Stability of Einstein metrics

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Einstein metrics can be characterised as critical points of the (normalised) total scalar curvature functional. They are always saddle points. However, there are Einstein metrics which are local maxima of the functional restricted to metrics of fixed volume and constant scalar curvature. These are by definition stable Einstein metrics. Stability can equivalently be characterised by a spectral condition for the Lichnerowicz Laplacian on divergence- and trace-free symmetric 2-tensors, i.e. on so-called tt-tensors: an Einstein metric is stable if twice the Einstein constant is a lower bound for this operator. Stability is also related to Perelman's ν entropy and dynamical stability with respect to the Ricci flow.

In my talk I will discuss the stability condition. I will present a recent result obtained with G. Weingart, which completes the work of Koiso on the classification of stable compact symmetric spaces. Moreover, I will describe an interesting relation between instability and the existence of harmonic forms. This is done in the case of nearly Kähler, Einstein-Sasaki and nearly G_2 manifolds. If time permits I will also explain the instability of the Berger space $SO(5)/SO(3)$, which is a homology sphere. In this case instability surprisingly is related to the existence of Killing tensors. The latter results are contained in joint work with M. Wang and C. Wang.