A Local Singularity Analysis for the Ricci Flow

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The Ricci Flow is the most famous and most successful geometric flow, having led to resolutions of the Poincaré and Geometrisation Conjectures, as well as proofs of the Differentiable Sphere Theorem and the Generalised Smale Conjecture. For many of these applications, it is important to understand precisely how singularities form along the flow - which is a notoriously difficult task, in particular in dimensions strictly greater than three. In this talk, we develop a new and refined singularity analysis for the Ricci Flow by investigating curvature blow-up rates locally. We introduce general definitions of Type I and Type II singular points and show that these are indeed the only possible types of singular points in a Ricci Flow. In particular, near any singular point the Riemannian curvature tensor has to blow up at least at a Type I rate, generalising a result previously obtained with Enders and Topping under a global Type I assumption. We also prove analogous results for the Ricci tensor, as well as a localised version of Sesum's result, namely that the Ricci curvature must blow up near every singular point of a Ricci flow, again at least at a Type I rate. If time permits, we will also see some applications of the theory to Ricci flows with bounded scalar curvature.