Best Ulam constant of a linear differential operator

Dorian Popa
Technical University of Cluj-Napoca,
Popa.Dorian@math.utcluj.ro

The Ulam stability of an operator L acting in Banach spaces is equivalent with the stability of the associated equation $Lx = y$. An equation is called Ulam stable if for every approximate solution of it there exists an exact solution near it. We present some results on Ulam stability for some linear differential operators.

The linear differential operator with constant coefficients

$$D(y) = y^{(n)} + a_1 y^{(n-1)} + \ldots + a_n y, \quad y \in \mathcal{C}^n(\mathbb{R}, X)$$

acting in a Banach space X is Ulam stable if and only if its characteristic equation has no roots on the imaginary axis. We prove that if the characteristic equation of D has distinct roots r_k satisfying $\Re r_k > 0$, $1 \leq k \leq n$, then the best Ulam constant of D is $K_D = \frac{1}{|V|} \int_0^\infty \left| \sum_{k=1}^{n} (-1)^k V_k e^{-r_k x} \right| dx$, where $V = V(r_1, r_2, \ldots, r_n)$ and $V_k = V(r_1, \ldots, r_{k-1}, r_{k+1}, \ldots, r_n)$, $1 \leq k \leq n$, are Vandermonde determinants.