Neural network approximations for high-dimensional PDEs

Diyora Salimova
ETH Zurich
sdiyora@mins.ee.ethz.ch

Most of the numerical approximation methods for PDEs in the scientific literature suffer from the so-called curse of dimensionality (CoD) in the sense that the number of computational operations employed in the corresponding approximation scheme to obtain an approximation precision $\varepsilon > 0$ grows exponentially in the PDE dimension and/or the reciprocal of ε. Recently, certain deep learning based approximation methods for PDEs have been proposed and various numerical simulations for such methods suggest that deep neural network (DNN) approximations might have the capacity to indeed overcome the CoD in the sense that the number of real parameters used to describe the approximating DNNs grows at most polynomially in both the PDE dimension $d \in \mathbb{N}$ and the reciprocal of the prescribed approximation accuracy $\varepsilon > 0$. In this talk we show that for every $a \in \mathbb{R}$, $b \in (a, \infty)$ solutions of suitable Kolmogorov PDEs can be approximated by DNNs on the entire space-time region $[0, T] \times [a, b]^d$ without the CoD.