Extending Thomason's Algorithm

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Carsten Thomassen and I proved that in any graph G, the number of cycles containing a specified edge as well as all the odd-degree vertices is odd if and only if G is eulerian. Where all vertices have even degree this is a theorem of Shunichi Toida and where all vertices have odd degree it is Andrew Thomason's extension of Smith's Theorem. Andrew Thomason proved his theorem by constructing a graph X(G) in which the odd-degree vertices correspond precisely to the things he wants to show there are an even number of, namely the hamiltonian cycles containing the specified edge. This provides an algorithm for given one of the objects, finding another. I have extended Thomason's algorithm to one which, in a non-eulerian graph, finds a second cycle containing a specified edge and all the odd-degree vertices. I will discuss some other parity theorems about paths, cycles, and trees in graphs; in particular, attempts to find proofs of them by showing that the objects of interest are the odd-degree vertices of an associated (generally large) graph.