# Perfect 2-colourings of Cayley graphs

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Let $\Gamma = (V, E)$ be a graph. A partition $\pi = \{V_1, \ldots, V_m\}$ of $V$ is called an equitable partition or a perfect $m$-colouring of $\Gamma$ if there exists an $m \times m$ matrix $(b_{ij})$, called the quotient matrix of $\pi$, such that every vertex in $V_i$ has exactly $b_{ij}$ neighbours in $V_j$. In particular, if $\{C, V \setminus C\}$ is a perfect 2-colouring of a $d$-regular graph $\Gamma$ with quotient matrix $\begin{pmatrix} 0 & d \\ 1 & d-1 \end{pmatrix}$, then $C$ is called a perfect 1-code in $\Gamma$. In general, for an integer $t \geq 1$, a perfect $t$-code in $\Gamma$ is a subset $C$ of $V$ such that every vertex of $\Gamma$ is at distance no more than $t$ to exactly one vertex in $C$. Perfect $t$-codes in Hamming graph $H(n, q)$ and in the Cartesian product of $n$ copies of cycle $C_q$ are precisely $q$-ary perfect $t$-codes of length $n$ under the Hamming and Lee metrics, respectively. Thus perfect codes in Cayley graphs are a generalization of perfect codes in classical coding theory.

I will talk about some recent and not-so-recent results on perfect 2-colourings of Cayley graphs with an emphasis on perfect 1-codes in Cayley graphs.