Dual incidences and t-designs in elementary abelian groups

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Let q be a prime and E_{q^n} is an elementary abelian group of order q^n. Let $\mathcal{H}$ be a collection of some subgroups of E_{q^n} of order q^k. A pair $(E_{q^n}, \mathcal{H})$ is a $t-(n, k, \lambda)_q$ design if every subgroup of E_{q^n} of order q^t is contained in exactly λ groups from $\mathcal{H}$. This definition corresponds to the classical definition of a q-analog design.

We introduce two incidence structures denoted by $\mathcal{D}_{max}$ and $\mathcal{D}_{min}$ with $\mathcal{H}$ as set of points. The blocks of $\mathcal{D}_{max}$ are labeled by maximal subgroups of E_{q^n}, while the blocks of $\mathcal{D}_{min}$ are labeled by groups of order q.

We fully describe a duality between $\mathcal{D}_{max}$ and $\mathcal{D}_{min}$ by proving some identities over group rings. The proven results are used to provide a full description of incidence matrices of $\mathcal{D}_{max}$ and $\mathcal{D}_{min}$ and their mutual dependance.