Mathematical Modeling of Inflammatory Processes of Atherosclerosis

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Atherosclerosis is a chronic disease which involves the build up of cholesterol and fatty deposits within the inner lining of the artery. It is associated with a progressive thickening and hardening of the arterial wall that result in narrowing of the vessel lumen and restriction of blood flow to vital organs. These events may cause heart attack or stroke, the commonest causes of death worldwide. We study the early stages of atherosclerosis via a mathematical model of partial differential equations of reaction-diffusion type. The model includes several key species and identifies endothelial hyperpermeability, believed to be a precursor on the onset of atherosclerosis. For simplicity, we reduce the system to a monotone system and provide a biological interpretation for the stability analysis according to endothelial functionality. The existence of solutions of traveling waves type are as well investigated along with numerical simulations. The results obtained are in good agreement with current biological knowledge. Likewise, they confirm and generalize results of mathematical models previously performed in literature. Then, we study the non monotone reduced model and prove the existence of perturbed solutions and perturbed waves, particularly in the bistable case. Finally, we extend the study by considering the complete model proposed initially, perform numerical simulations and provide more specific results. We examine the consistency between the reduced and complete model analysis for a certain range of parameters, we elaborate bifurcation diagrams showing the evolution of inflammation upon endothelial permeability and LDL accumulation and we consider the effect of anti-inflammatory process on the system behavior. The

study of the model shows that the regulation of atherosclerosis progression is mediated by anti-inflammatory responses that, up to certain extent, lead to plaque regression.