Universality of nodal count statistics for large quantum graphs.

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An eigenfunction of the Laplacian on a metric (quantum) graph has an excess number of zeros due to the graph's notrivial topology. This number, called the nodal surplus, is an integer between 0 and the first Betti number of the graph. We study the value distribution of the nodal surplus within the countably infinite spectrum of the graph. We conjecture that this distribution converges to Gaussian in any sequence of graphs of growing Betti number. We prove this conjecture for several special graph sequences and test it numerically for some other well-known types of graphs. An accurate computation of the distribution is made possible by a formula expressing the distribution as an integral over a high-dimensional torus with uniform measure.