

Hopf Algebras in Studying Graph and Embedded Graph Polynomials

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Study of Hopf algebra structures on spaces spanned by graphs was initiated by S. Joni and G.-C. Rota in 1979 and was later unified with umbral calculus. Since then, a lot of combinatorial objects similar to graphs were shown to generate natural Hopf algebras. Embedded graphs are not among them, but this is true for closely related to them binary delta-matroids as defined by A. Bouchét in 1987. These general Hopf algebras have interesting Hopf subalgebras the study of which is sometimes easier and leads to effective explicit computations.

Many polynomial invariants of graphs, embedded graphs, and binary delta-matroids demonstrate a nice behavior with respect not only to the multiplicative structure, but to comultiplication as well. Examples include chromatic polynomial, characteristic polynomial, matching polynomial, Stanley's symmetrized chromatic polynomial, and many others.

Invariants of abstract graphs are closely related to those of chord diagrams (which are embedded graphs with a single vertex). In the framework of Vassiliev' theory of finite order knot invariants, chord diagrams serve as a tool to describe the latter. Similarly, certain invariants of binary delta-matroids and embedded graphs produce finite invariants of links. The Hopf algebra point of view leads to unexpected approaches to extending graph invariants to embedded graphs and binary delta-matroids.

The talk will be based on recent results of my students, colleagues, and myself.