The quintic NLS on the tadpole graph

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The tadpole graph consists of a circle and a half-line attached at a vertex. We analyze standing waves of the nonlinear Schrödinger equation with quintic power nonlinearity and Kirchhoff boundary conditions at the vertex. The profile of a standing wave with frequency $\omega \in (-\infty, 0)$ is characterized as a global minimizer of the quadratic part of energy constrained to the unit sphere in L^6. The set of standing waves so defined strictly includes the set of ground states, i.e. the global minimizers of the energy at constant mass (L^2-norm), but it is actually wider. While ground states exist only for a certain interval of masses, the above standing waves exist for every $\omega \in (-\infty, 0)$ and correspond to a bigger interval of masses. It is proven that there exist critical frequencies ω_1 and ω_0 with $-\infty < \omega_1 < \omega_0 < 0$ such that the standing waves are the ground state for $\omega \in [\omega_0, 0)$, local constrained minima of the energy for $\omega \in (\omega_1, \omega_0)$ and saddle points of the energy at constant mass for $\omega \in (-\infty, \omega_1)$.
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