpha_i \geq 1$ ) or equivalently

$$2\left((2i)^2 + (2j+1)^2\right) \quad \text{with } i, j \in \{0, 1, 2, \dots\} \text{ and } (i, j) \neq (0, 0).$$

$\Gamma \equiv \bar{\Gamma}$  is NOT a lexicographic product of two graphs with at least 2 vertices.

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If  $\Gamma$  is vertex-transitive s.c. graph on  $n > 5$  vertices, then  $\Gamma \equiv \bar{\Gamma}$  is Hamiltonian-connected.

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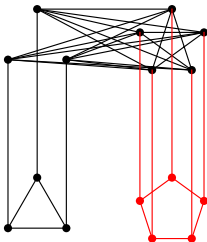
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## Lemma

If  $\Gamma \not\cong K_2, \bar{K}_2$ , then there are only 5 possibilities for  $\text{core}(\Gamma \equiv \bar{\Gamma})$ :

- 1  $\Gamma \equiv \bar{\Gamma}$  is a core
- 2  $V(\text{core}(\Gamma \equiv \bar{\Gamma})) \subseteq W_1$  and  $\text{core}(\Gamma \equiv \bar{\Gamma}) \cong \text{core}(\Gamma)$
- 3  $V(\text{core}(\Gamma \equiv \bar{\Gamma})) \subseteq W_2$  and  $\text{core}(\Gamma \equiv \bar{\Gamma}) \cong \text{core}(\bar{\Gamma})$
- 4 complicated but highly constrained structure
- 5 complicated but highly constrained structure

Example of possibility (4), (5):  $\Gamma = C_3 + C_5$



### Corollary (Orel 2021+)

If  $\Gamma$  is  $(\frac{n-1}{2})$ -regular, where  $|V(\Gamma)| = n$ , then only possibilities (1), (2), (3) can occur.

The same result is true for each graph  $\Gamma$  with at least 4 vertices if we assume that  $\text{core}(\Gamma \equiv \bar{\Gamma})$  is regular.

### Theorem (Orel 2021+)

If  $\Gamma$  is strongly regular s.c. graph, then  $\Gamma \equiv \bar{\Gamma}$  is a core.

### Theorem (Orel 2021+)

If  $\Gamma$  is vertex-transitive s.c. graph, and  $\Gamma$  is either a core or its core is complete, then  $\Gamma \equiv \bar{\Gamma}$  is a core.

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