p-ellipticity, generalized convexity and applications

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I will review some recent applications of the notions of p-ellipticity and generalized convexity introduced by O. Dragičević and myself [3]. M. Dindoš and J. Pipher [7], simultaneously and independently of us, found that p-ellipticity is a critical tool in different elliptic regularity problems they were studying. A condition weaker than p-ellipticity appeared in a different formulation in the 2005 work by A. Cialdea and V. Maz'ya [6].

The applications I will discuss include: (i) optimal holomorphic functional calculus for generators of symmetric contraction semigroups [1] and for non-symmetric Ornstein–Uhlenbeck operators [2] (ii) L^p-contractivity of the semigroups generated by divergence-form operators with complex coefficients [3,4,8,6] and (iii) maximal parabolic regularity of the generators subject to mixed boundary conditions on generic open subsets of $\mathbb{R}^d$ [4] (iv) trilinear estimates and Kato-Ponce-type inequalities [5].

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