Slice regular functions and orthogonal complex structures in eight dimensions

Caterina Stoppato
Università di Firenze
caterina.stoppato@unifi.it
Riccardo Ghiloni
Università di Trento
riccardo.ghiloni@unitn.it
Alessandro Perotti
Università di Trento
alessandro.perotti@unitn.it

The theory of slice regular functions, introduced by Gentili and Struppa in 2006, is a successful quaternionic analog of the theory of holomorphic functions of one complex variable. It includes new interesting phenomena, due to the noncommutative setting.

This theory has been applied to the problem of classifying Orthogonal Complex Structures on open dense subsets $\mathbb{R}^4 \setminus \Lambda$ of $\mathbb{R}^4$. Traditionally, this problem had been addressed with a toolset limited to quaternionic linear fractional transformations: only the case when Λ has Hausdorff dimension less than 1 and the case when Λ is a circle or a straight line could be addressed. Then the class of injective quaternionic slice regular functions became available as a tool for classification, which made other cases approachable. The construction of this new toolset required a detailed study of the differential topology of quaternionic slice regular functions. This study was a joint work with Gentili and Salamon published in 2014.

The talk will look at the theory of *octonionic* slice regular functions, introduced by Gentili and Struppa in 2010, through the lens of differential topology. This study has an independent interest, because of the peculiar features of the nonassociative setting of octonions. It leads to a full-fledged version of the Open Mapping Theorem for octonionic slice regular functions. Moreover, it opens the path for a possible use of slice regular functions in the study of almost-complex structures in eight dimensions.