Taxonomy of Three-Qubit Doilies

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We study doilies (i.e., $W(3,2)$'s) living in $W(5,2)$, when the points of the latter space are parametrized by canonical three-fold products of Pauli matrices and the associated identity matrix (i.e., by three-qubit observables). Key characteristics of such a doily are: the number of its negative lines, distribution of types of observables, character of the geometric hyperplane the doily shares with the distinguished (non-singular) quadric of $W(5,2)$ and the structure of its Veldkamp space. $W(5,2)$ is endowed with 90 negative lines of two types and its 1344 doilies fall into 13 types. 279 out of 480 doilies with three negative lines are composite, i.e. they all originate from the two-qubit doily by selecting in the latter a geometric hyperplane and formally adding to each two-qubit observable, at the same position, the identity matrix if an observable lies on the hyperplane and the same Pauli matrix for any other observable. Further, given a doily and any of its geometric hyperplanes, there are other three doilies possessing the same hyperplane. There is also a particular type of doilies a representative of which features a point each line through which is negative.