Isometries of Wasserstein spaces

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I will report on our systematic study of isometries of classical Wasserstein spaces with various underlying spaces. The starting point was the case of the discrete underlying space, where we found a rich family of non-surjective isometries. However, we proved isometric rigidity on the contrary, which means that every surjective isometry is governed by a permutation of the underlying space [G.-T.-V., J. Math. Anal. Appl. 480 (2019), 123435].

The next step was the description of the isometries of $\mathcal{W}_p(\mathbb{R})$ for $p \neq 2$. Here, we proved isometric rigidity and classified the non-surjective isometries for $p > 1$, as well. The study of $\mathcal{W}_p([0,1])$ led to the discovery of a mass-splitting isometry for $p = 1$, which turned out to be also a key step in giving an affirmative answer to Kloeckner's questions from 2010 concerning the existence of exotic and mass-splitting isometries on quadratic Wasserstein spaces [G.-T.-V., Trans. Amer. Math. Soc. 373 (2020), 5855-5883].

The most recent work of ours concerns Wasserstein-Hilbert spaces. We extended Kloeckner's result on the quadratic case to the infinite-dimensional setting and proved isometric rigidity for the non-quadratic cases. The main tool we introduced to study the non-quadratic cases is the Wasserstein potential of measures. As a byproduct of our results, we showed that $\mathcal{W}_p(X)$ is isometrically rigid for every Polish space X and parameter $0 < p < 1$ [G.-T.-V., arXiv:2102.02037 (2021)].