Coloring quadrangulations of the projective space

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A *quadrangulation* on a surface F^2 is a fixed embedding of a simple graph such that each face is quadrilateral. It is known that every quadrangulation on the sphere is bipartite, but every non-spherical surface admits non-bipartite quadrangulations. For the projective plane P^2, Young pointed out an interesting fact that every non-bipartite quadrangulation is 4-chromatic. Kaiser and Stehlík considered a higher dimensional quadrangulations in the projective space, and proved that every non-bipartite d-dimensional quadrangulation in the d-dimensional projective space P^d has chromatic number exactly $d + 2$. In our talk, we will give another proof to Young's result, focusing the dual map of quadrangulations. Moreover, giving a new definition of a higher dimensional quadrangulations different from those by Kaiser and Stehlík, we prove that 3-dimensional quadrangulations of P^3 in a certain class have chromatic number 4, and conjecture that this can be extended to all of our quadrangulations in P^3.