$\overline{\partial}$-Harmonic forms on compact almost Hermitian manifolds

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Let M be a smooth manifold of dimension $2n$ and let J be an almost-complex structure on M. Then, J induces on the space of forms $A^\bullet(M)$ a natural bigrading, namely

$$A^\bullet(M) = \bigoplus_{p+q=\bullet} A^{p,q}(M)\,.$$

Accordingly, the exterior derivative d splits into four operators

$$d : A^{p,q}(M) \to A^{p+2,q-1}(M) \oplus A^{p+1,q}(M) \oplus A^{p,q+1}(X) \oplus A^{p-1,q+2}(M)$$

$$d = \mu + \partial + \overline{\partial} + \bar{\mu}\,,$$

where μ and $\bar{\mu}$ are differential operators that are linear over functions.

Let g be a Hermitian metric on (M, J). Denote by

$$\Delta_{\overline{\partial}} := \overline{\partial}\,\overline{\partial}^* + \overline{\partial}^*\overline{\partial}$$

the $\overline{\partial}$-Laplacian. Then $\Delta_{\overline{\partial}}$ is an elliptic differential operator. We study the space of $\overline{\partial}$-harmonic forms on (M, J, g). Special results are obtained for $\dim_{\mathbb{R}} M = 4$. This a joint work with Nicoletta Tardini.