Embeddings with Eulerian faces I: context and parities

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Closed walks that cover all the edges of a graph arise in many settings. In DNA self-assembly experiments, they give a route for a special strand of DNA through the assembled molecule. A reporter strand walk covers all the edges of a graph at least once, and if twice then in opposite directions, without doubling back on any edge. A graph is edge-outer embeddable if it has an orientable embedding with a special face whose boundary uses every edge at least once. Thus a reporter strand walk is a facial walk around an outer face of such an edge-outer embedding. While every graph has such an edge-outer embedding, finding one with a minimum size outer face is NP-hard. Furthermore, genus-related questions naturally arise in this new edge-outer embeddability setting. Here we focus on a min-max question: What graphs have edge-outer embeddings with both a minimum size outer face and maximum genus? This question is particularly interesting in the case of Eulerian graphs. Does there always exist an oriented embedding of a given Eulerian graph with two faces, each bounded by an Euler circuit, possibly even with one circuit specified in advance? We answer this by addressing the cases, in order, of yes, no, sometimes, and (at the time of writing) we have a conjecture.