Graphs with two moplexes are more than perfect

Meike Hatzel
TU Berlin
meike.hatzel@tu-berlin.de

Clément Dallard
University of Primorska
clement.dallard@famnit.upr.si

Robert Ganian
TU Wien
rganian@ac.tuwien.ac.at

Matjaž Krnc
University of Primorska
matjaz.krnc@famnit.upr.si

Martin Milanič
University of Primorska
martin.milanic@upr.si

A well-known result by Dirac (1961) states that every chordal graph contains a simplicial vertex. This theorem proved to be very useful for structural and algorithmic applications. Moplexes are a generalisation of simplicial vertices in chordal graphs to the setting of general graphs, as Berry and Bordat (1998) proved that every non-complete graph contains at least two moplexes.

There are results on the structure of chordal graphs with a bounded number of simplicial modules, for example the chordal graphs having at most two simplicial modules are interval. This motivates the research of graphs with a bounded number of moplexes. As only complete graphs have exactly one moplex, we consider the smallest interesting case: the class of graphs with at most two moplexes. Berry and Bordat (2001) proved that this class of graphs contains all connected proper interval graphs and is contained in the class of AT-free graphs. We strengthen the latter inclusion in two ways. First, we generalise it by proving that the asteroidal number yields a lower bound on the number of moplexes. Second, as our main structural result, we show that graphs with at most two moplexes are cocomparability.

So, as the class of connected graphs with at most two moplexes is sandwiched between the connected proper interval graphs and cocomparability graphs, this leads to the natural question of whether the presence of at most two moplexes guarantees a sufficient amount of structure to efficiently solve

problems that are known to be intractable on cocomparability graphs, but not on proper interval graphs. For two such problems, namely Graph Isomorphism and Max-Cut, we show that they stay hard on the graphs with two moplexes. On the other hand, we prove that every connected graph with two moplexes contains a Hamiltonian path.