Asymptotic estimates for one class of homeomorphisms

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Let Γ be a family of curves γ in $\mathbb{R}^n$, $n \geqslant 2$. A Borel measurable function $\rho : \mathbb{R}^n \to [0, \infty]$ is called *admissible* for Γ, (abbr. $\rho \in \operatorname{adm} \Gamma$), if

$$\int\limits_{\gamma} \rho(x)\, ds \ \geqslant \ 1$$

for any curve $\gamma \in \Gamma$. Let $p \in (1, \infty)$.

The quantity

$$M_p(\Gamma) \ = \ \inf_{\rho \in \operatorname{adm} \Gamma} \int\limits_{\mathbb{R}^n} \rho^p(x)\, dm(x)$$

is called *p–modulus* of the family Γ.

For arbitrary sets E, F and G of $\mathbb{R}^n$ we denote by $\Delta(E, F, G)$ a set of all continuous curves $\gamma : [a, b] \to \mathbb{R}^n$ that connect E and F in G, i.å., such that $\gamma(a) \in E$, $\gamma(b) \in F$ and $\gamma(t) \in G$ for $a < t < b$.

Let D be a domain in $\mathbb{R}^n$, $n \geqslant 2$, $x_0 \in D$ and $d_0 = \operatorname{dist}(x_0, \partial D)$. Set

$$\mathbb{A}(x_0, r_1, r_2) = \{x \in \mathbb{R}^n : r_1 < |x - x_0| < r_2\}\,,$$

$$S_i = S(x_0, r_i) = \{x \in \mathbb{R}^n : \ |x - x_0| = r_i\}\,, \quad i = 1,\, 2\,.$$

Let a function $Q : D \to [0, \infty]$ be Lebesgue measurable. We say that a homeomorphism $f : D \to \mathbb{R}^n$ is ring Q-homeomorphism with respect to p-modulus at $x_0 \in D$ if the relation

$$M_p(\Delta(fS_1, fS_2, fD)) \ \leqslant \ \int\limits_{\mathbb{A}} Q(x)\, \eta^p(|x - x_0|)\, dm(x)$$

holds for any ring $\mathbb{A} = \mathbb{A}(x_0, r_1, r_2)\,,\ 0 < r_1 < r_2 < d_0$, $d_0 = \operatorname{dist}(x_0, \partial D)$ and for any measurable function $\eta : (r_1, r_2) \to [0, \infty]$ such that

$$\int\limits_{r_1}^{r_2} \eta(r)\, dr = 1\,.$$

Let

$$L(x_0, f, R) = \sup_{|x-x_0|\leqslant R} |f(x) - f(x_0)|\,.$$

Theorem. *Suppose that $f : \mathbb{R}^n \to \mathbb{R}^n$ is a ring Q-homeomorphism with respect to p-modulus at a point x_0 with $p > n$ where x_0 is some point in $\mathbb{R}^n$ and for some numbers $c > 0$, $\kappa \leqslant p$, $r_0 > 0$ the condition*

$$\int\limits_{\mathbb{A}(x_0,r_0,R)} Q(x)\, \psi^p(|x - x_0|)\, dm(x) \ \leqslant\ c\, I^{\kappa}(r_0, R) \quad \forall\, R > r_0\,,$$

holds, where $\psi(t)$ is a nonnegative measurable function on $(0, +\infty)$ such that

$$0 < I(r_0, R) = \int\limits_{r_0}^{R} \psi(t)dt < \infty \quad \forall\, R > r_0\,,$$

then

$$\varliminf_{R\to\infty} L(x_0, f, R)\, I^{\frac{\kappa-p}{p-n}}(r_0, R) \geqslant \omega_{n-1}^{\frac{1}{n-p}} \left(\frac{p-n}{p-1}\right)^{\frac{p-1}{p-n}} c^{\frac{1}{n-p}}\,,$$

where ω_{n-1} is an area of the unit sphere $\mathbb{S}^{n-1} = \{x \in \mathbb{R}^n : |x| = 1\}$ in $\mathbb{R}^n$.