Analogues of the Dolbeault resolution in higher dimensions

Vladimír Souček

Faculty of Mathematics and Physics, Charles University

soucek@karlin.mff.cuni.cz

Abstract:

Higher dimensional analogues of complex function theory are based on a suitable choice of an elliptic system of PDEs of the first order. The most developed case is theory of several complex variables. The old one-variable example is the Fueter equation for regular functions of a quaternionic variable, the new example of that sort is the Dirac equation for spinor-valued fields. Higher spin generalizations includes massless fields equations in dimension 4 and its higher dimensional analogues.

Theory of several complex variables includes the Dolbeault resolution as a proper generalization of the de Rham complex. In a similar way, function theories mentioned above can be considered in its several variables versions and the key tool here is a generalization of the Dolbeault complex. There is the general scheme of BGG sequences of invariant differential operators on manifolds with a given parabolic structure. They can be constructed for any regular infinitesimal character. Such sequences form complexes in homogeneous situation but in non-flat case, there is an obstruction given by nontrivial curvatures. Particularly nice examples are complexes of invariant differential operators on quaternionic manifolds introduced and studied by S. Salamon and R. Baston. They form complexes even in curved situation (for quaternionic manifolds) due to the fact that they correspond to singular infinitesimal character.

An understanding of constructions and properties of such complexes advanced a lot in case of Hermitian symmetric spaces mainly due to work of Enright and Shelton. Interest in cases outside this setting was iniciated by development of function theory of several Clifford variables. This corresponds to the case of $|2|$-graded parabolic geometry in singular infinitesimal character. Using methods of integral geometry (in particular of the Penrose transform), it was possible to construct analogues of the Dolbeault resolution in even dimensions and in the stable range.

The aim of the lecture is to introduce first main known results, to describe relations of resolutions with different type of symmetry (Clifford analysis in dimension 4 and several quaternionic variables) and to describe new results in stable as well as non stable range.