Particle methods for local mean-field games

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We study a particle approximation for one-dimensional first-order meanfield games (MFGs) with local interactions with planning conditions. Our problem comprises a system of a Hamilton-Jacobi equation coupled with a transport equation. As we are dealing with the planning problem, we prescribe initial and terminal distributions for the transport equation. The particle approximation builds on a semi-discrete variational problem. First, we address the existence and uniqueness of the semi-discrete variational problem. Next, we show that our discretization preserves some conserved quantities. Finally, we prove that the approximation by particle systems preserves displacement convexity. We use this last property to establish uniform estimates for the discrete problem. All results for the discrete problem are illustrated with numerical examples.