The Witten Conjecture for homology $S^1 \times S^3$

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The Witten conjecture (1994) poses that the Seiberg–Witten invariants contain all of the topological information of the Donaldson polynomials. The natural domain of this conjecture comprises closed simply connected oriented smooth 4-manifolds with $b_+ > 1$, where the Seiberg–Witten invariants are obtained by a straightforward count of irreducible solutions to the Seiberg–Witten equations. The Seiberg-Witten invariants have also been extended to manifolds with $b_+ = 1$ using wall-crossing formulas. In our work with Mrowka and Ruberman (2009) we defined the Seiberg–Witten invariant for a class of manifolds X with $b_+ = 0$ having homology of $S^1 \times S^3$. The usual count of irreducible solutions in this case depends on metric and perturbation but we succeeded in countering this dependence by a correction term to obtain a diffeomorphism invariant of X. In the spirit of the Witten conjecture, we conjectured that the degree zero Donaldson polynomial of X can be expressed in terms of this invariant. I will describe the special cases in which the conjecture has been verified, together with some applications. This is a joint project with Jianfeng Lin and Daniel Ruberman.