A new look at the Blaschke-Leichtweiss theorem

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The Blaschke-Leichtweiss theorem (Abh. Math. Sem. Univ. Hamburg 75: 257–284, 2005) states that the smallest area convex domain of constant width w in the 2-dimensional spherical space $\mathbb{S}^2$ is the spherical Reuleaux triangle for all $0 < w \leq \frac{\pi}{2}$. In this paper we extend this result to the family of wide r-disk domains of $\mathbb{S}^2$, where $0 < r \leq \frac{\pi}{2}$. Here a wide r-disk domain is an intersection of spherical disks of radius r with centers contained in their intersection. This gives a new and short proof for the Blaschke-Leichtweiss theorem. Furthermore, we investigate the higher dimensional analogue of wide r-disk domains called wide r-ball bodies. In particular, we determine their minimum spherical width (resp., inradius) in the spherical d-space $\mathbb{S}^d$ for all $d \geq 2$. Also, it is shown that any minimum volume wide r-ball body is of constant width r in $\mathbb{S}^d$, $d \geq 2$.