

**Multiplicative and linear dependence in finite fields
and on elliptic curves modulo primes**

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In 2008 Maurin proved that given n multiplicatively independent rational functions $\varphi_1(x), \dots, \varphi_n(x) \in \mathbb{Q}(x)$, there are at most finitely many $\alpha \in \overline{\mathbb{Q}}$ such that $\varphi_1(\alpha), \dots, \varphi_n(\alpha)$ satisfy two independent multiplicative relations. This statement is an instance of more general conjectures of *unlikely intersections* over tori made by Bombieri, Masser and Zannier and independently by Zilber. We consider a positive characteristic variant of this problem, proving that, for sufficiently large primes, the cardinality of the set of $\alpha \in \overline{\mathbb{F}}_p$ such that $\varphi_1(\alpha), \dots, \varphi_n(\alpha)$ satisfy two independent multiplicative relations with exponents bounded by a certain constant K is bounded independently of K and p . We prove analogous results for products of elliptic curves and for split semiabelian varieties $E^n \times \mathbb{G}_m^k$.