Regularity of solutions of complex coefficient elliptic systems: the p-ellipticity condition

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Solving boundary value problems for divergence form real elliptic equations has been an active and productive area of research ever since the foundational work of De Giorgi - Nash - Moser established Hölder continuity of solutions when the operator coefficients are merely bounded and measurable. The solutions to such real-valued equations share some important properties with harmonic functions: maximum principles, Harnack principles, and estimates up to the boundary that enable one to solve Dirichlet problems in the classical sense of nontangential convergence. Weak solutions of complex elliptic equations and elliptic systems do not necessarily share these good properties of continuity or maximum principles.

In joint work with M. Dindoš, we introduced in 2017 a structural condition (p-ellipticity) on divergence form complex elliptic equations that was inspired by a condition related to L^p contractivity due to Cialdea and Maz'ya. The p-ellipticity condition was simultaneously discovered by Carbonaro and Dragičević to prove a bilinear embedding result. Subsequently, the condition has proven useful in the study of well-posedness of a degenerate elliptic operator associated with domains with lower-dimensional boundary.

In this talk we discuss p-ellipticity for complex divergence form equations, and then describe recent work, joint with J. Li and M. Dindoš, extending this condition to elliptic systems. In particular, we discuss applications to the Dirichlet problem for the Lamé systems.