Removable singularities for anisotropic porous medium equations

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This paper is devoted to the obtaining conditions for removable singularity at the point for solutions of quasilinear parabolic equations model of which are

$$u_t - \sum_{i=1}^{n} \left(u^{m_i-1}u_{x_i}\right)_{x_i} = 0, \tag{1}$$

$$u_t - \sum_{i=1}^{n} \left(u^{m_i-1}u_{x_i}\right)_{x_i} + f(u) = 0, \tag{2}$$

$$\frac{\partial u}{\partial t} - \sum_{i=1}^{n} \left(u^{m_i-1}u_{x_i}\right)_{x_i} + \sum_{i=1}^{n} |u_{x_i}|^{q_i} = 0, \tag{3}$$

We focus on the solutions which satisfy the initial condition

$$u(x,0) = 0, \;\; x \in \Omega \setminus \{0\}, \tag{4}$$

where Ω is a bounded domain in R^n, $n \geq 2, t \in (0,T), 0 < T < +\infty$, $0 \in \Omega$.

We suppose that the exponents $m_i, q_i \; i = \overline{1,n}$ satisfy the following condition

$$1 - \frac{2}{n} < m_1 \leq m_2 \leq \ldots \leq m_n < m + \frac{2}{n}, m = \frac{1}{n}\sum_{i=1}^{n} m_i,$$

$$\frac{2+nm}{1+n} \leq q < 2, \quad \max_{0\leq i\leq n} q_i < q\left(1+\frac{1}{n}\right), \quad \frac{1}{q} = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{q_i}.$$

The main difficulty lies in the fact that part of $m_i < 1$ (singular case), and another part of $m_i > 1$ (degenerate case). We found a universal approach to study the properties of solutions of the anisotropic porous medium equation which not depends on the values of the anisotropic exponents m_i. We established the pointwise condition for removability of the singularity for solutions of the equation (1) [1]. We also obtained the pointwise estimates of solutions, depending on the relations between the exponents m_i and q_i (for the equation (3) [3]), m_i and q (for the equation (2) in case $f(u) = u^q$ [2]) which

guarantee that the point singularity is removable. The proof of removability result is based on the new a priori estimates of "large" type solutions. In particular, we obtain the Keller-Osserman type estimate of the solution to the problems (2), (4) and (3), (4).

References

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