Refined Doob Inequalities for σ-Integrable Submartingales

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Using the notion of conditional expectation extended to σ-integrable functions and σ-finite measure spaces, we consider σ-integrable submartingales with totally ordered index sets. Without requiring path properties, using the essential supremum of the process, we prove refined versions of Doob's maximum inequality and Doob's L^p-inequality for the cases $p > 1$, $p = 1$ and $p \in (0, 1)$. Tightness of bounds is shown by examples involving stopped Brownian motion or a martingale increasing continuously until jumping down and stopped.