Stahl–Totik regularity for continuum Schrödinger operators

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We develop a theory of Stahl–Totik regularity for half-line Schrödinger operators $-\partial_x^2 + V$ with bounded potentials (in a local L^1 sense). We prove a universal thickness result for the essential spectrum, E, in the language of potential theory. Namely, E is an Akhiezer-Levin set and the Martin function of the complementary domain at ∞ obeys an asymptotic expansion $\sqrt{-z} + \frac{a_E}{\sqrt{-z}} + o(\frac{1}{\sqrt{-z}})$ as $z \to -\infty$. The constant a_E plays the role of a Robin constant suited for Schödinger operators. Stahl-Totik regularity is characterized in terms of the behavior of the averages $\frac{1}{x}\int_0^x V(t)dt$ and root asymptotics of the Dirichlet solutions as $x \to \infty$. Moreover, it is connected to the zero counting measure for finite truncations. Applications to decaying and ergodic potentials will be discussed.