Heffter Arrays and Biembbedings of Cycle Systems on Orientable Surfaces

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In this talk we will review the recent developments on square Heffter arrays, $H(n;k)$, and their applications on face 2-colourable embeddings of the complete graph K_{2nk+1} on an orientable surfaces.

Square Heffter arrays, $H(n;k)$, are $n \times n$ arrays such that each row and each column contains k filled cells, each row and column sum is divisible by $2nk+1$ and either x or $-x$ appears in the array for each integer $1 \leq x \leq nk$.

Archdeacon noted that a Heffter array, satisfying two additional conditions, yields a face 2-colourable embedding of the complete graph K_{2nk+1} on an orientable surface, where for each colour, the faces give a k-cycle system. These necessary conditions pertain to cyclic orderings of the entries in each row and each column of the Heffter array and are: (1) for each row and each column the sequential partial sums determined by the cyclic ordering must be distinct modulo $2nk+1$; (2) the composition of the cyclic orderings of the rows and columns is equivalent to a single cycle permutation on the entries in the array.

We construct Heffter arrays that satisfy condition (1) whenever (a) $k \equiv 0 \ mod\ 4$; or (b) $n \equiv 1 \ mod\ 4$ and $k \equiv 3 \ mod\ 4$; or (c) $n \equiv 0 \ mod\ 4$, $k \equiv 3 \ mod\ 4$ and $n \gg k$. As a corollary to the above we obtain pairs of orthogonal k-cycle decompositions of K_{2nk+1}.

Furthermore we study when these arrays satisfy condition (2). We show the existence of face 2-colourable embeddings of cycle decompositions of the complete graph when $n \equiv 1 \ mod\ 4$ and $k \equiv 3 \ mod\ 4$, $n \gg k \geq 7$ (provided that when $n \equiv 0 \ mod \ 3$ then $k \equiv 7 \ mod \ 12$).