Use of singular perturbation theory in the analysis of epidemic models

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We analyse models of SIR and SIRS type [3], using singular perturbation theory exploiting the difference in time-scales between the infection period and the typical period of susceptible recruitment by new births or immunity loss. The systems are not in standard form with slow and fast equations, but it is still possible to divide the flow into slow and fast parts, and to use the entry-exit map [2] to characterize the exit from the fast flow. When applied to classical SIR and SIRS model, we recover through this method the well-known properties of convergence to equilibrium, but we also able to characterize the sequence of susceptible densities at which recurrent epidemics start. When applied to the SIRWS model, in which immunity of W (weakly immune) individuals can be boosted by encounter with infectives, we are able to prove, by joining analytical and numerical results, the existence of periodic solutions, already shown numerically in [1]. Finally, we studied an SIRS system resulting from pair approximation of a network model; although the system is 5-dimensional, the method allows to reduce the study of the system to that of a sequence of 2-dimensional maps; we are then able to prove existence of periodic solutions of the system when the degree of network nodes is sufficiently small.

References

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