Bound states for nonlinear Dirac equations on metric graphs with localized nonlinearities

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The investigation of evolution equations on metric graphs has become very popular nowadays, as they represent effective models for the dynamics of physical systems confined in branched spatial domains. In particular, the Dirac operator

$$\mathcal{D} := -\imath c\sigma_1 \frac{d}{dx} + mc^2\sigma_3, \,, \qquad \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{1}$$

on metric graphs has attracted a growing interest for the description of systems where confined particles exhibit a 'relativistic behavior'. Here $m > 0$ is the mass of the particle whose (effective) hamiltonian is given by (1) and $c > 0$ is a phenomenological parameter, playing the role of the speed of light. In this talk, I will discuss nonlinear Dirac equations with localized nonlinearity, namely

$$\mathcal{D}\psi - \chi_{\kappa}|\psi|^{p-2}\,\psi = \omega\psi\,, \qquad \psi : \mathcal{G} \to \mathbb{C}^2\,, \quad p > 2\,, \tag{2}$$

where $\mathcal{G}$ is a metric graph with finitely many edges and $\mathcal{K} \neq \emptyset$ is its *compact core*, i.e. the set of bounded edges, and $\chi_{\mathcal{K}}$ is its characteristic function. The reduction to this simplified model arises if one assumes that the nonlinearity affects only the compact core of the graph. This idea was originally exploited in the case of Schrödinger equation in and it represents a preliminary step toward the investigation of the case with the "extended" nonlinearity.

Considering the operator endowed with *Kirchoff-type* vertex conditions, we proved that (2) possesses infinitely many solutions with $\omega \in (-mc^2, mc^2)$, converging, after a suitable renormalization, in H^1-sense to solutions to the analogous Schrödinger equation $-u'' - \chi_{\mathcal{K}}|u|^{p-2}u = \lambda u$ for some $\lambda < 0$., in the *non-relativistic limit* as $c \to +\infty$.