Thin-film problems with dynamic contact angle

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The evolution of a thin viscous fluid over a solid surface is often described using the thin-film equations

$$\partial_t h - \nabla \cdot \left(m|h|^\alpha \nabla \frac{\delta E}{\delta h} \right) = 0, \qquad E = \int_\omega \frac{1}{2}|\nabla h|^2 + W(x,h)\,\mathrm{d}x,$$

which are fourth-order degenerate parabolic equations for the height h of the fluid layer with the support $\omega(t) = \{x : h(t,x) > 0\}$. The motion of the liquid layer is driven by an energy E, which in addition to a surface energy contains other sources of internal energy in W. By complementing this PDE with suitable boundary conditions on $\partial\omega(t)$, this becomes a free boundary problem with a moving contact line.

In this talk I will introduce the gradient structure underlying the thin-film problem, detail the variational structure of the bulk-interface coupling that leads to dynamic contact angles, and investigate different limiting cases of low and high viscosities, i.e., $m \to 0$ and $m \to \infty$ for $0 < \alpha < 3$.