How to beat the $1/e$-strategy of best choice (the random arrivals problem)

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Elimination of dominated strategies is the fundamental technique used to reduce the size of a finite zero-sum game. For infinite games, however, the dominance phenomenon may occur within the set of minimax strategies. See [1] and references therein for the Blackwell-Hill-Cover paradox of this kind. In this talk we consider a more involved optimal stopping game.

In the best choice problem with random arrivals, an unknown number n of rankable items arrive at times sampled from the uniform distribution. As is well known, a real-time player can ensure stopping at the overall best item with probability at least $1/e$ by means of the strategy τ^* that waits until time $1/e$ then selects the first relatively best item to appear (if any). The number $1/e$ is also the value of the game against an adversary in charge of the variable n.

We show that the adversary has no minimax strategy and
(i) for every u there exist stopping strategies that strictly improve upon τ^* simultaneously for all $n \leq u$,
(ii) there exists a simple strategy outperforming τ^* simultaneously for all $n > 1$ (strictly for $n > 2$),
(iii) there exist more complex strategies strictly outperforming τ^* simultaneously for all $n > 2$,
(iv) for every $\ell \geq 1$ there exist still more complex strategies that guarantee the winning probability at least $1/e$ for all n, and are outperforming τ^* simultaneously for all $n > \ell$.
(v) in the other direction (as the $\ell = \infty$ case of (iv)), there exist stopping strategies that guarantee the winning chance $1/e$, but are strictly dominated by τ^*.

The stopping strategies we employ are defined in terms of multiple time cutoffs, and rely decisions on both the arrival time of relatively best item in question and the number of arrivals seen so far.

References

Gnedin, A. (2016) Guess the larger number. *Math. Appl. (Warsaw)* **44**, 183–207.