On decay rates of the solutions of parabolic Cauchy problems

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We consider the Cauchy problem in the Euclidean space $\mathbb{R}^N \ni x$ for the parabolic equation $\partial_t u(x,t) = Au(x,t)$, where the operator A (e.g. the Laplacian) is assumed, among other things, to be a generator of a C_0 semigroup in a weighted L^p-space $L^p_w(\mathbb{R}^N)$ with $1 \leq p < \infty$ and a fast growing weight w. We show that there is a Schauder basis $(e_n)_{n=1}^\infty$ in $L^p_w(\mathbb{R}^N)$ with the following property: given an arbitrary positive integer m there exists $n_m > 0$ such that, if the initial data f belongs to the closed linear span of e_n with $n \geq n_m$, then the decay rate of the solution of the problem is at least t^{-m} for large times t. In other words, the Banach space of the initial data can be split into two components, where the data in the infinite-dimensional component leads to decay with any pre-determined speed t^{-m}, and the exceptional component is finite dimensional.

We discuss in detail the needed assumptions of the integral kernel of the semigroup e^{tA}. We present variants of the result having different methods of proofs and also consider finite polynomial decay rates instead of unlimited m.

The results are contained in the following of papers published together with José Bonet (Valencia) and Wolfgang Lusky (Paderborn).

[1] J.Bonet, W.Lusky, J.Taskinen, Schauder basis and the decay rate of the heat equation. J. Evol. Equations 19 (2019), 717–728.
[2] J.Bonet, W.Lusky, J.Taskinen: On decay rates of the solutions of parabolic Cauchy problems, Proc. Royal Soc. Edinburgh, to appear. DOI:10.1017/prm.2020.48